Implications from $B \to K^* \ell^+ \ell^-$ observables using LHCb data.

Abinash Kumar Nayak, Rusa Mandal, Rahul Sinha

The Institute of Mathematical Sciences, Taramani, Chennai 600113, India

DOI: http://dx.doi.org/10.3204/DESY-PROC-2016-04/Sina

The decay $B \to K^* \ell^+ \ell^-$ is regarded as one of the most important modes to search for physics beyond the standard model as the angular distribution enables the independent measurement of a plethora of observables. To disentangle new physics from the standard model effect an "exact" test of standard model is needed. This drive us to derive a relation including all short-distance and long-distance effects, factorizable and non-factorizable contributions, complete electromagnetic corrections to hadronic operators up to all orders, resonance contributions and the finite lepton masses in a complete model independent approach. The violation of this relation will provide a smoking gun signal of new physics. The model independent framework has also been implemented in the maximum q^2 limit to highlight strong evidence of right-handed currents, which are absent in the SM. The conclusions derived are free from hadronic corrections. Our approach differs from other approaches that probe new physics at low q^2 as it does not require estimates of hadronic parameters but relies instead on heavy quark symmetries that are reliable at the maximum q^2 kinematic endpoint.

The mode $B \to K^* \ell^+ \ell^-$

The Standard Model(SM) is a gauge theory capable of explaining interactions between the observed particles. Most ingredients completed by 1974 and with the discovery of the Higgs, the SM is complete. Though this gauge theory has been remarkably successful in explaining a whole lot of physical phenomenons, it also fails to answer various others(e.g. the origin of its parameters, Naturalness problem, dark matter, dark energy etc.). Hence a lot of attempts have been made to extend this highly successful theory, which we call the Physics beyond SM or New Physics(NP). However, no conclusive NP signal has been seen till now. Which brings us to the question that how does one search for NP. It can be discovered either by direct production of new particles which are not accommodated in SM or by indirect searches at high luminosity facilities where NP can contribute virtually through loop processes.

A lot of effort has been put in observation of new particles through direct production. But if we were to follow the later approach, then we should focus on modes that are potentially sensitive to NP. One such mode is the mode $B \to K^* \ell^+ \ell^-$. The underlying process is a Flavor Changing Neutral Current (FCNC) decay which happens through a penguin process. It has a large number of related observables. In addition, this mode can get contribution from variety of operators through the loop. Hence, this is very good candidate in probing NP. But this being a hadronic decay mode, we have to be careful in dealing with the hadronic uncertainties. In this work we show that the hadronic uncertainties can be eliminated by carefully taking relation between observables. Then we use these relations to probe for NP.

Model independent framework

The decays $B \to K^* \ell^+\ell^-$ occurs at the quark level via a $b \to s\ell^+\ell^-$ flavor changing neutral current transition. The short-distance effective Hamiltonian for the inclusive process $b \to s\ell^+\ell^-$ is given in the SM by [1, 2, 3],

$$\mathcal{H}_{\text{eff}} = -\,4\frac{G_F}{\sqrt{2}}\Big[V_{tb}V_{ts}^*\Big(C_1\mathcal{O}_1^c + C_2\mathcal{O}_2^c + \sum_{i=3}^{10} C_i\mathcal{O}_i\Big) + V_{ub}V_{us}^*\Big(C_1(\mathcal{O}_1^c - \mathcal{O}_1^u) + C_2(\mathcal{O}_2^c - \mathcal{O}_2^u)\Big)\Big]. \tag{1}$$

The local operators $\mathcal{O}_i$ are as given in Ref. [2], however, for completeness we present the relevant operators that are dominant:

$$\begin{aligned}
\mathcal{O}_7 &= \frac{e}{g^2}\left[\bar{s}\sigma_{\mu\nu}(m_b P_R + m_s P_L)b\right]F^{\mu\nu},\\
\mathcal{O}_9 &= \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)\,\bar{\ell}\gamma^\mu\ell,\\
\mathcal{O}_{10} &= \frac{e^2}{g^2}(\bar{s}\gamma_\mu P_L b)\,\bar{\ell}\gamma^\mu\gamma_5\ell,
\end{aligned}$$

where $g\,(e)$ is the strong(electromagnetic) coupling constant, $P_{L,R} = (1 \mp \gamma_5)/2$ are the left and right chiral projection operators and $m_b\,(m_s)$ are the running $b\,(s)$ quark mass in the $\overline{\text{MS}}$ scheme. The Wilson coefficients C_i encode all the short-distance effects and are calculated in perturbation theory at a matching scale $\mu = M_W$ up to desired order in the strong coupling constant α_s before being evolved down to the scale $\mu = m_b \approx 4.8$GeV. All NP contributions to $B \to K^*\ell^+\ell^-$ contribute exclusively to C_i; this includes new Wilson coefficients corresponding to new operators that arise from NP.

