External injection into a laser-driven plasma accelerator with sub-femtosecond timing jitter
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Abstract. The use of external injection in plasma acceleration is attractive due to the high control over the electron beam parameters, which can be tailored to meet the plasma requirements and therefore preserve its quality during acceleration. However, using this technique requires an extremely fine synchronization between the driver and witness beams. In this paper, we present a new scheme for external injection in a laser-driven plasma accelerator that would allow, for the first time, sub-femtosecond timing jitter between laser pulse and electron beam.

1. Introduction
Over the last decades, laser-driven plasma wakefield acceleration (LWFA) [1] has been positioned as a possible alternative to conventional RF technology for future compact accelerators. A series of milestones like the experimental measurement of gradients on the hundreds of GV/m [2, 3], the development of controlled injection techniques [4, 5, 6] or the production of quasi-monoenergetic beams [7, 8, 9] showcase its potential.

Unfortunately, in terms of stability, emittance and energy spread, the quality of these electron beams is still inferior to those produced at classical RF accelerators.

One of the proposed methods for improving the beam quality is to use an external injection scheme, where the electron beam is provided by an external RF linac, rather than produced within the plasma. This allows a better control of the beam properties so that the emittance and energy spread growth during acceleration can be minimized by properly shaping [10] and matching [11, 12] the beam into the plasma.

However, operating a plasma accelerator with external injection brings in new technical challenges. In particular, it requires an extremely low timing jitter between the drive laser and witness beam. Due to the small size of the accelerating plasma buckets (typically on the order of 10 – 100 µm) and the steep field gradients in them, an accuracy on the femtosecond level is required, smaller than what is available today in state-of-the-art setups.

2. A scheme for timing jitter compensation
In this paper, we propose a new concept for external injection that would allow to achieve unprecedented sub-femtosecond timing jitter between laser and witness beam. The scheme, as
seen in Figure 1, relies on adding an intermediate synchronizing stage before the main plasma acceleration module.

In this concept, the laser pulse is initially split in two, with one of them containing only a small fraction of the energy (e.g. \( \sim 5\% \)). These two pulses are intrinsically synchronized and will serve different purposes: the weak one will be used in the synchronizing stage, while the strong pulse will be used as driver in the main plasma acceleration module.

As seen in Figure 1, the main idea is to use the weak pulse to drive a first plasma stage in which the electron beam will get a small energy change proportional to its arrival time. In the speed-of-light frame, \( \xi = z - ct \), beams with an offset \( \Delta \xi_i = \xi_i - \xi_0 \) with respect to the zero crossing, \( \xi_0 \), of the accelerating field, \( E_z \), will gain or lose energy depending on whether they arrived too late (\( \Delta \xi_i < 0 \)) or too soon (\( \Delta \xi_i > 0 \)), therefore correlating arrival time and beam energy.

Due to this correlation, a magnetic chicane will then be able to correct the initial offsets thanks to the differences in path length for each beam energy, virtually removing the timing jitter between electron beam and main laser pulse for injection into the second plasma stage.

Additionally, the scheme contains two quadrupole sections to match the beam into and out of the plasma modules.

The laser parameters should be such that the weak pulse is able to excite a linear wake in the first plasma stage, so powers on the order of 1 – 10 TW should suffice. For the strong one, the necessary power will depend on the desired final energy of the beam after acceleration.

The design energy of the chicane, \( E_0 \), should be selected as the energy, after the first plasma, of a beam with \( \Delta \xi = 0 \). In case of no beam loading, this means that \( E_0 = E_i \), where \( E_i \) is the initial energy of the beams coming from the linac.

![Figure 1. Schematic view of the synchronizing stage.](image)

### 3. Linear model

In the first plasma stage, beams will get an energy deviation \( \delta = \frac{E - E_0}{E_0} \) depending on their longitudinal position. For small offsets this can be approximated as

\[
\delta = -\frac{eE'_z L_p}{E_0} (\xi_i - \xi_0) ,
\]

where \( e \) is the electron charge, \( E'_z = \partial_e E_z \) is the slope of the accelerating field around \( \xi_0 \) and \( L_p \) is the length of the plasma stage.

