Limits from LEP Data on *CP*-Violating Nonminimal Higgs Sectors

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We derive a sum rule which shows how to extend limits from LEP data on the masses of the lightest *CP*-even and *CP*-odd Higgs bosons of a *CP*-conserving two-Higgs doublet model to any two Higgs bosons of a general *CP*-violating two-Higgs-doublet model. We generalize the analysis to a Higgs sector consisting of an arbitrary number of Higgs doublets and singlets, giving explicit limits for the *CP*-conserving and *CP*-violating two-doublet plus one-singlet Higgs sectors. [S0031-9007(97)03786-1]

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Models of electroweak symmetry breaking driven by elementary scalar dynamics predict the existence of one or more physical Higgs bosons. One can use LEP data to place significant bounds on Higgs boson masses. The minimal model consists of a one-doublet Higgs sector as employed in the standard model (SM), which gives rise to a single *CP*-even scalar Higgs, $h_{\rm SM}$. The absence of any $e^+e^- \to Zh_{\rm SM}$ signal in LEP1 data (where the Z is virtual) and LEP2 data (where the Z is real) translates into a lower limit on $m_{h_{\rm SM}}$ which has been increasing as higher energy data become available. For example, the latest (preliminary) ALEPH data imply $m_{h_{\rm SM}} \gtrsim 70.7$ GeV [1]; similar preliminary results are available from L3, OPAL, and DELPHI [2]. (Currently published limits, which do not include data from the $\sqrt{s} = 172$ GeV run, are weaker [3].) Ultimately, the strongest limit will be obtained by combining results from all four experiments. The simplest and most attractive generalization of the SM Higgs sector is a two-Higgs-doublet model (2HDM), the *CP*-conserving version of which has received considerable attention, especially in the context of the minimal supersymmetric model (MSSM) [4]. A *CP*-conserving two-Higgs-doublet model predicts the existence of two neutral *CP*-even Higgs bosons (h^0 and H^0, with $m_{h^0} \leq m_{H^0}$), one neutral *CP*-odd Higgs (A^0), and a charged Higgs pair ($H^\pm$). The negative results of Higgs bosons searches at LEP can be formulated as restrictions on the parameter space of this and more general Higgs sector models. For the most general *CP*-conserving two-doublet model one can exclude the (m_{h^0}, m_{A^0}) and (m_{h^0}, m_{H^0}) regions shown in Fig. 1 on the basis of $e^+e^- \to Zh^0$, $e^+e^- \to ZH^0$, and $e^+e^- \to h^0A^0$ event rate limits; see the Appendix (stronger limits are possible in models such as the MSSM where there are relations between the Higgs masses and their couplings). The ability to exclude the illustrated (m_{h^0}, m_{A^0}) region derives from a coupling constant sum rule which implies that the two production processes, $e^+e^- \to Zh^0$ and $e^+e^- \to h^0A^0$, cannot both be simultaneously suppressed when kinematically allowed. Similarly, the illustrated (m_{h^0}, m_{H^0}) region is excluded by virtue of a second sum rule which implies that the couplings responsible for the $e^+e^- \to Zh^0$ and $e^+e^- \to ZH^0$ processes cannot be simultaneously suppressed.

However, there is no reason to assume that the Higgs sector is *CP* conserving; *CP* violation is still very much a mystery from both an experimental and theoretical point of view. The possibility that *CP* violation derives largely from the Higgs sector is especially intriguing [5]. In a general 2HDM, *CP* violation can arise either explicitly or spontaneously and leads to three electrically neutral physical Higgs mass eigenstates, h_i ($i = 1, 2, 3$), that have

