Cosmological constant, gauge hierarchy, and warped geometry

Stephan J. Huber*

Deutsches Elektronen-Synchrotron DESY, Hamburg, Germany

Qaisar Shafi[†]

Bartol Research Institute, University of Delaware, Newark, Delaware 19716, USA

It is suggested that the mechanism responsible for the resolution of the gauge hierarchy problem within the warped geometry framework can be generalized to provide a new explanation of the extremely tiny vacuum energy density ρ_V suggested by recent observations. We illustrate the mechanism with some 5D examples in which the true vacuum energy is assumed to vanish, and ρ_V is associated with a false vacuum energy such that $\rho_V^{1/4} \sim \text{TeV}^2/M_{\text{Pl}} \sim 10^{-3}$ eV, where M_{Pl} denotes the reduced Planck mass. We also consider a quintessence-like solution to the dark energy problem.

I. INTRODUCTION

Evidence for a tiny but non-zero cosmological vacuum energy density has steadily mounted in recent years [1]. With information from a variety of observations put together, it appears that we are living in a flat universe, such that $\Omega_\Lambda + \Omega_m = 1$, where $\Omega_\Lambda = 0.7$ and $\Omega_m = 0.3$ denote the density parameters associated with the vacuum energy and matter respectively. These observations pose at least two formidable theoretical challenges: namely, what physics determines the magnitude of the vacuum energy density to be of order

$$\rho_V \sim 10^{-120} M_{\text{Pl}}^4 \sim (10^{-3} \text{ eV})^4, \tag{1}$$

and why are ρ_V and ρ_m in such close proximity in magnitude, considering that their evolution, at least for constant cosmological constant, can be so different? Here $M_{\text{Pl}} = 2.4 \times 10^{18}$ GeV denotes the reduced Planck mass.

A certain amount of theoretical prejudice suggests that the true vacuum energy density of the universe could be exactly zero. Needless to say, how the zero value occurs remains a complete mystery, despite many attempts [2]. Nonetheless, following this reasoning, it has been speculated that the present universe happens to lie in a false vacuum, separated from the true vacuum by the observed energy density ρ_V [3]. The modest goal of this approach is to try to identify the physics underlying the origin of ρ_V, as well as ensure that the metastable vacuum is sufficiently long lived.

In this paper we propose a new mechanism for realizing ρ_V of the desired magnitude which is based on warped geometry. While the original motivation of warped geometry was to resolve the notorious gauge hierarchy problem [4,5], it has subsequently been exploited to provide at least a qualitatively new understanding of fermion mass hierarchies and their mixings, especially within the context of neutrino oscillations [6–9]. We will show here that the warped geometry can be further exploited to provide a new mechanism for

generating ρ_V of the desired magnitude. Earlier attempts in this direction have been made in Ref. [10]. Recall that the TeV scale in warped geometry arises from the presence of the warp factor $\Omega^{-1} = \exp(-\pi k R) = \text{TeV}/M_P$, where $1/k$ denotes the AdS curvature radius and R is the radius of the orbifold S^1/Z_2. We will see that ρ_V can be associated with the vacuum energy density of a suitable scalar field, such that

$$\rho_V^{1/4} \sim \Omega^{-2} M_{\text{Pl}} = \text{TeV}^2/M_{\text{Pl}} \sim 10^{-3} \text{ eV}.$$

We also discuss how the quintessence scenario [11] can be realized within this approach.