To first order, the final position of the beam after the chicane will be given by

\[
\xi_f = \xi_i - R_\delta \delta ,
\]

where the minus sign comes from the fact that in the comoving coordinate \( \xi \), as commonly used in plasma acceleration, the bunch head is on the right.
Then, in order to correct the initial offsets, $\xi_f = \xi_0$ should be imposed. This, together with Equation (1), yields

$$R_{56} = -\frac{E_0}{eE_z L_p}.$$  \hspace{1cm} (3)

For a plasma in the linear regime ($a_0 \ll 1$) \cite{13, 14} and a circularly polarized Gaussian pulse, $a^2 = a_0^2 \exp(-\xi^2/2\sigma_z^2)$, the slope of the field $E_z$ around its zero crossing is

$$E'_z = \frac{mc^2}{e} \sqrt{\frac{\pi}{2}} a_0^2 k_p^3 \sigma_z e^{-\frac{\xi^2}{\sigma_z^2}},$$ \hspace{1cm} (4)

where $m$ is the electron mass, $c$ is the speed of light, $a_0$ is the peak amplitude of $a$, the laser’s normalized vector potential, $k_p$ is the plasma wavenumber and $\sigma_z$ is the laser rms length. In case of linear polarization $a_0^2$ should be replaced by $a_0^2/2$. By substituting this into Equation (3) one finds the general expression

$$R_{56} = -\sqrt{\frac{2}{\pi}} \frac{E_0}{mc^2 a_0^2 k_p^3 \sigma_z L_p} e^{-\frac{\xi^2}{\sigma_z^2}}.$$ \hspace{1cm} (5)

Now, if the laser pulse length satisfies the resonant condition ($\sigma_z k_p = 1$), one can find a simplified equation which, written in a convenient way, reads

$$R_{56} \text{[mm]} = -7.27 \times 10^{13} \frac{E_0 \text{[MeV]}}{a_0^2 n_p \text{[cm}^{-3}\text{]} L_p \text{[mm]}}.$$ \hspace{1cm} (6)

In reality, depending on the beam energy, it might be necessary to take into account the path length effect due to the deviation from the speed of light not being negligible. This would decrease the necessary $R_{56}$ at the chicane, but a precise value can only be given once the full geometry of the beamline up to the second plasma has been decided. Also, Equations (5) and (6) do not take into account beam loading so, if this effect becomes significant, $R_{56}$ might have to be determined from the chirp of $\delta$ as obtained from simulations.

Another important remark is that the quality of the jitter correction will depend on the stability of the parameters in Equation (5), as well as the pointing jitter of the electron beam after the first plasma. For the case presented below, assuming the pointing jitter does not exceed the natural beam divergence, the resulting maximum timing jitter would be around 0.2 fs per meter of beamline downstream of the plasma, still on the sub-fs range.

Moreover, $\delta$ should always be much bigger than the intrinsic energy spread of the beam, otherwise its longitudinal phase space might be perturbed in the chicane.

4. Numerical simulations

In order to test this method, a series of start-to-end simulations going from the injection of the beam into the first plasma until the end of the chicane have been performed.

The simulation of the plasma stage was performed in 3D with the fully relativistic Particle-in-Cell code OSIRIS \cite{15} using the Ponderomotive Guiding Center (PGC) algorithm \cite{16}. WinAGILE \cite{17} and MAD-X \cite{18} were used for the optimization of the quadrupole section, and the tracking of the electron beam from the plasma exit until the end of the chicane was done with ELEGANT \cite{19}.

Effects derived from Coherent Synchrotron Radiation (CSR) at the dipoles have been taken into account, but a full treatment including space charge after exiting the plasma has not yet been possible. However, due to the low charge density of the beam after the plasma and its relativistic energy, strong space charge effects are not expected.
4.1. Main Parameters
The electron beam after the linac is Gaussian with a charge of 0.1 pC in 1 fs, an energy of 100 MeV with 0.1% relative energy spread, a normalized emittance $\epsilon_{nx,y}$ of 0.3 $\mu$m and initial offsets due to timing jitter between $-20$ and 20 fs. These parameters were chosen in order to avoid significantly perturbing the plasma wakefield with the beam (see Figure 2) and are within the range of those expected to be produced at the SINBAD facility at DESY [20, 21, 22]. The first plasma stage has a length of 2 mm with a density of $10^{17}$ cm$^{-3}$, and the laser parameters are based on those considered for the EuPRAXIA conceptual design [23], using only around 3.5% of its energy for the synchronizing stage. This means an 800 nm linearly polarized pulse of 3.5 J with an $a_0$ of 0.6, a waist $w_0$ of 54 $\mu$m and a length of 93 fs (FWHM in intensity), having a peak power of 35 TW. With these parameters, the matched beam size for emittance preservation [12] is $\sigma_x = 1.3$ $\mu$m for the focusing fields around $\xi_0$.