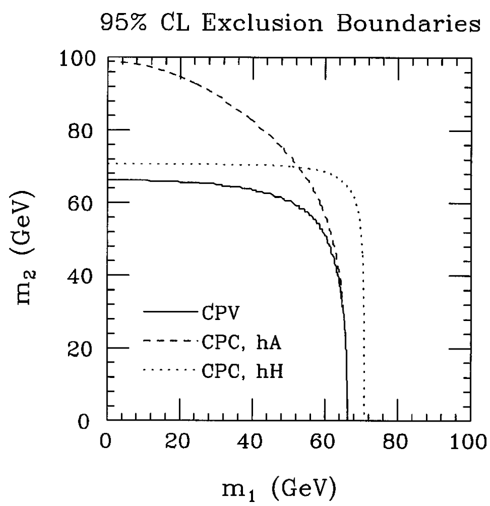

FIG. 1. The LEP 95% confidence level boundaries (based on our use of the latest ALEPH results [1] and earlier LEP1 results as contained in Refs. [3,8]) in the (m_1, m_2) plane for three two-Higgs-doublet model cases: (i) The *CP*-conserving (CPC) model where $h_1 = h^0, h_2 = H^0$ (dotted curve); (ii) the CPC model where $h_1 = h^0, h_2 = A^0$ (dashed curve); (iii) the *CP*-violating (CPV) model where h_1 and h_2 are the two lightest neutral Higgs bosons (solid curve). This figure is based on including just two constraints: A particular choice of masses is excluded (a) if some ZZh_i coupling must lie above the 95% C.L. upper limit for the assumed m_{h_i} or (b) if one or more events are expected in $e^+e^- \to h_ih_j$ production at the 95% C.L. for some choice of $i \neq j$. For details, see the Appendix.

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undefined CP properties. To date, only the obvious limit on each ZZh_i coupling as a function of m_{h_i}, deriving from nonobservation of $e^+e^- \rightarrow Zh_i$ at LEP, has been noted. In this Letter, we generalize the coupling constant sum rules appropriate in the CP-conserving model to a single sum rule in the CP-violating case that requires at least one of the ZZh_i, ZZh_j, and Zh_ih_j (*any* $i \neq j$, $i, j = 1, 2, 3$) couplings to be substantial in size. The LEP 95% confidence level exclusion region in the (m_{h_1}, m_{h_2}) plane that results from the general sum rule is quite significant, as illustrated in Fig. 1.

In this Letter, we also present an analysis of both the CP-conserving and the CP-violating versions of the two-doublet plus one-singlet (2D1S) extension of the 2HDM. A 2D1S model yields five neutral Higgs bosons. We derive sum rules that can be used to demonstrate that LEP data exclude the possibility that three of these neutral Higgs bosons can be light. (There is no sum rule which allows exclusion of the possibility that only two of the neutral Higgs bosons of the 2D1S model are light.) The 95% confidence level boundaries in three-Higgs-boson mass space for the CP-conserving and CP-violating cases, based on the procedures described in the Appendix, are presented in Figs. 2 and 3, respectively.

We now turn to a derivation of the sum rules required and a discussion of how they lead to the experimental constraints outlined above. In the 2HDM, the two complex neutral Higgs fields contain four neutral degrees of freedom. One is eaten by the Z gauge boson; the others mix to yield three physical neutral Higgs bosons, h_i $(i = 1, 2, 3)$. We shall denote their couplings to the Z boson by

$$g_{ZZh_i} \equiv \frac{g m_Z}{c_W} C_i, \qquad g_{Zh_ih_j} \equiv \frac{g}{2c_W} C_{ij}, \qquad g_{ZZh_ih_j} = \frac{g^2}{2c_W^2} \delta_{ij}, \tag{1}$$

where C_i and $C_{ij} = C_{ji}$ $(i \neq j)$ (for $i = j$, $C_{ij} = 0$ by Bose symmetry) are model-dependent coupling strengths and $c_W \equiv \cos\theta_W$.