II. SCALAR FIELDS IN A WARPED BACKGROUND

We take the fifth dimension to be an S_1/Z_2 orbifold with a negative bulk cosmological constant, bordered by two 3-branes with opposite tensions and separated by distance R. Einstein's equations are satisfied by the non-factorizable metric [4]

$$ds^2 = e^{-2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad \sigma(y) = k|y|, \tag{2}$$

which describes a slice of AdS$_5$. The 4-dimensional metric is $\eta_{\mu\nu} = \text{diag}(-1,1,1,1)$, k is the AdS curvature related to the bulk cosmological constant and brane tensions, and y denotes the fifth coordinate. The AdS curvature and 5D Planck mass M_5 are both assumed to be of order M_{Pl}. The AdS warp factor $\Omega = e^{\pi k y}$ generates an exponential hierarchy of energy scales. If the brane separation is $kR \simeq 11$, the scale at the negative tension brane, located at $y = \pi R$, is of TeV size, while the scale at the brane at $y = 0$ is of order M_{Pl}. At the TeV-brane gravity is weak because the zero mode corresponding to the 4D graviton is localized at the positive tension brane (Planck-brane).

Let us consider the equation of motion of a real scalar field in the background (2) [12,7]

$$\frac{1}{\sqrt{-g}} \partial_M (\sqrt{-g} g^{MN} \partial_N \Phi) - M^2 \Phi = 0 \tag{3}$$

which follows from the action

----

*Email address: stephan.huber@desy.de

[†]Email address: shafi@bartol.udel.edu

$$S_5 = -\int d^4x \int dy \ \sqrt{-g} \left(\frac{1}{2} (\partial_M \Phi)(\partial^M \Phi) + \frac{1}{2} M^2 \Phi^2 \right).$$

$$(4)$$

Here g_{MN} denotes the 5D metric and $g = e^{-4\sigma}$ its determinant. In general, the 5D scalar mass consists of bulk and brane contributions

$$M^2(y) = b^2 k^2 + a^2 k \delta(y - \pi R) + \tilde{a}^2 k \delta(y), \qquad (5)$$

where b, a and $\tilde{a}$ are dimensionless parameters. In the case of supersymmetry the relations

$$\tilde{a}^2 = -a^2$$

$$b^2 = \frac{1}{4} a^2 (a^2 + 8) \qquad (6)$$

have to be satisfied [7], and Φ becomes a complex scalar field which, together with a second complex scalar and a Dirac fermion, forms a hypermultiplet. The connection to the 5D mass parameter of the Dirac fermion, $M_\Psi = c\sigma'$, is given by $c = 3/2 + a^2/2$. A special case of Eqs. (6) is the massless scalar field where $a^2 = \tilde{a}^2 = b = 0$ (and $c = 3/2$).

The effective 4D theory arises from the Kaluza-Klein (KK) decomposition

$$\Phi(x^\mu, y) = \frac{1}{\sqrt{2\pi R}} \sum_{n=0}^{\infty} \Phi^{(n)}(x^\mu) f_n(y), \qquad (7)$$

where the wave functions $f_n(y)$ satisfy the differential equation [12,7]

$$(-\partial_5^2 + 4\sigma' \partial_5 + M^2) f_n = e^{2\sigma} m_n^2 f_n, \qquad (8)$$

and m_n are the masses of the KK excitations. The solution to Eq. (8) is given by

$$f_n(y) = \frac{e^{2\sigma}}{N_n} \left[J_\alpha \left(\frac{m_n}{k} e^\sigma \right) + \beta_\alpha(m_n) Y_\alpha \left(\frac{m_n}{k} e^\sigma \right) \right], \qquad (9)$$

with $\alpha = \sqrt{4 + b^2}$. Assuming Φ is even under the Z_2 orbifold transformation, i.e. $f_n(-y) = f_n(y)$, the coefficient β and the KK spectrum follow from the matching conditions at the branes,

$$\beta_\alpha(x_n, \tilde{a}^2) = -\frac{(-\tilde{a}^2 + 4 - 2\alpha) J_\alpha(x_n) + 2x_n J_{\alpha-1}(x_n)}{(-\tilde{a}^2 + 4 - 2\alpha) Y_\alpha(x_n) + 2x_n Y_{\alpha-1}(x_n)}, \qquad (10)$$

$$\beta_\alpha(x_n, \tilde{a}^2) = \beta_\alpha(\Omega x_n, -a^2), \qquad (11)$$

where $x_n = m_n \Omega / k$. Note that for non-vanishing boundary mass terms the derivative of f_n becomes discontinuous on the boundaries. The normalization constants N_n in Eq. (9) are defined such that

$$\frac{1}{2\pi R} \int_{-\pi R}^{\pi R} dy \, e^{-2\sigma(y)} f_n^2(y) = 1. \qquad (12)$$

The mass splitting between the KK modes is $\Delta m_n \sim \pi k \Omega^{-1} \sim$ TeV.

