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Threshold and jet radius joint resummation for single-inclusive jet production

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We present the first threshold and jet radius jointly resummed cross section for single-inclusive hadronic jet production. We work at next-to-leading logarithmic accuracy and our framework allows for a systematic extension beyond the currently achieved precision. Longstanding numerical issues are overcome by performing the resummation directly in momentum space within Soft Collinear Effective Theory. We present the first numerical results for the LHC and observe an improved description of the available data. Our results are of immediate relevance for LHC precision phenomenology including the extraction of parton distribution functions and the QCD strong coupling constant.

Introduction. The inclusive production of jets plays a crucial role at the LHC and the corresponding cross section has been measured with great accuracy by ALICE, ATLAS and CMS [1–3]. From the theoretical point of view, inclusive jet production constitutes a benchmark process that is used to determine universal nonperturbative quantities like parton distribution functions (PDFs) and the QCD strong coupling constant α_s. In this sense, a very good understanding of the relevant QCD dynamics for inclusive jet production at the LHC is crucial as it will impact the comparisons between theory and data for other processes as well. Furthermore, high transverse momentum jets are promising observables for the search of physics beyond the standard model.

In order to match the achieved experimental precision for the process $pp \to \text{jet} + X$, ongoing theory efforts have recently succeeded in calculating the fully differential cross section at next-to-next-to leading order (NNLO) [4, 5]. The results were presented for all partonic processes in the leading-color approximation for the α_s^2 coefficient. While the completion of the NNLO results marks a new milestone for high precision QCD calculations, there are, nevertheless, remaining theoretical uncertainties. Recent comparisons of the NNLO predictions with the ATLAS measurements suggest that even at NNLO the results still heavily rely on the scale choice [3]. Slightly different scale choices can lead to quite different NNLO predictions which indicates large higher-order perturbative corrections as well as an underestimation of the QCD scale dependence as pointed out in [6, 7]. From a practical point of view, any information beyond fixed NNLO accuracy can only be accessed by using resummation techniques, where dominant classes of logarithms are summed up to all orders in the strong coupling constant. In this work, we focus specifically on the joint resummation of the following two numerically important classes relevant for the current experimental kinematics: threshold logarithms and logarithms in the jet-size parameter R.

The importance of resumming single logarithms in the jet-size parameter $\alpha_s^n \ln^n R$ was addressed in [7–10]. The so-called threshold logarithms arise near the exclusive phase space boundary, where the production of the signal-jet just becomes possible. At threshold, the invariant mass $\sqrt{s_4}$ of the unobserved partonic system recoiling against the signal-jet vanishes. Note that the signal-jet retains a finite invariant mass at threshold allowing for radiation inside the jet cone [11, 12]. The cancelation of infrared divergences leaves behind logarithms of the form $\alpha_s^n (\ln^k(z)/z)_+$, with $k \leq 2n-1$, and $z = s_4/s$, where s is the partonic center-of-mass (CM) energy. In the threshold limit as $z \to 0$, these terms become large and need to be resummed to all orders so as to obtain reliable perturbative results. In [12], it was shown that threshold logarithms dominate indeed over a wide range of the jet-p_T even far away from the hadronic threshold due to the steeply falling hadron luminosity functions.

Even though the threshold resummed cross section for hadronically produced jets was addressed before [13, 14], it has so far eluded a numerical evaluation. Traditionally, threshold resummation is derived in Mellin moment space [15–17] and was applied to the rapidity integrated inclusive jet cross section in [18] at next-to-leading logarithmic (NLL) accuracy. However, in order to allow for a meaningful comparison to the available data, the complete kinematics of the jet have to be taken into account. The traditional methods failed to apply in this case so far. The reasons are twofold and can be traced back to the factorization structure of the resummed cross section and the specific properties of the Mellin transformation. Instead, only fixed-order (FO) expansions of the threshold resummed cross section are currently available in the literature [12, 19, 20]. Note that these problems do not necessarily occur for observables with identified final state hadrons [21–24].