The decay amplitude in terms of hadronic matrix elements must therefore include direct contributions proportional to C_7, C_9 and C_{10} multiplied by $B \to K^*$ form factors and contributions from non local hadronic matrix elements $\mathcal{H}_i$ such that [4, 5],

$$\begin{aligned}
A(B(p) \to K^*(k)\ell^+\ell^-) = \frac{G_F\alpha}{\sqrt{2}\pi}V_{tb}V_{ts}^*\Bigg[\Bigg\{&\widehat{C}_9\langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle\\
&- \frac{2\widehat{C}_7}{q^2}\langle K^*|\bar{s}i\sigma^{\mu\nu}q_\nu(m_b P_R + m_s P_L)b|\bar{B}\rangle\\
&- \frac{16\pi^2}{q^2}\sum_{i=\{1-6,8\}}\widehat{C}_i\mathcal{H}_i^\mu\Bigg\}\bar{\ell}\gamma_\mu\ell\\
&+ \widehat{C}_{10}\langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}\rangle\,\bar{\ell}\gamma_\mu\gamma_5\ell\Bigg],
\end{aligned} \tag{2}$$

where, $p = q + k$ with q being the dilepton invariant momentum and the non local hadron matrix element $\mathcal{H}_i^\mu$ is given by

$$\mathcal{H}_i^\mu = \langle K^*(k)|i \int d^4x\, e^{iq\cdot x} T\{j^\mu_{em}(x), \mathcal{O}_i(0)\}|\bar{B}(p)\rangle.$$

In Eq. (2), we have introduced *new notional theoretical parameters $\widehat{C}_7$, $\widehat{C}_9$ and $\widehat{C}_{10}$ to indicate the true values of Wilson coefficients, which are by definition not dependent on the order of the perturbative calculation to which they are evaluated.* Our definition is explicit and should not be confused with those defined earlier in literature. The amplitude expressed in Eq. (2) is *notionally complete and free from any approximations.* In this paper we do not attempt to estimate the hadronic matrix element involved in Eq. (2), instead we use Lorentz invariance to write out the most general form of the hadron matrix elements $\langle K^*|\bar{s}\gamma^\mu P_L b|\bar{B}(p)\rangle$ and $\langle K^*|\bar{s}i\sigma^{\mu\nu}q_\nu P_{R,L}b|\bar{B}(p)\rangle$ which may be defined as

$$\langle K^*(\epsilon^*,k))|\bar{s}\gamma^\mu P_L b|B(p)\rangle = \epsilon^*_\nu\Big(\mathcal{X}_0\, q^\mu q^\nu + \mathcal{X}_1\,(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \mathcal{X}_2\,(k^\mu - \frac{k.q}{q^2}q^\mu)q^\nu + i\mathcal{X}_3\,\epsilon^{\mu\nu\rho\sigma}\,k_\rho q_\sigma\Big), \tag{3}$$

$$\langle K^*(\epsilon^*,k))|i\bar{s}\sigma^{\mu\nu}q_\nu P_{R,L}b|B(p)\rangle = \epsilon^*_\nu\Big(\pm\mathcal{Y}_1\,(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) \pm \mathcal{Y}_2\,(k^\mu - \frac{k.q}{q^2}q^\mu)q^\nu + i\mathcal{Y}_3\,\epsilon^{\mu\nu\rho\sigma}\,k_\rho q_\sigma\Big). \tag{4}$$

We have written Eq. (3) such that the vector part of the current in $\langle K^*(\epsilon^*,k))|\bar{s}\gamma^\mu P_L b|B(p)\rangle$ is conserved and only the $\mathcal{X}_0$ term in the divergence of the axial part survives. Equation (4) is also written so as to ensure that $\langle K^*|i\bar{s}\sigma^{\mu\nu}q_\nu P_{R,L}b|B\rangle q_\mu = 0$. The relations between $\mathcal{X}_{0,1,2,3}$ and $\mathcal{Y}_{1,2,3}$ and the form factors conventionally defined for on-shell K^* are discussed in [6]. It should be noted that form factors $\mathcal{X}_{0,1,2,3}$ and $\mathcal{Y}_{1,2,3}$ are functions of q^2 and k^2, but we suppress the explicit dependence for simplicity of notation. The subsequent decay of the K^*, i.e., $K^*(k) \to K(k_1)\pi(k_2)$ can be easily taken into account [7, 2] resulting in the hadronic matrix element $\langle [K(k_1)\pi(k_2)]_{K^*}|\bar{s}\gamma^\mu P_L b|B(p)\rangle$ being written as

$$\begin{aligned}\langle [K(k_1)\pi(k_2)]_{K^*}|\bar{s}\gamma^\mu P_L b|B(p)\rangle = D_{K^*}(k^2)W_\nu\Big(&\mathcal{X}_0\, q^\mu q^\nu + \mathcal{X}_1(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})\\ &+ \mathcal{X}_2\,(k^\mu - \frac{k.q}{q^2}q^\mu)q^\nu + i\mathcal{X}_3\,\epsilon^{\mu\nu\rho\sigma}\,k_\rho q_\sigma\Big),\end{aligned} \tag{5}$$