In the massless case the zero mode wave function is constant, i.e. $f_0(y) = 1/N_0 \approx \sqrt{2\pi kR}$. The KK states have TeV scale masses and are localized at the TeV-brane. The mass of the first excited state is $m_1 \approx 3.83 k\Omega^{-1}$. The effects of the bulk and brane mass terms on the mass and wave function of the lowest mode are quite different. If one switches on a small bulk mass, one finds $m_0 \approx bk/\sqrt{2}$ for $b \lesssim \Omega^{-1}$ ($bk \lesssim$ TeV). Thus, there is no warp factor suppression. The zero mode wave function is still approximately constant, with a small enhancement at the TeV-brane. Increasing the bulk mass to $b \lesssim 1$ ($bk \lesssim M_{Pl}$) the mass of the lightest mode remains frozen at $x_0 = 3.83$, which is practically the former first excited state. For $b \gtrsim 1$ ($bk \gtrsim M_{Pl}$) one finds that m_0 also increases. The mass term at the Planck-brane induces a similar behavior of the zero mode, except there is no increase of m_0 for $\tilde{a} \gtrsim 1$ ($\tilde{a}k \gtrsim M_{Pl}$).

In the case of a TeV-brane mass the result is very different. Because of the factor $\sqrt{-g}$ in the action (4), the mass of the zero mode is warped by *two* powers of Ω. With this double warping we have

$$m_0 \approx ak\Omega^{-2}, \quad a \lesssim 1. \qquad (13)$$

The wave function is nearly constant, with a dip at the TeV-brane, as shown in Fig. 1(a) for $kR = 11.13$ ($M_{Pl}e^{-\pi kR} = 1.6$ TeV). For $a \gtrsim 1$ the dip in the wave function slows down the increase of m_0. For large values of a any further increase is completely compensated by this, so that the zero mode mass saturates at $m_0 = 2\sqrt{2}k\Omega^{-2}$. The dependence of m_0 on the TeV-brane mass is displayed in Fig. 1(b). We note that gauge bosons show a similar behavior as we discussed in Ref. [13]. Numerically, for a TeV-brane mass of order M_{Pl}, m_0 is comparable to the energy scale associated with dark energy (1). Of course, at this stage m_0 is not yet related to some vacuum energy, but merely represents a tiny particle mass. In the next section we will show how such a link can be established.

At the end of this section, let us briefly summarize the supersymmetric case which was extensively discussed in Refs. [7,14]. Supersymmetry requires the bulk and brane mass terms to be simultaneously present. Because of the special relations, Eqs. (6), the zero mode remains strictly massless, despite the non-zero bulk and brane masses. The wave function has an exponential form [7]