In this work, we present for the first time the results for the threshold and small-R jointly resummed inclusive jet cross section in proton-proton collisions. The shortcomings of the traditional approaches to threshold resummation are overcome by making use of techniques developed in the context of Soft Collinear Effective Theory (SCET) [25–29], which allows for the resummation to be carried out directly in momentum space [30]. Since there are no numerical results available for the threshold resummed inclusive jet cross section using traditional methods, it is here, where the SCET approach exhibits its full potential. In addition, our framework allows for a systematic extension to next-to-next-to-leading-logarithmic (NNLL) accuracy or beyond for the resummation of both threshold and the small-R logarithms, which we briefly discuss below and address in detail in a future publication.

Theoretical framework. The double differential cross section for the process $pp \to \text{jet} + X$ can be written as

$$\frac{p_T^2 \mathrm{d}^2\sigma}{\mathrm{d}p_T^2 \mathrm{d}\eta} = \sum_{i_1 i_2} \int_0^{V(1-W)} \mathrm{d}z \int_{\frac{VW}{1-z}}^{1-\frac{1-V}{1-z}} \mathrm{d}v \, x_1^2 \, f_{i_1}(x_1) \, x_2^2 \, f_{i_2}(x_2) \times \frac{\mathrm{d}^2\hat{\sigma}_{i_1 i_2}}{\mathrm{d}v \, \mathrm{d}z}(v, z, p_T, R) \,, \tag{1}$$

where p_T and η are the transverse momentum and rapidity of the signal-jet, respectively, and we have $V = 1 - p_T e^{-\eta}/\sqrt{S}$, $VW = p_T e^{\eta}/\sqrt{S}$ and the hadronic CM energy is denoted by $\sqrt{S}$. The sum runs over all partonic channels initiating the process whose cross sections are given by $\hat{\sigma}_{i_1 i_2}$. Besides depending on p_T, the partonic cross sections $\hat{\sigma}_{i_1 i_2}$ are functions of the partonic kinematic variables $s = x_1 x_2 S$, $v = u/(u+t)$ and z. Here we have introduced $t = (p_1 - p_3)^2$ and $u = (p_2 - p_3)^2$, where $p_{1,2}$ are the momenta of the two incoming partons and p_3 is the momentum of the parton initiating the signal-jet. The PDFs are denoted by f_i evaluated at the momentum fractions $x_1 = VW/v/(1-z)$ and $x_2 = (1-V)/(1-v)/(1-z)$.

In the small-R and $z \to 0$ threshold limit, the partonic cross sections can be further factorized as

$$\frac{\mathrm{d}^2\hat{\sigma}_{i_1 i_2}}{\mathrm{d}v \, \mathrm{d}z} = s \int \mathrm{d}s_X \, \mathrm{d}s_c \mathrm{d}s_G \, \delta(zs - s_X - s_G - s_c) \times \mathrm{Tr}\left[\mathbf{H}_{i_1 i_2}(v, p_T\,, \mu_h\,, \mu) \, \mathbf{S}_G(s_G\,, \mu_{sG}\,, \mu)\right] J_X(s_X\,, \mu_X\,, \mu) \times \sum_m \mathrm{Tr}\left[J_m(p_T R\,, \mu_J\,, \mu) \otimes_\Omega S_{c,m}(s_c R\,, \mu_{sc}\,, \mu)\right] \,, \tag{2}$$

where the traces are taken in color space. The sum runs over all collinear splittings and '$\otimes_\Omega$' denotes the associated angular integrals [31]. Here we have assumed that the jet is constructed using the anti-k_T algorithm [32], $z \sim R$, and we allow for a finite mass of the signal-jet. The factorization formula is established within the framework of SCET, where $\mathbf{H}_{i_1 i_2}$ are the hard functions for $2 \to 2$ scattering, which are known to 2-loops [33]. The inclusive jet function $J_X(s_X)$ depends on the invariant mass s_X of the recoiling collimated radiation, and it is also known to order α_s^2 [34, 35]. The global soft function $\mathbf{S}_G$ takes into account wide-angle soft radiation which cannot resolve the small jet radius R. At NLO, the bare global soft function $\mathbf{S}_G$ is found to be