$$\begin{aligned}\langle [K(k_1)\pi(k_2)]_{K^*}|i\bar{s}\sigma^{\mu\nu}q_\nu P_{R,L}b|B(p)\rangle = D_{K^*}(k^2)W_\nu\Big(&\pm\mathcal{Y}_1\,(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2})\\ &\pm\mathcal{Y}_2\,(k^\mu - \frac{k.q}{q^2}q^\mu)q^\nu + i\mathcal{Y}_3\,\epsilon^{\mu\nu\rho\sigma}\,k_\rho q_\sigma\Big),\end{aligned} \tag{6}$$

where, the subscript K^* in $[K(k_1)\pi(k_2)]_{K^*}$ indicates that the final sate is produced by the decay of a K^*. $D_{K^*}(k^2)$ is the K^* propagator, so that

$$|D_{K^*}(k^2)|^2 = \frac{g^2_{K^*K\pi}}{(k^2 - m^2_{K^*})^2 + (m_{K^*}\Gamma_{K^*})^2}, \tag{7}$$

with $g_{K^*K\pi}$ being the $K^*K\pi$ coupling and the other parameters introduced are

$$W_\nu = K_\nu - \xi k_\nu,\ K = k_1 - k_2,\ k = k_1 + k_2,\ \xi = \frac{k_1^2 - k_2^2}{k^2}.$$

The most general expression for the hadronic matrix element $\mathcal{H}_i^\mu$ can also be written using Lorentz invariance. Since this hadronic matrix element arises from non local contributions at the quark level, it involves introducing "new" form factors $\mathcal{Z}_1^i$, $\mathcal{Z}_2^i$ and $\mathcal{Z}_3^i$ corresponding to nonfactorizable contribution from each $\mathcal{H}_i^\mu$ in analogy with those introduced in Eq. (3) as follows:

$$\begin{aligned}\mathcal{H}_i^\mu &= \langle K^*(\epsilon^*, k)|i\int d^4x\, e^{iq\cdot x} T\{j_{em}^\mu(x), \mathcal{O}_i(0)\}|\bar{B}(p)\rangle \\ &= \epsilon_\nu^*\Big(\mathcal{Z}_1^i\,(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}) + \mathcal{Z}_2^i\,(k^\mu - \frac{k.q}{q^2}q^\mu)q^\nu + i\mathcal{Z}_3^i\,\epsilon^{\mu\nu\rho\sigma}\,k_\rho q_\sigma\Big).\end{aligned} \tag{8}$$

Our definition follows Ref. [8] of "nonfactorizable " and includes those corrections that are not contained in the definition of form factors introduced in Eqs. (3) and (4). Here the most general form of $\mathcal{H}_i^\mu$ is written to ensure the conservation of EM current i.e, $q_\mu\mathcal{H}_i^\mu = 0$.

The non local effects represented by $\mathcal{H}_i^\mu$ can be taken into account by absorbing the contributions into redefined $\widehat{C}_9$ and modifying the contribution from the electromagnetic dipole operator $\mathcal{O}_7$. The electromagnetic corrections to operators $\mathcal{O}_{1-6,8}$ can also contribute to $B \to K^*\gamma$ at $q^2 = 0$. Since only the Wilson coefficient $\widehat{C}_7$ contributes to $B \to K^*\gamma$, the charm loops at $q^2 = 0$ must contribute to $\widehat{C}_7$ in order for the Wilson coefficient to be process independent. It is easily seen that the effect of this is to modify the $\widehat{C}_7\langle K\pi|\bar{s}i\sigma^{\mu\nu}q_\nu(m_bP_R+m_sP_L)b|\bar{B}\rangle$ terms such that the form factors and Wilson coefficients mix in an essentially inseparable fashion. This holds true even for the leading logarithmic contributions [8, 9]. Both factorizable and nonfactorizable contributions arising from electromagnetic corrections to hadronic operators up to all orders can in principle be included in this approach. The remaining contributions can easily be absorbed into a redefined "effective" Wilson coefficient $\widehat{C}_9$ defined such that

$$\widehat{C}_9 \to \widetilde{C}_9^{(j)} = \widehat{C}_9 + \Delta C_9^{(\text{fac})}(q^2) + \Delta C_9^{(j),(\text{non-fac})}(q^2) \tag{9}$$

where, $j = 1, 2, 3$ and $\Delta C_9^{(\text{fac})}(q^2)$, $\Delta C_9^{(\text{non-fac})}(q^2)$ correspond to factorizable and soft gluon nonfactorizable contributions respectively. Note that the nonfactorizable contributions necessitates the introduction of new form factors $\mathcal{Z}_j$ and the explicit dependence on $\mathcal{Z}_j/\mathcal{X}_j$ is absorbed in defining

$$\Delta C_9^{(\text{fac})} + \Delta C_9^{(j),(\text{non-fac})} = -\frac{16\pi^2}{q^2}\sum_{i=\{1-6,8\}}\widehat{C}_i\,\frac{\mathcal{Z}_j^i}{\mathcal{X}_j}, \tag{10}$$

resulting in the j dependence of the term as indicated. We also mention that there is no nonfactorizable correction term in Eq. (8) analogous to $\mathcal{X}_0$ (in Eq. (3)) due EM current conservation as discussed above.