$$f_0(y) = \frac{1}{N_0} e^{-(a^2/2)\sigma}. \qquad (14)$$

For a positive (negative) mass squared term at the TeV-brane the scalar field is localized towards the Planck-brane (TeV-brane). The supersymmetric scalar fields thus behave similar to fermions in the warped background, which also possess massless exponential zero modes in the presence of bulk mass terms [6,7].
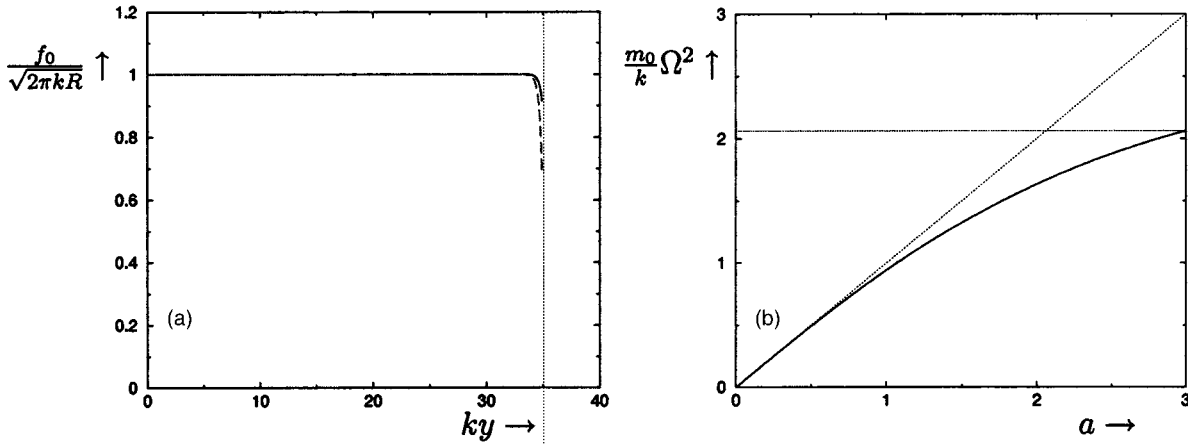
FIG. 1. (a) Wave function of the zero mode for $a=1$ (solid line) and $a=2$ (dashed line). (b) Dependence of the zero mode mass on the TeV-brane mass term (solid line) compared to the linear approximation of Eq. (13) (dotted line).

III. EXPONENTIALLY SMALL CONTRIBUTIONS TO THE SCALAR POTENTIAL

In the last section we learned that a Planck-size mass term at the TeV-brane is translated into a sub-eV mass of the scalar particle. We now generalize this result to include a scalar potential on the TeV-brane. This will allow us to generate tiny energy splittings in the minima of the 4D effective potential, which we hope to identify with the observed dark energy.

We start with the simplest case of a massless scalar field with $a=b=\tilde{a}=0$ which, for instance, could be a consequence of supersymmetry. If we now switch on some general potential on the TeV-brane, related, for example, to supersymmetry breaking, the scalar zero mode remains, as discussed in the last section. Consider the following contribution to the 5D action:

$$\Delta S_5 = -\int d^4x \int dy \ \sqrt{-g}\left(\frac{1}{2}a^2k\Phi^2 + \frac{1}{3}E_5k^{-1/2}\Phi^3\right.$$
$$\left. + \frac{1}{4}\lambda_5k^{-2}\Phi^4\right)\delta(y-\pi R). \qquad (15)$$

We approximate f_0 by a constant, and after integrating over the extra dimension, we arrive at the 4D effective potential for the zero mode ϕ:

$$V_4 = \frac{1}{2}m^2\phi^2 + \frac{1}{3}E\phi^3 + \frac{1}{4}\lambda\phi^4, \qquad (16)$$

where

$$m^2 = a^2k^2\Omega^{-4}$$

$$E = E_5k\Omega^{-4}$$

$$\lambda = \lambda_5\Omega^{-4}. \qquad (17)$$

Thus all operators on the TeV-brane are redshifted by four powers of the warp factor. The warped geometry not only generates tiny masses but also exponentially small coupling constants. This result may seem surprising since the zero mode f_0 is constant in the extra dimension and thus has a large overlap with the TeV-brane. However, as we observe from Eq. (12), in the integration over the extra dimension the region close to the TeV-brane is exponentially suppressed. The rescaled zero mode $\tilde{f}_0 = e^{-\sigma}f_0$ is the more appropriate object to look at. For a constant f_0, the rescaled zero mode is exponentially localized towards the Planck-brane, which explains the tiny quantities in Eq. (17). For a very large mass squared term on the TeV-brane, i.e. $a\gg1$, there is an additional suppression of the brane operators (15) induced by the dip in the zero mode. We note that scalar couplings in the bulk or on the Planck-brane are not scaled down by the warped geometry.