$$\mathbf{S}_G^{(1)} = \frac{\alpha_s}{\pi \, \epsilon} \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \sum_{i \neq j \neq 4} \mathbf{T}_i \cdot \mathbf{T}_j \frac{n_{ij}}{\mu_{sG}} \left(\frac{s_G n_{ij}}{\mu_{sG}}\right)^{-1-2\epsilon} \,, \tag{3}$$

with $n_{ij} = \sqrt{s_{ij}/s_{i4}/s_{j4}}$ and $s_{ij} = 2p_i \cdot p_j$. After performing the renormalization in the $\overline{\text{MS}}$ scheme, the NLO global soft function can be obtained as well as the anomalous dimension governing its renormalization group (RG) evolution. The signal-jet function $J(p_T R)$ and the soft-collinear ("coft") function $S_c(s_c R)$ [31, 36] account for the energetic radiation inside the jet and the soft radiation near the jet boundary, respectively. Due to the fact that the soft-collinear radiation can resolve the splitting details of the collinear radiation inside the signal-jet, one has to perform an infinite sum over the collinear splitting history inside the jet and keep the angular correlations between the jet and soft-collinear radiation which account for the non-global logarithms (NGLs) [37], as addressed in [31]. We note that the signal-jet and the soft-collinear functions can be viewed as the threshold limit of the semi-inclusive jet function [7, 10]. If we ignore the NGLs, which usually have a relatively small phenomenological impact, the infinite sum and the angular correlation structure can be approximated by a product of the jet and soft-collinear functions. The NLO jet function can be extracted from [38] and for the NLO bare soft-collinear function, we find (see also [10])

$$S_c^{(1)} = \mathbf{T}_3^2 \frac{\alpha_s}{\pi \, \epsilon} \frac{e^{\epsilon \gamma_E}}{\Gamma(1-\epsilon)} \frac{p_T R}{s \mu_{sc}} \left(\frac{s_c \, p_T R}{s \, \mu_{sc}}\right)^{-1-2\epsilon} \,, \tag{4}$$

from which the renormalized soft-collinear function and its anomalous dimension can be readily obtained.

In order to evaluate the cross section in Eq. (2), all functions are evolved from their natural scales μ_i to the scale μ according their RG equations which leads to the resummation of the large logarithms. Here, we do not elaborate on the solution of the various RG equations as this has been studied extensively in the literature, see for instance [30]. With all currently available ingredients, Eq. (2) allows us to achieve the NLL resummation for hadronic single-inclusive jet production. In order to go beyond NLL accuracy, the relevant anomalous dimensions need to be extracted from explicit 2-loop calculations which are in principle within reach. For example, the 2-loop hard and inclusive jet functions are both known and the 2-loop global soft and the soft-collinear functions can be obtained following [39] and [40].

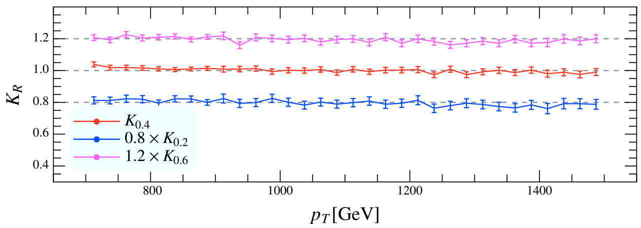

FIG. 1. Ratios K_R of the $\mathrm{NLO_{sin}}$ result which is obtained by expanding Eq. (2) and the full NLO QCD result for different values of R as a function of the jet-p_T, for $1.5 < |\eta| < 2$ at $\sqrt{S} = 13$ TeV. The error bars show the numerical uncertainty.