The corresponding corrections to $\widehat{C}_7$ are taken into by the replacement,

$$\frac{2(m_b{+}m_s)}{q^2}\widehat{C}_7\,\mathcal{Y}_j \to \widetilde{\mathcal{Y}}_j = \frac{2(m_b + m_s)}{q^2}\widehat{C}_7\,\mathcal{Y}_j + \cdots, \tag{11}$$

where the dots indicate other factorizable and nonfactorizable contributions and the factor $2(m_b + m_s)/q^2$ has been absorbed in the form factors $\widetilde{\mathcal{Y}}_j$. Note that the $\widetilde{\mathcal{Y}}_j$'s are in general complex because of the nonfactorizable contributions to the Wilson coefficient $\widehat{C}_7$, but on-shell quarks and resonances do not contribute to them. It should be noted that $\widetilde{C}_9^{(j)}$ includes contributions from both factorizable and nonfactorizable effects, whereas $\widehat{C}_{10}$ is unaffected by strong interaction effects coming from electromagnetic corrections to hadronic operators. The use of a 'widetilde' versus 'widehat' throughout the paper is also meant as a notation to indicate this fact. It should be noted that $\widehat{C}_{10}$ is real in the SM, whereas, $\widetilde{C}_9^{(j)}$ and $\widetilde{\mathcal{Y}}_j$ are in general complex within the SM. The amplitude in Eq. (2) can therefore be written as

$$\mathcal{A}_\lambda^{L,R} = C_{L,R}^\lambda \mathcal{F}_\lambda - \widetilde{\mathcal{G}}_\lambda = \big(\widetilde{C}_9^\lambda \mp \widehat{C}_{10}\big)\mathcal{F}_\lambda - \widetilde{\mathcal{G}}_\lambda = (\mp\widehat{C}_{10} - r_\lambda)\mathcal{F}_\lambda + i\varepsilon_\lambda, \tag{12}$$

r_λ and ε_λ are defined as,

$$r_\lambda = \frac{\mathrm{Re}(\widetilde{\mathcal{G}}_\lambda)}{\mathcal{F}_\lambda} - \mathrm{Re}(\widetilde{C}_9^\lambda), \quad \varepsilon_\lambda \equiv \mathrm{Im}(\widetilde{C}_9^\lambda)\mathcal{F}_\lambda - \mathrm{Im}(\widetilde{\mathcal{G}}_\lambda). \tag{13}$$

Observables and relations

With these amplitudes we can construct several observables. For us the relevant observables are the three helicity fractions and six asymmetries. The helicity fractions are defined as

$$F_L = \frac{|\mathcal{A}_0^L|^2 + |\mathcal{A}_0^R|^2}{\Gamma_f}\ , \tag{14a}$$

$$F_\parallel = \frac{|\mathcal{A}_\parallel^L|^2 + |\mathcal{A}_\parallel^R|^2}{\Gamma_f}\ , \tag{14b}$$

$$F_\perp = \frac{|\mathcal{A}_\perp^L|^2 + |\mathcal{A}_\perp^R|^2}{\Gamma_f}\ , \tag{14c}$$

where $\Gamma_f \equiv \sum_\lambda(|\mathcal{A}_\lambda^L|^2 + |\mathcal{A}_\lambda^R|^2)$ such that $F_L + F_\parallel + F_\perp = 1$. The rest six asymmetries are defined below.

$$A_{\mathrm{F}B} = \frac{3}{2}\frac{\mathrm{Re}(\mathcal{A}_\parallel^L\mathcal{A}_\perp^{L^*} - \mathcal{A}_\parallel^R\mathcal{A}_\perp^{R^*})}{\Gamma_f}, \tag{15}$$

$$A_4 = \frac{\sqrt{2}}{\pi}\frac{\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_\parallel^{L^*} + \mathcal{A}_0^R\mathcal{A}_\parallel^{R^*})}{\Gamma_f}, \tag{16}$$

$$A_5 = \frac{3}{2\sqrt{2}}\frac{\mathrm{Re}(\mathcal{A}_0^L\mathcal{A}_\perp^{L^*} - \mathcal{A}_0^R\mathcal{A}_\perp^{R^*})}{\Gamma_f}, \tag{17}$$

$$A_7 = \frac{3}{2\sqrt{2}}\frac{\mathrm{Im}(\mathcal{A}_0^L\mathcal{A}_\parallel^{L^*} - \mathcal{A}_0^R\mathcal{A}_\parallel^{R^*})}{\Gamma_f}, \tag{18}$$

$$A_8 = \frac{\sqrt{2}}{\pi}\frac{\mathrm{Im}(\mathcal{A}_0^L\mathcal{A}_\perp^{L^*} + \mathcal{A}_0^R\mathcal{A}_\perp^{R^*})}{\Gamma_f}, \tag{19}$$

$$A_9 = \frac{3}{2\pi}\frac{\mathrm{Im}(\mathcal{A}_\parallel^{L^*}\mathcal{A}_\perp^L + \mathcal{A}_\parallel^{R^*}\mathcal{A}_\perp^R)}{\Gamma_f}. \tag{20}$$