In the CP-conserving 2HDM, the h^0 and H^0 are mixtures of the real parts of the neutral Higgs fields (the diagonalizing mixing angle is denoted by α) while the CP-odd state A^0 derives from the imaginary components not eaten by the Z. One finds $C_{h^0} = C_{H^0A^0} = \sin(\beta - \alpha)$, $C_{H^0} = C_{h^0A^0} = \cos(\beta - \alpha)$, and $C_{A^0} = C_{h^0H^0} = 0$; $\tan\beta = v_2/v_1$ is the ratio of the vacuum expectation values for the neutral components of the two Higgs doublet fields. For any two Higgs bosons, we wish to obtain an excluded mass region that does not depend upon knowledge of the third Higgs boson. There are two pairs of interest: (a) h^0H^0 and (b) h^0A^0. In case (a) the excluded region in (m_{h^0}, m_{H^0}) parameter space is illustrated by the dotted curve in Fig. 1. Inside the excluded region (plotted symmetrically, even though by definition $m_{H^0} \geq m_{h^0}$), nonobservation of $e^+e^- \rightarrow Zh^0, ZH^0$ at LEP implies 95% C.L. limits on $C_{h^0}^2$ and $C_{H^0}^2$ such that $C_{h^0}^2 + C_{H^0}^2 < 1$, whereas the relevant couplings obey the sum rule

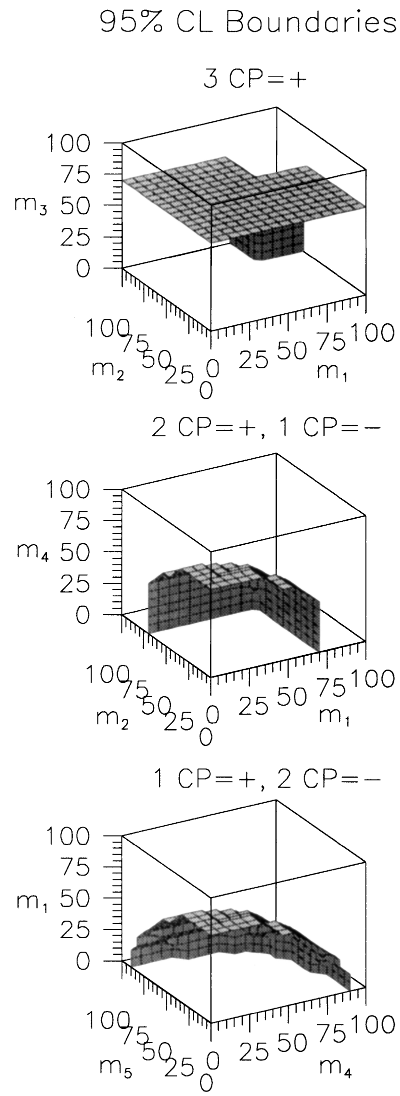

FIG. 2. The LEP 95% confidence level boundaries for the two-doublet plus one-singlet CP-conserving Higgs sector in the (m_1, m_2, m_3), (m_1, m_2, m_4), and (m_4, m_5, m_1) spaces for three CP-even, two CP-even plus one CP-odd, and two CP-odd plus one CP-even Higgs bosons, respectively. Mass axes are in GeV units. Constraints are as in Fig. 1.

$$C_{h^0}^2 + C_{H^0}^2 = \sin^2(\beta - \alpha) + \cos^2(\beta - \alpha) = 1\,. \tag{2}$$

In case (b) the excluded region in (m_{h^0}, m_{A^0}) parameter space is indicated by the dashed line in Fig. 1. Inside the excluded region, failure to observe $e^+e^- \rightarrow Zh^0$ and $e^+e^- \rightarrow h^0A^0$ events implies 95% C.L. upper limits on $C_{h^0}^2$ and $C_{h^0A^0}^2$, respectively, such that $C_{h^0}^2 + C_{h^0A^0}^2 < 1$, whereas these couplings obey the sum rule

$$C_{h^0}^2 + C_{h^0A^0}^2 = \sin^2(\beta - \alpha) + \cos^2(\beta - \alpha) = 1\,. \tag{3}$$

Thus, LEP data exclude the possibility that both the scalar and pseudoscalar Higgs bosons of a CP-conserving 2HDM are light. The asymmetry of the (m_{h^0}, m_{A^0}) excluded region arises as follows. If m_{h^0} is small, then nonobservation of Zh^0 events implies that $C_{h^0}^2 \ll 1$, implying via Eq. (3) that $C_{h^0A^0}^2 \sim 1$. Large m_{A^0} is then required for the predicted number of h^0A^0 events to be consistent with 0. In