Phenomenology. To proceed, we first validate the factorization formalism by comparing the predictions of Eq. (2) expanded to NLO, denoted by $\mathrm{NLO_{sin}}$ in the following, with the full NLO QCD calculation in the threshold region. Since Eq. (2) is derived in the strict threshold limit, the scale choice related to the jet-p_T can only be the leading-jet transverse momentum $p_T^{\max}$, since no jets in the event can be harder than the signal-jet in this limit. Therefore, when comparing the two results, we choose the renormalization and factorization scales as $\mu = \mu_F = \mu_R = p_T^{\max}$ for the full NLO QCD calculation instead of using the so-called individual jet-p_T which probes a softer scale than $p_T^{\max}$. We use the MMHT2014nlo PDF set of [41] and focus on $\sqrt{S} = 13$ TeV. To enforce the threshold limit, we demand that $p_T > 700$ GeV and $1.5 < |\eta| < 2$. Fig. 1 displays the ratios K_R of the $\mathrm{NLO_{sin}}$ result to the full NLO QCD calculation [42] for $R = 0.2$, 0.4 and 0.6. We find very good agreement between these two calculations for all choices of R which validates our factorization theorem.

We further separate the $\mathrm{NLO_{sin}}$ result into a "virtual" $\delta(z)$ term and the logarithmic terms $(\ln^k(z)/z)_+$ with $k = 0, 1$. We observe that the "virtual" term gives a large positive correction. The net logarithmic contribution decreases the cross section, where the $(\ln(z)/z)_+$ term is positive whereas the $(1/z)_+$ term is negative and large due to its coefficient in the kinematic regime under study.

Now we turn to the phenomenology at the LHC. We match the NLL resummed results with the full NLO calculation using

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{NLL}} - \mathrm{d}\sigma_{\mathrm{NLO_{sin}}} + \mathrm{d}\sigma_{\mathrm{NLO}} \,, \tag{5}$$

and we set $\mu = p_T^{\max}$ for the reasons discussed above. We make the central scale choices $\mu_h = p_T^{\max}$ and $\mu_J = p_T R$ for the hard and the signal-jet functions, respectively. The naive scale for the recoiling jet function is of order $\mu_X \sim \kappa \sqrt{s}\left(1 - \frac{2p_T}{\sqrt{S}}\right)$ with $\kappa \sim 1$. However, due to the steeply falling shape of the luminosity function, κ can deviate from 1 and approach a smaller value. We determine κ dynamically following [43] and we set $\mu_X = \kappa \times 2p_T\left(1 - \frac{2p_T}{\sqrt{S}}\right)$ with $\kappa = 1/2$. The other scales are determined in the seesaw way: $\mu_{sG} = \mu_X^2/\mu_h$ and $\mu_{sc} = \mu_J \times \mu_{sG}/\mu_h$. Our uncertainty estimates are obtained by varying μ, μ_h and μ_J independently while keeping the seesaw relations for μ_X, μ_{sG} and μ_{sc}. For all scales we consider variations by a factor of 2 around their central values and the final scale uncertainty is obtained by taking the envelope.

We first present the results for the single-inclusive jet cross sections at $\sqrt{S} = 2.76$ TeV which was measured by the CMS Collaboration for different jet radii [2]. In Fig. 2, we show the resummed calculations using the CT10nlo PDFs [44] for $|\eta| < 2$ along with the experimental data both normalized to the full NLO results. We observe a significant improvement of the description of the data for all values R once the joint resummation is taken into account.

Next, we turn to single-inclusive jet production at $\sqrt{S} = 13$ TeV. The cross section was measured by ATLAS for a jet radius of $R = 0.4$ for various bins of the jet-rapidity η [3]. For the scale choice $\mu = p_T^{\max}$, the NLO predictions slightly overshoot the data by about 7% to 10% using the MMHT2014nlo PDF set for $|\eta| > 1$. Nevertheless, the NLO calculation is still within the experimental errors bars. The NNLO corrections further enhance the cross section leading to a more significant disagreement with the data [3]. In Fig. 3, we show the results for the p_T spectrum of our jointly resummed calculation. As an example, we consider the rapidity region $1.5 < |\eta| < 2$ and we plot the ratio of the NLL improved result to the NLO prediction. We find that the joint resummation decreases the cross section relative to the NLO result and thus improves the agreement with the data. A similar trend was observed in the CMS single jet analysis [2].