Using Eq. (12) and (13) we express the observables in terms of $\widehat{C}_{10}$, r_λ, $\mathcal{F}_\lambda$ and ε_λ as follows:

$$F_L\Gamma_f = 2\mathcal{F}_0^2\big(r_0^2+\widehat{C}_{10}^2\big)+2\varepsilon_0^2, \tag{21}$$

$$F_\parallel\Gamma_f = 2\mathcal{F}_\parallel^2\big(r_\parallel^2+\widehat{C}_{10}^2\big)+2\varepsilon_\parallel^2, \tag{22}$$

$$F_\perp\Gamma_f = 2\mathcal{F}_\perp^2\big(r_\perp^2+\widehat{C}_{10}^2\big)+2\varepsilon_\perp^2, \tag{23}$$

$$\sqrt{2}\pi A_4\Gamma_f = 4\mathcal{F}_0\mathcal{F}_\parallel\big(r_0r_\parallel+\widehat{C}_{10}^2\big)+4\varepsilon_0\varepsilon_\parallel, \tag{24}$$

$$\sqrt{2}A_5\Gamma_f = 3\mathcal{F}_0\mathcal{F}_\perp\widehat{C}_{10}\big(r_0+r_\perp\big), \tag{25}$$

$$A_{\rm FB}\Gamma_f = 3\mathcal{F}_\parallel\mathcal{F}_\perp\widehat{C}_{10}\big(r_\parallel+r_\perp\big), \tag{26}$$

$$\sqrt{2}A_7\Gamma_f = 3\widehat{C}_{10}\big(\mathcal{F}_0\varepsilon_\parallel-\mathcal{F}_\parallel\varepsilon_0\big), \tag{27}$$

$$\pi A_8\Gamma_f = 2\sqrt{2}\big(\mathcal{F}_0r_0\varepsilon_\perp-\mathcal{F}_\perp r_\perp\varepsilon_0\big), \tag{28}$$

$$\pi A_9\Gamma_f = 3\big(\mathcal{F}_\perp r_\perp\varepsilon_\parallel-\mathcal{F}_\parallel r_\parallel\varepsilon_\perp\big). \tag{29}$$

The ε_λ's can be solved using A_7, A_8 and A_9 from Eqs. (27)–(29) to give

$$\varepsilon_\perp = \frac{\sqrt{2}\pi\Gamma_f}{(r_0-r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9\mathsf{P}_1}{3\sqrt{2}}+\frac{A_8\mathsf{P}_2}{4}-\frac{A_7\mathsf{P}_1\mathsf{P}_2r_\perp}{3\pi\widehat{C}_{10}}\right], \tag{30}$$

$$\varepsilon_\parallel = \frac{\sqrt{2}\pi\Gamma_f}{(r_0-r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9r_0}{3\sqrt{2}r_\perp}+\frac{A_8\mathsf{P}_2r_\parallel}{4\mathsf{P}_1r_\perp}-\frac{A_7\mathsf{P}_2r_\parallel}{3\pi\widehat{C}_{10}}\right], \tag{31}$$

$$\varepsilon_0 = \frac{\sqrt{2}\pi\Gamma_f}{(r_0-r_\parallel)\mathcal{F}_\perp}\left[\frac{A_9\mathsf{P}_1r_0}{3\sqrt{2}\mathsf{P}_2r_\perp}+\frac{A_8r_\parallel}{4r_\perp}-\frac{A_7\mathsf{P}_1r_0}{3\pi\widehat{C}_{10}}\right]. \tag{32}$$

Where,

$$\mathsf{P}_1=\frac{\mathcal{F}_\perp}{\mathcal{F}_\parallel},\quad \mathsf{P}_2=\frac{\mathcal{F}_\perp}{\mathcal{F}_0}. \tag{33}$$

As expected, we also get some relations between observables as there are only 6 amplitudes but several observables. Implying, all the observables are not independent. These relations are obtained by interplaying with the Eqs. (21)-(26). One such relation, with the imaginary contribution to the amplitudes, is given below

$$A_4 = \frac{2\sqrt{2}\varepsilon_\parallel\varepsilon_0}{\pi\Gamma_f}+\frac{8A_5A_{\rm FB}}{9\pi\Big(F_\perp-\dfrac{2\varepsilon_\perp^2}{\Gamma_f}\Big)} +\sqrt{2}\,\frac{\sqrt{\Big(F_L-\dfrac{2\varepsilon_0^2}{\Gamma_f}\Big)\Big(F_\perp-\dfrac{2\varepsilon_\perp^2}{\Gamma_f}\Big)-\dfrac{8}{9}A_5^2}\,\sqrt{\Big(F_\parallel-\dfrac{2\varepsilon_\parallel^2}{\Gamma_f}\Big)\Big(F_\perp-\dfrac{2\varepsilon_\perp^2}{\Gamma_f}\Big)-\dfrac{4}{9}A_{\rm FB}^2}}{\pi\Big(F_\perp-\dfrac{2\varepsilon_\perp^2}{\Gamma_f}\Big)}. \tag{34}$$