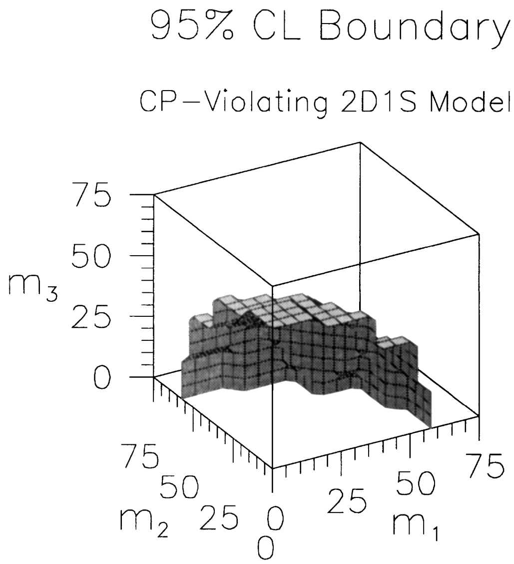

FIG. 3. The LEP 95% confidence level boundary for the two-doublet plus one-singlet CP-violating Higgs sector in the (m_1, m_2, m_3) parameter space. Mass axes are in GeV units. Constraints are as in Fig. 1.

contrast, if m_{h^0} is close to (or above) $\sqrt{s} - m_Z$, then Zh^0 events would not be observed even if $C^2_{h^0} = 1$: At the same time, $C^2_{h^0} = 1$ implies via Eq. (3) that $C^2_{h^0A^0} = 0$ so that there is no constraint from nonobservation of h^0A^0 events.

Below, we shall demonstrate that if CP is not conserved the sum rules of Eqs. (2) and (3) can be generalized in the 2HDM case to [6]:

$$C_i^2 + C_j^2 + C_{ij}^2 = 1\,, \tag{4}$$

where $i \neq j$ are any two of the three possible indices. [Note that Eq. (4) reduces to Eq. (3) in the CP-conserving limit when we identify $h_i = h^0$ and $h_j = A^0$ and use $C_{A^0} = 0$. Similarly, Eq. (4) reduces to Eq. (2) in the CP-conserving limit when we identify $h_i = h^0$ and $h_j = H^0$ and use $C_{h^0H^0} = 0$.] The power of the Eq. (4) sum rule derives from the facts that it involves only two of the neutral Higgs bosons and that the experimental upper limit on any one C_i^2 derived from $e^+e^- \to Zh_i$ data is very strong: $C_i^2 \leq 0.1$ for $m_{h_i} \leq 50$ GeV. Thus, if h_i and h_j are both below about 50 GeV in mass then Eq. (4) requires that $C_{ij}^2 \sim 1$, whereas for such masses limits on $e^+e^- \to h_ih_j$ from $\sqrt{s} = 161$ and 172 GeV data require $C_{ij}^2 \ll 1$. The excluded region in the (m_{h_i}, m_{h_j}) plane that results is illustrated in Fig. 1—there cannot be two light Higgs bosons even in the general CP-violating case.

The sum rule of Eq. (4) is required in order to have unitary high energy behavior at tree level for the $W^+W^- \to W^+W^-$, $ZZ \to W^+W^-$, and $ZZ \to h_ih_i$ scattering amplitudes; see Eqs. (4.1), (4.2), and (A.18) of Ref. [7], respectively. In the context of a Higgs sector containing only doublet and singlet fields, the cited equations of Ref. [7] reduce to the requirements

$$\sum_i C_i^2 = 1\,, \tag{5}$$

$$C_i^2 + \sum_{k\neq i} C_{ik}^2 = 1\,, \tag{6}$$

where the 1 on the right hand side of Eq. (6) arises from the ZZh_ih_i four-point interaction of Eq. (1) contributing to $ZZ \to h_ih_i$ scattering. To derive Eq. (4) in the 2HDM [for which $i, k = 1,2,3$ in Eqs. (5) and (6)], we sum the $i = 1,2$ and the $i = 1,2,3$ cases of Eq. (6), respectively, to obtain