Conclusions. In this work, we presented for the first time a joint resummation framework for single-inclusive jet production in the threshold and small-R limit using SCET. Due to the small jet-size parameter used in the experimental analyses and the shape of the steeply falling luminosity functions, the threshold and the small-R logarithmic terms make up the dominant bulk of the FO contributions in the kinematic range from moderate to large jet-p_T. Therefore, in order to provide reliable theoretical calculations, these classes of logarithmic corrections have to be resummed to all orders in perturbation theory. The fact that the full NNLO calculation depends significantly on the scale choice [3] makes the importance of including higher-order corrections beyond NNLO even more evident.

Using our framework, we obtained the resummed results for single-inclusive jet production at the LHC differential in both the jet-p_T and the rapidity η. We demonstrated the validity of our framework by finding very good agreement with the full NLO results for various values of R and cuts on p_T and η. We calculated all necessary ingredients for the resummation to NLL accuracy and we

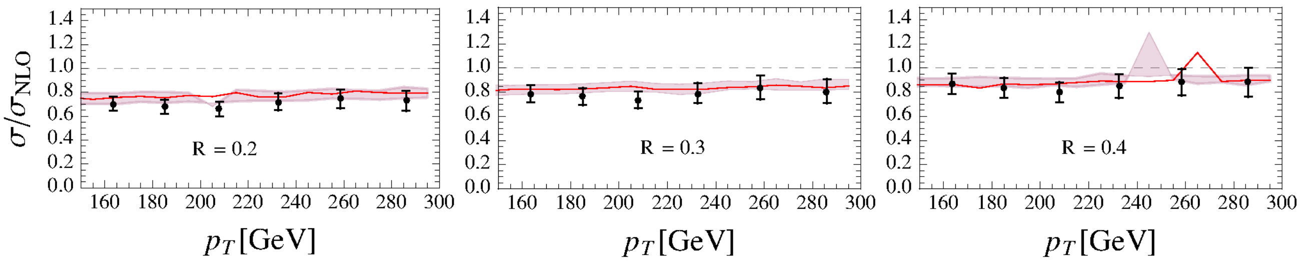

FIG. 2. The resummed calculation for inclusive jet production for $|\eta| < 2$ at $\sqrt{S} = 2.76$ TeV for different values of R and the CMS data (black dots) of [2] both normalized to the NLO results.

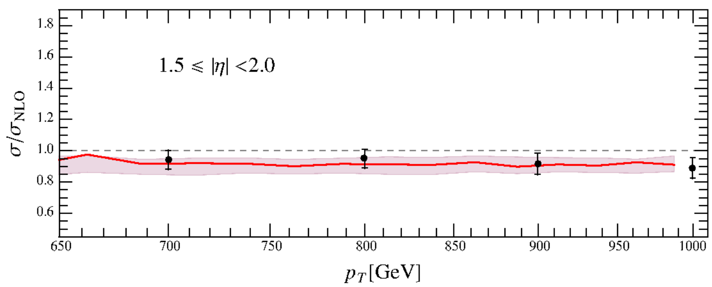

FIG. 3. The resummed calculation for inclusive jet production with $R = 0.4$ at $\sqrt{S} = 13$ TeV and the preliminary ATLAS data (black dots) extracted from [3] both normalized to the NLO result.

presented phenomenological results for LHC kinematics at CM energies of $\sqrt{S} = 2.76$ and 13 TeV. In both cases, we found an improved agreement of the theoretical calculations with the LHC data after complementing the NLO calculations with the NLL resummation.

The results presented in this work will have a direct impact on various aspects of QCD precision phenomenology at the LHC. This includes the precise extraction of PDFs [45] and the QCD strong coupling constant [46] as well as the improvement of parton shower Monte Carlo event generators [47]. In the future, we plan to further explore the phenomenology at the LHC and to perform more detailed studies of the matching procedure between the resummed cross section and the FO results. In addition, as outlined briefly above, it is possible to extend our framework beyond NLL accuracy which we are going to address in a forthcoming publication.

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