In real limit(i.e. $\varepsilon_\lambda\to 0$), the asymmetries $A_{7,8,9}\to 0$ from Eq.(27)-(29). Whereas the rest of the asymmetries are related to other observables through the following relations,

$$A_4 = \frac{8A_5A_{\rm FB}}{9\pi F_\perp} + \frac{\sqrt{4F_\| F_\perp - \frac{16}{9}A_{\rm FB}^2}\sqrt{4F_L F_\perp - \frac{32}{9}A_5^2}}{2\sqrt{2}\pi F_\perp}, \tag{35}$$

$$A_5 = \frac{\pi A_4 A_{\rm FB}}{2F_\|} \pm \frac{3\sqrt{4F_\| F_\perp - \frac{16}{9}A_{\rm FB}^2}\sqrt{2F_\| F_L - \pi^2 A_4^2}}{8F_\|}, \tag{36}$$

$$A_{\rm FB} = \frac{\pi A_4 A_5}{F_L} \pm \frac{3\sqrt{4F_L F_\perp - \frac{32}{9}A_5^2}\sqrt{2F_\| F_L - \pi^2 A_4^2}}{4\sqrt{2}F_L}. \tag{37}$$

The detailed derivations of the above relations are given in [6, 10]. We note that these relations do not depend on any hadronic parameters except the observables. Hence these relations should hold true unless some new operators, or New Physics operators to be precise, end up altering the whole angular distribution such that the above mentioned relations will no longer hold. So we emphasize that these relations can be used as a very clean and indirect probe for New Physics. However in the presence of right-handed currents and any extra vector current such as Z' which does not change the angular distributions, the relations will remain valid. At this point we'd also like to mention that the kind of New Physics contributing to this mode are not that arbitrary as there will be several constraints, on several kinds of New Physics, coming from several other hadronic as well as non-hadronic decay modes.

In Fig.1, we test the validity of Eq.(35)-(37). We get the values of the observable on LHS by putting experimentally observed observable values on the RHS in each q^2 bin. Then we plot this result against the experimentally observed values of the corresponding observables. The mean values and $\pm 1\sigma$ uncertainty bands for asymmetries $A_{\rm FB}$, A_4, A_5 and P_5' calculated using Eqs. (35)-(37), are shown in yellow, gray, green and brown bands, respectively. The error bars in red (dark) correspond to the LHCb measured [11] central values and errors for each observable for the respective q^2 bins. Sizable discrepancies are found for $A_{\rm FB}$ in $11.0 \le q^2 \le 12.5$ GeV^2 and $15 \le q^2 \le 17$ GeV^2 bins and for A_4 in the range $0.1 \le q^2 \le 0.98$ GeV^2.

Looking for New Physics

The rare decay $B \to K^*\ell^+\ell^-$, which involves a $b \to s$ flavor changing loop induced quark transition at the quark level, provides an indirect but very sensitive probe of new physics (NP) beyond the standard model (SM). The angular distribution of the decay products provides a large number of observables [7] and thus can be used to reduce hadronic uncertainties making the mode a very special tool to probe for NP. We have already indicated some irregularities between the SM and the data in the previous discussion. While it implies the presence of NP, it says very little about the exact nature of the possible New Physics. In the following work we discover a specific kind of NP(i.e. Right-Handed Currents) at the kinematic endpoint of $B \to K^*\ell^+\ell^-$. We emphasize that even with these RH-currents, which does not alter the angular distribution, the relations in Eq. (35)-(37) would still hold and some other kind of NP would be required to explain the discrepancies found in Fig.1.

Significant work has been done to probe NP in this mode. Most previous attempts have focused [12] on the low dilepton invariant mass squared region $q^2 = 1-6$ GeV^2. An alternative approach that probes the maximum q^2 limit has also been studied in literature [5, 13]. We show