We now assume that $m^2 < 0$ in order to generate a vacuum expectation value (VEV) for the scalar field. Without additional interactions from the bulk, the tiny self-coupling λ induced by the brane can stabilize the field only at a Planck-size VEV $\langle\phi\rangle\sim k$. We therefore have to introduce some additional interaction. A quartic coupling in the bulk does not feel the warping and therefore generates $\lambda\sim1$ in the effective 4D action. A quartic coupling on the Planck-brane would have the same effect. The scalar field then acquires a tiny VEV $\langle\phi\rangle\sim k\Omega^{-2}$. If there is now more than one minimum with a similar VEV, the energy splitting between the two minima will be on the order of $\Delta V\sim m^2\langle\phi\rangle^2\sim k^4\Omega^{-8}$. This is exactly what is required for the dark energy. Two minima can easily arise, for example, in the case of two interacting scalar fields with a polynomial potential. We will show later that the false vacuum has a lifetime much longer than the age of the universe. In the next section we will also discuss how this setup can be stabilized against radiative corrections in a supersymmetric framework.

A different possibility for the scalar field to be stabilized is by some non-renormalizable interaction in the bulk, for instance by a $(1/Q^2)\phi^6$ operator. As discussed in Refs. [7,8], non-renormalizable interactions may be suppressed by much smaller energy scales than M_{Pl} because of the warped geom-

etry. However, as explained above, in some sense the flat zero mode is localized towards the Planck-brane. Therefore non-renormalizable operators are not enhanced, and we expect $Q \sim M_{\text{Pl}} = k$ in the 4D action. The scalar would then be stabilized at $\langle \phi \rangle = \sqrt{mQ} \sim k\Omega^{-1}$, which is the KK scale. Assuming that the Z_2 symmetry in the scalar potential is broken by the cubic term on the brane, the induced energy splitting between the vacua at $\pm \sqrt{mQ}$ is $\Delta V \sim E\langle \phi \rangle^3 \sim k^4 \Omega^{-7}$. This is a factor of Ω larger than what we found in the previous example. To make it compatible with the observational value (1) we either would have to increase the warp factor and end up with a KK scale of about 10 GeV (!) or reduce the suppression scale to about $Q = k\Omega^{-1/3}$. Both possibilities do not look realistic. In the next section we discuss the supersymmetric case (with a non-vanishing bulk mass) which gives us additional possibilities.

The false vacuum energy not only leads to an exponential expansion of the universe, but will also modify the geometry of the extra dimension. To estimate the size of the back reaction we compare the false vacuum energy density to the bulk cosmological constant and brane tensions of the unperturbed Randall-Sundrum solution [4]. We consider a somewhat simplified setup where the scalar VEV is induced by a negative squared mass term $(1/2)a^2 k\Phi^2$ at the TeV-brane, as in Eq. (15). The potential is stabilized by a quartic self-interaction in the bulk, $(1/4)\lambda_b \Phi^4$. Because the scalar potential changes along the extra dimension, the vacuum configuration $\langle \Phi(y) \rangle$ does as well and has to be computed numerically. We find the profile to be $\langle \Phi(y) \rangle \sim ak\Omega^{-2}/\sqrt{\lambda_b} \sim k^{3/2}\Omega^{-2}$ and almost flat in the extra dimension. In the second step we assumed $a \sim 1$ and $\lambda_b \sim k^{-1}$. Close to the TeV-brane, where the instability occurs, the profile $\langle \Phi(y) \rangle$ increases by about 10% relative to its almost constant value in the bulk. On the TeV-brane the scalar potential is $V = (1/2)a^2 k\Phi^2 \sim k^4 \Omega^{-4}$. In the bulk we obtain $V = (1/4)\lambda_b \Phi^4 \sim k^5 \Omega^{-8}$. This means that Einstein's equations are still completely dominated by the Planck-size bulk cosmological constant and brane tensions of the Randall-Sundrum solution [4]. Thus we can safely ignore the back reaction, at most of order Ω^{-4}, on the wave functions (9) and (14).