![Figure 2. On-axis longitudinal electric field after 1.2 mm of propagation in the plasma with the electron beam placed at the zero crossing (case with $\Delta \xi = 0$).](image)

4.2. Quadrupole Section
The set of magnets is placed 4 cm after the plasma and consists on 4 permanent quadrupoles with a width of 1 cm separated by drifts of 4, 6 and 4 cm. Their strengths were left as free parameters for optimization in order to achieve, as seen in Figure 3, a beam waist at the center of the chicane and a beta function, $\beta_{x,y} = \gamma_r \sigma_{x,y}^2 / \epsilon_{nx,y}$, as constant as possible, where $\gamma_r$ is the Lorentz factor of the beam.

The resulting geometric strengths of the quadrupoles read, from first to last, -878, 1406, -1497 and 823 in units of m$^{-2}$ which, for this beam energy, translate into field gradients of up to 500 T/m, requiring tight apertures.

![Figure 3. Beta function and emittance evolution for beams with different offsets. The solid line corresponds to $\Delta \xi = 0$, while the dashed and dotted lines represent offsets of 20 and -20 fs, respectively.](image)
4.3. Chicane Setup
For this case, Equation (5) predicts an $R_{56} = -0.222$ mm. However, for higher accuracy, the slope of $\delta$ was directly measured from the simulation results (see Figure 4), obtaining an $R_{56} = -0.267$ mm. This could be due to the laser $a_0$ being 0.6, slightly high for the linear theory to hold, since $a_0 \ll 1$ is assumed.

The bending angle $\theta$ of the dipoles was determined from

$$R_{56} = -2\theta^2 \left( L_d + \frac{2}{3} L_m \right)$$

by assuming a magnet length $L_m = 10$ cm and a drift length $L_d = 2.5$ cm, obtaining $\theta = 2.19^\circ$.

4.4. Simulation Results
As seen in Figure 4, the plasma stage correctly imprints a linear kick to the beam energy depending on its initial offset, allowing the chicane to compensate the jitter. All beam offsets between $\pm 10$ fs have been reduced to sub-fs level, showcasing the potential of this method.

![Figure 4](image)

**Figure 4.** Jitter correction thanks to plasma stage and chicane. The vertical lines delimit a 1 fs jitter.

In terms of quality, as seen in Figure 3, the beam experiences an increase in emittance of around $10 - 20\%$ due to chromatic effects at the quadrupoles. Additionally, due to the strong focusing, the beam develops a slightly curved shape in the transverse plane, increasing its rms length in about 0.5 fs.

Also, as seen in Figure 3, beams with $\Delta \xi_i \neq 0$ will present variations in the evolution of the beta function due to their different energies, so some way of compensating this chromatic effect before the second plasma stage will be required.

5. Conclusion
We have presented a new scheme for arrival time jitter minimization between laser pulse and witness beam for LWFA with external injection. Simulation results show that jitters of 10 fs can be reduced to the sub-femtosecond level with minimal loss of beam quality.

Further studies are required to determine the stability and tolerances of the setup, as well as the influence of space charge effects and finding ways of matching the beam into the second plasma stage taking into account the variations in beta function seen in Figure 3.

Also, although only ultrashort beams have been tested, it is possible that for longer beams ($\sigma_z \sim 10$ fs) this scheme could be used both for synchronization and compression.

Acknowledgments
The authors would like to give special thanks to Jun Zhu and Daniel Marx for fruitful discussions regarding the beamline design and its simulation.

This project has received funding from the European Union’s Horizon 2020 research and innovation programme under grant agreement No. 653782.
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