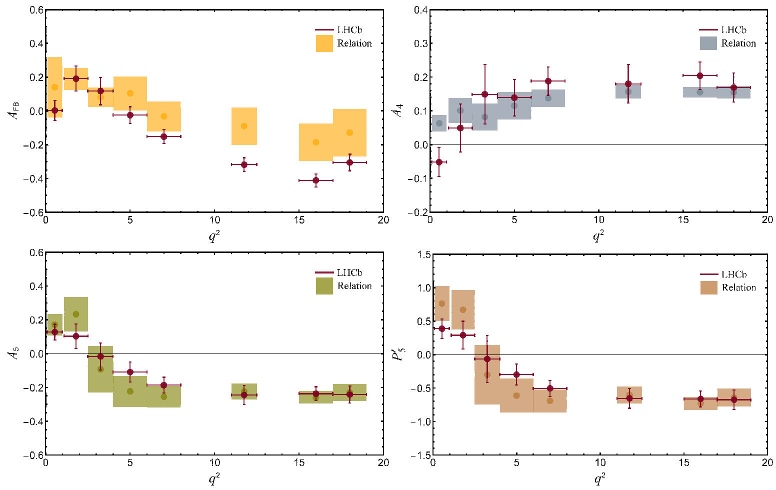

Figure 1: (color online) The mean values and $\pm 1\sigma$ uncertainty bands for asymmetries $A_{\rm FB}$, A_4, A_5 and P_5' calculated using Eqs. (35) – (37) are shown in yellow, gray, green and brown bands, respectively. The error bars in red (dark) correspond to the LHCb measured [11] central values and errors for each observable for the respective q^2 bins. The predictions for the asymmetries are obtained using the relations among observables which are independent of any hadronic parameters and depend on experimental measurements of the other observables remaining in the corresponding relations. Sizable discrepancies are shown for $A_{\rm FB}$ in $11.0 \le q^2 \le 12.5$ GeV^2 and $15 \le q^2 \le 17$ GeV^2 bins and for A_4 in the range $0.1 \le q^2 \le 0.98$ GeV^2. We note that the relations (Eqs. (35) – (37)) remain valid except in the presence of NP operators that result in modified angular distribution. Hence the presence of right-handed currents and any extra vector current such as Z' the relations will remain valid.

that this limit holds significant promise for clean probes of NP. A previous study suggested a possible signal of NP in the large q^2 region [10]. In this work we show that LHCb data implies a 5σ signal for the existence of NP. While the evidence for right handed currents is clear, other NP contributions are also possible. Our conclusions are derived in the maximum q^2 limit ($q^2_{\rm max}$) and are free from hadronic corrections. Our approach differs from other approaches that probe NP at low q^2 by not requiring estimates of hadronic parameters but relying instead on heavy quark symmetries that are completely reliable at the kinematic endpoint $q^2_{\rm max}$ [5, 14]. While the observables themselves remain unaltered from their SM values, their derivatives and second derivatives at the endpoint are sensitive to NP effects.

Right-Handed currents

As mentioned previously the Right-Handed currents do not alter the angular distributions. However, they introduce two new operators namely O_9' and O_{10}'. These operators O_9' and O_{10}', with respective couplings C_9' and C_{10}', modify the amplitudes in Eq. (12) as follows

$$\begin{aligned}\mathcal{A}_\perp^{L,R} &= \big((\widetilde{C}_9^\perp + C_9') \mp (C_{10} + C_{10}')\big)\mathcal{F}_\perp - \widetilde{\mathcal{G}}_\perp,\\ \mathcal{A}_{\|,0}^{L,R} &= \big((\widetilde{C}_9^{\|,0} - C_9') \mp (C_{10} - C_{10}')\big)\mathcal{F}_{\|,0} - \widetilde{\mathcal{G}}_{\|,0}\end{aligned} \tag{38}$$

We define two new variables

$$\xi = \frac{C'_{10}}{C_{10}} \quad \text{and} \quad \xi' = \frac{C'_9}{C_{10}}. \tag{39}$$

Ignoring the imaginary contributions for the time being, the observables in Eq.(21)-(26) can be rewritten as,

$$F_\perp = 2\zeta\,(1+\xi)^2(1+R_\perp^2) \tag{40}$$
$$F_\| \mathsf{P}_1^2 = 2\zeta\,(1-\xi)^2(1+R_\|^2) \tag{41}$$
$$F_L \mathsf{P}_2^2 = 2\zeta\,(1-\xi)^2(1+R_0^2) \tag{42}$$
$$A_{\rm FB}\mathsf{P}_1 = 3\zeta\,(1-\xi^2)\big(R_\| + R_\perp\big) \tag{43}$$
$$\sqrt{2}A_5\mathsf{P}_2 = 3\zeta\,(1-\xi^2)\big(R_0 + R_\perp\big) \tag{44}$$

where $\mathsf{P}_1 = \dfrac{\mathcal{F}_\perp}{\mathcal{F}_\|}$, $\quad \mathsf{P}_2 = \dfrac{\mathcal{F}_\perp}{\mathcal{F}_0}$, $\quad \zeta = \dfrac{\mathcal{F}_\perp^2 C_{10}^2}{\Gamma_f}$,

$$R_\perp = \frac{\dfrac{r_\perp}{C_{10}} - \xi'}{1+\xi}, \; R_\| = \frac{\dfrac{r_\|}{C_{10}} + \xi'}{1-\xi}, \; R_0 = \frac{\dfrac{r_0}{C_{10}} + \xi'}{1-\xi}. \tag{45}$$