IV. SUPERSYMMETRIC CASE

A crucial point is that a perturbation of the mass relations (6) by some supersymmetry breaking mass term on the TeV-brane with $\Delta a^2 \lesssim 1$ leaves the zero mode (14) practically unchanged. For $\Delta a^2 \gtrsim 1$ we observe an additional reduction of f_0 close to the TeV-brane, as in the case of a vanishing bulk mass. To obtain the 4D couplings from the brane action (15) we approximate the zero mode by its supersymmetric shape (14). We find

$$m^2 \approx \Delta a^2 k^2 \Omega^{-4-a^2}$$

$$E \approx E_5 k\Omega^{-4-(3/2)a^2}$$

$$\lambda \approx \lambda_5 \Omega^{-4-2a^2}, \tag{18}$$

where a^2 is now the TeV-brane mass term required by supersymmetry and determines the shape of the zero mode. We have omitted a factor $N_0 \sqrt{2\pi kR}$ to some power which is of order unity. Localizing the scalar zero mode towards the Planck-brane by taking $a^2 > 0$, we can suppress the induced 4D couplings even stronger than in Eq. (17). This opens up new possibilities to generate a small vacuum energy density.

The $(N=2)$ 5D supersymmetry prohibits a self-interaction of the scalar field in the bulk. Self-interactions can be provided, however, by superpotentials on the branes, where the boundary conditions break the supersymmetry to $N=1$. Planck-brane operators are not warped down and induce order unity couplings in the 4D potential, in contrast to their TeV-brane counterparts (18).

Localizing the scalar field close to the Planck-brane, we can implement a quintessence-like solution to the dark energy problem. The cosmological evolution is assumed to leave the scalar field displaced from its minimum towards which it is slowly evolving. For polynomial potentials, the necessary slow rolling conditions are satisfied only for very large VEVs $\langle \phi \rangle \sim M_{\text{Pl}}$. A small vacuum energy can therefore be explained only by extraordinarily small couplings. To generate ρ_V from a term $m^2 \langle \phi \rangle^2 \sim m^2 M_{\text{Pl}}^2$ requires $m^2 \sim \Omega^{-8} M_{\text{Pl}}^2$. From Eq. (18) we see that this requires $a^2 \approx 4$. We can also implement this scenario by relying on the quartic term of the brane potential, $\rho_v \sim \lambda \langle \phi \rangle^4$. For $a^2 \sim 2$ we obtain the required small $\lambda \approx \Omega^{-8}$. The other terms in Eq. (15) as well as possible bulk self-interactions have to vanish to accommodate this solution. It remains to be seen whether a realistic scenario of this type can be constructed in which the required small masses remain stable under quantum corrections [15].

A true cosmological constant, instead of quintessence, can be generated from a supersymmetry breaking cubic coupling which shifts previously degenerate minima. We assume that a scalar self-interaction on the Planck-brane leads to $\lambda \sim 1$ in the 4D action. If the scalar potential is now destabilized by a supersymmetry breaking brane mass (18), the induced VEV is $\langle \phi \rangle \sim m \sim k\Omega^{-2-a^2/2}$. The degeneracy of the minima at $\pm \langle \phi \rangle$ is lifted by the supersymmetry breaking cubic term on the brane (18). The induced energy splitting between the vacua is then of order $\Delta V \sim E\langle \phi \rangle^3 \sim k^4 \Omega^{-10-3a^2}$. The preferred value of the cosmological constant (1) then leads to $a^2 \sim -2/3$; i.e., the scalar field is localized somewhat towards the TeV-brane. The mass of the scalar particles associated with Φ is of order $\langle \phi \rangle \sim k\Omega^{-5/3} \sim 10^2$ eV.