$$C_1^2 + C_2^2 + C_{12}^2 = 2 - (C_{12}^2 + C_{13}^2 + C_{23}^2)\,, \tag{7}$$

$$C_1^2 + C_2^2 + C_3^2 = 3 - 2(C_{12}^2 + C_{13}^2 + C_{23}^2)\,, \tag{8}$$

respectively. We then employ the relation $C_1^2 + C_2^2 + C_3^2 = 1$ from Eq. (5) in Eq. (8) to show that $C_{12}^2 + C_{13}^2 + C_{23}^2 = 1$ and substitute this result into Eq. (7) to obtain $C_1^2 + C_2^2 + C_{12}^2 = 1$. Cyclic permutation gives the other cases of Eq. (4). Eq. (4) is much more useful for obtaining experimental limits than either Eqs. (5) or (6) since the latter two sum rules involve three Higgs bosons (in the 2HDM), whereas the former refers to just two. This distinction arises only in the CP-violating case. In the CP-conserving limit, Eqs. (5) and (6) can be used to derive Eqs. (2) and (3), respectively, while Eq. (4) implies both (as described earlier).

These considerations can be generalized to extensions of the 2HDM. The simplest extension is to add one complex singlet (neutral) Higgs field. In this case, it is no longer possible to place restrictions on two Higgs bosons. The best that one can do is to write the sum rules in such a way as to demonstrate that there cannot be three light neutral Higgs bosons. Consider first the case where CP is conserved. In the 2D1S model, there will then be three CP-even Higgs bosons (labeled $1,2,3$ in order of increasing mass) and two CP-odd Higgs bosons (labeled $4,5$ in order of increasing mass). Using the sum rules of Eqs. (5) and (6), with $C_4 = C_5 = C_{12} = C_{13} = C_{23} = C_{45} = 0$, we easily derive the three crucial sum rules:

$$C_1^2 + C_2^2 + C_3^2 = 1\,, \tag{9}$$

$$C_1^2 + C_2^2 + C_{14}^2 + C_{24}^2 = 1 + C_{35}^2\,, \tag{10}$$

$$C_1^2 + C_{14}^2 + C_{15}^2 = 1\,. \tag{11}$$

Equations (9)–(11) can be used to show that there cannot be three CP-even, two CP-even plus one CP-odd, and one CP-even plus two CP-odd Higgs bosons, respectively, that are all light. (In the second sum rule, since $C_{35}^2 \geq 0$ the region within which exclusion is certain is obtained by setting $C_{35}^2 = 0$.) The boundaries in the three-dimensional mass spaces that follow from these sum rules [with $C_{35}^2 = 0$ in Eq. (10)] are shown in Fig. 2.

In the case that CP is violated, the required sum rule for the 2D1S is obtained by generalizing the procedure sketched for the derivation of Eq. (4) in the 2HDM case. Focusing on Higgs bosons numbers 1, 2, and 3, one finds

$$C_1^2 + C_2^2 + C_3^2 + C_{12}^2 + C_{13}^2 + C_{23}^2 = 1 + C_{45}^2 . \qquad (12)$$

This sum rule implies a lower bound (obtained with $C_{45}^2 = 0$) for the sum of all the couplings squared responsible for production of Zh_1, Zh_2, Zh_3, h_1h_2, h_1h_3, and h_2h_3 in e^+e^- collisions. The portion of the (m_1, m_2, m_3) mass space excluded by LEP data in the *CP*-violating case, as implied by the sum rule of Eq. (12), is shown in Fig. 3.