V. QUANTUM CORRECTIONS

The case with a vanishing bulk mass discussed in Sec. III can readily be embedded in the supersymmetric framework by setting $a=0$ in Eq. (18). Then supersymmetry helps to tame radiative corrections. As in four dimensions the soft mass of the scalar zero mode, m^2, will receive a 1-loop quantum correction $\delta m^2 \sim (1/16\pi^2)m^2 \lambda \ln(\Lambda)$ by exchange of the scalar zero mode and its superpartner. Here Λ denotes the momentum cutoff and λ stems from the self-coupling localized at the Planck-brane. As discussed above, in the case

$a=0$ we have $m^2 \sim k^2 \Omega^{-4}$ and $\lambda \sim 1$. In the warped model there are additional radiative corrections to m^2 by the exchange of KK states of the scalar field. The KK states are localized towards the TeV-brane. Therefore they acquire a larger soft mass of order $k^2 \Omega^{-2}$. The quartic coupling between two zero modes and two KK states of the scalar field is of order Ω^{-2}. Thus loops with KK states of the scalar are of the same order as zero mode loops. It remains to be seen if performing the sum over the KK contributions, which are individually small, leads to a destabilization of the soft mass of the zero mode.

Since we assume the scalar field to be a gauge singlet, further quantum corrections can only come from gravity. Exchange of a zero mode gravitino in the loop gives rise to $\delta m^2 \sim (1/16\pi^2) m_{3/2}^2 (\Lambda/M_{\rm Pl})^2$, where $m_{3/2}$ is the gravitino mass. Since for the zero mode of the gravitino $m_{3/2} \sim k\Omega^{-3}$, we obtain a tiny correction even for a Planck-size cutoff. For the KK gravitinos the situation is different. Being localized towards the TeV-brane, their supersymmetry breaking mass is TeV size $(k\Omega^{-1})$ [14] and their dimension-6 interaction with the scalar zero mode is suppressed by $1/(\Omega M_{\rm Pl}^2)$. The corresponding correction to the scalar mass is then $\delta m^2 \sim (1/16\pi^2) k^2 \Omega^{-3} (\Lambda/M_{\rm Pl})^2$. Since the dimension-6 operator has its support at the TeV-brane, it seems plausible that the KK gravitino loop is cut off at the TeV scale. In this case its contribution to the scalar mass $\delta m^2 \sim (1/16\pi^2) k^2 \Omega^{-5}$ is safely suppressed. However, given the limitations and uncertainties which are inherent in our treatment of radiative corrections, the issue of the quantum stability of our approach to the dark energy problem is not yet satisfactorily settled.

VI. LIFETIME OF THE FALSE VACUUM

The false vacuum state is metastable and will finally relax to the true vacuum by thermal fluctuations and/or quantum tunneling. We must require the lifetime of the false vacuum to be longer than the age of the universe. Because of the low temperature of the present universe, quantum tunneling is the dominant decay process. In the WKB approximation the tunneling probability is given by [16]

$$p \sim \left(\frac{t_U}{R}\right)^4 e^{-B}, \qquad (19)$$

where t_U denotes the age of the universe, and $R \sim 1/m$ is the characteristic scale of the problem. In the thin wall approximation the Euclidean action of the tunneling configuration,

$$B = 27\pi^2 \frac{S_1^4}{2(\Delta V)^3}, \qquad (20)$$

depends on the energy splitting of the vacua ΔV and the "surface tension" of the bubble $S_1 = \int d\phi \sqrt{2V(\phi)}$.

Vacuum decay is most effective between minima which are close to each other in field space and separated only by a low energy barrier V_B. Among the examples discussed before the tunneling rate is largest in the case of Sec. III where $m \sim \langle \phi \rangle \sim 10^{-3}$ eV. From the constraint $p<1$ we obtain $B \gtrsim 280$. For a polynomial potential one finds $B \approx 7 \times 10^3 (V_B/\Delta V)^3/\lambda$; i.e., the false vacuum is metastable even for $V_B \sim \Delta V$, as we expect for a potential with only a single mass scale. In the other example we discussed, the barrier heights and scalar VEVs are even much larger. Thus, in all the cases discussed the false vacuum is sufficiently long lived.

VII. CONCLUSIONS

In conclusion, we have shown that a warped geometry setting could help provide a new explanation of the origin of the tiny vacuum energy density of the universe that is indicated by recent observations. This can either be in the form of a cosmological constant or, if supersymmetry is invoked, as a contribution arising from a slow rolling quintessence field. Although no explanation exists as to why all other contributions to the vacuum energy density effectively vanish, it is certainly intriguing to think that resolution of the gauge hierarchy and dark energy problems may have a common origin.

[1] S. Perlmutter *et al.*, Astrophys. J. **483**, 565 (1997); S. Perlmutter *et al.*, Bull. Am. Astron. Soc. **29**, 1351 (1997); B. Schmidt *et al.*, Astrophys. J. **507**, 46 (1998); A.J. Riess *et al.*, Astron. J. **116**, 1009 (1998); Astrophys. J. **560**, 49 (2001).

[2] For recent reviews of various suggestions for resolving the cosmological constant problems and additional references, see A. Vilenkin, hep-th/0106083; U. Ellwanger, hep-ph/0203252.

[3] S. Kachru, J. Kumar, and E. Silverstein, Phys. Rev. D **59**, 106004 (1999); E.I. Guendelman, Mod. Phys. Lett. A **14**, 1043 (1999); P.H. Frampton, hep-th/0002053; N. Arkani-Hamed, L.J. Hall, C. Colda, and H. Murayama, Phys. Rev. Lett. **85**, 4434 (2000); S.M. Barr and D. Seckel, Phys. Rev. D **64**,

123513 (2001); T. Banks and M. Dine, J. High Energy Phys. **10**, 012 (2001).

[4] L. Randall and R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999).

[5] M. Gogberashvili, Int. J. Mod. Phys. D **11**, 1635 (2002).

[6] Y. Grossman and M. Neubert, Phys. Lett. B **474**, 361 (2000).

[7] T. Gherghetta and A. Pomarol, Nucl. Phys. **B586**, 141 (2000).

[8] S.J. Huber and Q. Shafi, Phys. Lett. B **498**, 256 (2001).

[9] S.J. Huber and Q. Shafi, Phys. Lett. B **521**, 365 (2001); *ibid.* **544**, 295 (2002).

[10] S.H. Henry Tye and I. Wasserman Phys. Rev. Lett. **86**, 1682 (2001); J.M. Cline and H. Firouzjahi, Phys. Lett. B **514**, 205 (2001).

[11] C. Wetterich, Nucl. Phys. **B302**, 668 (1988); P.J.E. Peebles and B. Ratra, Astrophys. J., Lett. Ed. **325**, L17 (1988); R. R Caldwell, R. Dave, and P. J Steinhard, Phys. Rev. Lett. **80**, 1582 (1998); I. Zlatev, L. Wang, and P. J Steinhard, *ibid.* **85**, 4434 (2000).

[12] W.D. Goldberger and M.B. Wise, Phys. Rev. D **60**, 107505 (1999); Phys. Rev. Lett. **83**, 4922 (1999).

[13] S.J. Huber and Q. Shafi, Phys. Rev. D **63**, 045010 (2001).

[14] T. Gherghetta and A. Pomarol, Nucl. Phys. **B602**, 3 (2001).

[15] G.R. Dvali and A. Vilenkin, Phys. Rev. D **64**, 063509 (2001).

[16] S. Coleman, Phys. Rev. D **15**, 2929 (1977); S. Coleman and C. Callan, *ibid.* **16**, 1762 (1977).