The above considerations can be further generalized to a Higgs sector that contains ℓ doublets and m neutral complex singlets; the number of physical neutral Higgs mass eigenstates is $2(\ell + m) - 1$. The general sum rule will apply to any subset containing $n = (\ell + m)$ of these Higgs bosons. Let us label the members of the subset with indices $i = 1, \ldots, n$. Following the techniques illustrated in the 2D1S case, and assuming that *CP* violation is present, we derive the coupling constant sum rule

$$\sum_{i=1}^{n} C_i^2 + \sum_{\substack{i,j=1 \\ i<j}}^{n} C_{ij}^2 = 1 + \sum_{\substack{i,j=n+1 \\ i<j}}^{2n-1} C_{ij}^2 , \qquad (13)$$

where the most conservative bounds on the subset would be obtained by setting all the $C_{ij}^2 = 0$ on the right hand side. If *CP* violation is not present in the Higgs sector, then the above sum rule will reduce to a simpler form that depends upon the *CP* nature of the Higgs bosons included in the subset.

In this Letter, we have derived a new coupling constant sum rule which makes it possible to use LEP data to exclude a portion of (m_{h_1}, m_{h_2}) mass space for the lightest two neutral Higgs bosons of the most general *CP*-violating two-Higgs-doublet model. Although this region is not as large as that excluded in the (m_{h^0}, m_{A^0}) mass parameter space in the *CP*-conserving case, it is still very substantial. Thus, LEP data imply that it is not possible for two of the three neutral Higgs bosons of a general two-Higgs-doublet model to be light. We have further shown how to extend this type of analysis to both *CP*-conserving and *CP*-violating Higgs sectors with an arbitrary number of doublets and singlets. In the two-doublet plus one-singlet Higgs model, LEP data already exclude the possibility that three of the five neutral Higgs bosons are light.

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Appendix.—The limits presented in Figs. 1–3 have been obtained from LEP1 and LEP2 data on $e^+e^- \to Zh_i$ and $e^+e^- \to h_ih_j$ production using the following procedures. Consider first any given Zh_i channel. For $m_{h_i} < 50$ GeV, the 95% C.L. upper limit on C_i^2 [defined in Eq. (1)] is obtained by using the smaller of the values shown in the *LS* region, Ref. [3], and Fig. 29 from Ref. [8]. For $m_{h_i} \geq 50$ GeV, the 95% C.L. upper limit on C_i^2 is obtained as the ratio of the 95% C.L. upper limit on the number of events as observed by ALEPH, taken to be ~ 3 events from the graph in Ref. [1] (which includes data at $\sqrt{s} = m_Z$, $\sqrt{s} = 161$ GeV, and $\sqrt{s} \sim 172$ GeV), to the number of events expected at the given Higgs mass in the SM, as plotted in the same graph. If the assumed C_i^2 exceeds the 95% C.L. as defined above at the input m_{h_i}, then the parameter choice is taken to be excluded at the 95% C.L. For a two-Higgs channel h_ih_j, we approximate the ALEPH 95% C.L. limits implicit in Ref. [1] by employing integrated luminosities of $L = 11.08\ \mathrm{pb}^{-1}$ at $\sqrt{s} = 161$ GeV and $L = 10.5\ \mathrm{pb}^{-1}$ at $\sqrt{s} = 172$ GeV and the quoted efficiencies and branching ratios of $\epsilon = 0.55$, BR $= 0.83$ for the $4b$ channel and $\epsilon = 0.45$, BR $= 0.16$ for the $2b2\tau$ channel. We compute the expected number of events (combining the $4b$ and $2b2\tau$ channels) for the input value of the Zh_ih_j coupling strength and then evaluate the Poisson probability that no events are observed. If this probability is below 5% then the chosen parameter set for the h_i and h_j is said to be excluded at 95% C.L.

[1] We employ the preliminary ALEPH results presented by G. Cowan, in Proceedings of the Special CERN Particle Physics Seminar on Physics, 1997 (http://alephwww.cern.ch/ALPUB/seminar/Cowan-172-jam/cowan.html).

[2] Preliminary results similar to those of ALEPH were presented for DELPHI by F. Richard (http://delphiwww.cern.ch/delfigs/figures/search/higgs172/higgs.html), for OPAL by S. Komamiya (http://www.cern.ch/Opal/plots/komamiya/koma.html), and for L3 by M. Pieri (http://hpl3sn02.cern.ch/analysis/nplep2/plots.html).

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