We consider the observables F_L, $F_\|$, $F_\perp$, $A_{\rm FB}$ and A_5, with the constraint $F_L + F_\| + F_\perp = 1$. Using Eq. (40)–(44), we obtain expressions for $R_\perp$, $R_\|$, R_0 and P_2 in terms of the observables and P_1:

$$R_\perp = \pm\frac{3}{2}\,\frac{\left(\frac{1-\xi}{1+\xi}\right)F_\perp + \frac{1}{2}\mathsf{P}_1 Z_1}{\mathsf{P}_1 A_{\rm FB}} \tag{46}$$
$$R_\| = \pm\frac{3}{2}\,\frac{\left(\frac{1+\xi}{1-\xi}\right)\mathsf{P}_1 F_\| + \frac{1}{2}Z_1}{A_{\rm FB}} \tag{47}$$
$$R_0 = \pm\frac{3}{2\sqrt{2}}\,\frac{\left(\frac{1+\xi}{1-\xi}\right)\mathsf{P}_2 F_L + \frac{1}{2}Z_2}{A_5} \tag{48}$$
$$\mathsf{P}_2 = \frac{\left(\frac{1-\xi}{1+\xi}\right)2\mathsf{P}_1 A_{\rm FB} F_\perp}{\sqrt{2}A_5\left(\left(\frac{1-\xi}{1+\xi}\right)2F_\perp + Z_1\mathsf{P}_1\right) - Z_2\mathsf{P}_1 A_{\rm FB}} \tag{49}$$

where $Z_1 = (4F_\| F_\perp - \frac{16}{9}A_{\rm FB}^2)^{\frac{1}{2}}$ and $Z_2 = (4F_L F_\perp - \frac{32}{9}A_5^2)^{\frac{1}{2}}$. Since we have one extra parameter compared to observables, all of the above expressions depend on P_1. Fortunately in the large q^2 limit, the relations between form factors enable us to eliminate one parameter.

At the kinematic limit $q^2 = q^2_{\rm max} = (m_B - m_{K^*})^2$ the K^* meson is at rest and the two leptons travel back to back in the B meson rest frame. There is no preferred direction in the decay kinematics. Hence, the differential decay distribution in this kinematic limit must be independent of the angles θ_ℓ and ϕ, which can be integrated out. This imposes constraints on the amplitude $A_\lambda^{L,R}$ and hence the observables. The entire decay, including the decay $K^* \to K\pi$ takes place in a single plane, resulting in a vanishing contribution to the '$\perp$' helicity, or $F_\perp = 0$.

Since the K^* decays at rest, the distribution of $K\pi$ is isotropic and cannot depend on θ_K. It can easily be seen that this is only possible if $F_\parallel = 2F_L$ [14].

At $q^2 = q^2_{\rm max}$, $\Gamma_f \to 0$ as all the transversity amplitudes vanish in this limit. The constraints on the amplitudes described above result in unique values of the helicity fractions and the asymmetries at this kinematical endpoint. The values of the helicity fractions and asymmetries were derived in Ref. [14, 6] where it is explicitly shown that

$$F_L(q^2_{\rm max}) = \frac{1}{3}, \quad F_\parallel(q^2_{\rm max}) = \frac{2}{3}, \quad A_4(q^2_{\rm max}) = \frac{2}{3\pi},$$
$$F_\perp(q^2_{\rm max}) = 0, \quad A_{\rm FB}(q^2_{\rm max}) = 0, \quad A_{5,7,8,9}(q^2_{\rm max}) = 0. \tag{50}$$

At low recoil energy of K^* meson, only three independent form factors describe the whole $B \to K^*\ell^+\ell^-$ decay and there exist a relation among the form factors at leading order in $1/m_B$ expansion given by [5, 15],

$$\frac{\widetilde{\mathcal{G}}_\parallel}{\mathcal{F}_\parallel} = \frac{\widetilde{\mathcal{G}}_\perp}{\mathcal{F}_\perp} = \frac{\widetilde{\mathcal{G}}_0}{\mathcal{F}_0} = -\kappa\frac{2m_b m_B C_7}{q^2}, \tag{51}$$

where $\kappa \approx 1$. The helicity independence of the ratios $\widetilde{\mathcal{G}}_\lambda/\mathcal{F}_\lambda$ at $q^2_{\rm max}$ is easy to understand, since both the B and K^* mesons are at rest, resulting in a complete overlap of the wave functions of these two mesons and the absence of any preferred direction in the $K\pi$ distribution. Hence at the maximum point in q^2, i.e. the kinematic endpoint $q^2_{\rm max}$, one gets from Eq.(13) $r_0 = r_\parallel = r_\perp \equiv r$. Therefore Eq. (45) implies that in the presence of RH currents one should expect $R_0 = R_\parallel \neq R_\perp$ at $q^2 = q^2_{\rm max}$ without any approximation. Interestingly, this relation is unaltered by non-factorizable and resonance contributions [14] at this kinematic endpoint.

To test the relation among R_λ's in light of LHCb data, first defining $\delta \equiv q^2_{\rm max} - q^2$, we expand the observables F_L, $F_\perp$, $A_{\rm FB}$ and A_5 around $q^2_{\rm max}$ as follows:

$$F_L = \frac{1}{3} + F_L^{(1)}\delta + F_L^{(2)}\delta^2 + F_L^{(3)}\delta^3, \tag{52}$$

$$F_\perp = F_\perp^{(1)}\delta + F_\perp^{(2)}\delta^2 + F_\perp^{(3)}\delta^3, \tag{53}$$

$$A_{\rm FB} = A_{\rm FB}^{(1)}\delta^{\frac{1}{2}} + A_{\rm FB}^{(2)}\delta^{\frac{3}{2}} + A_{\rm FB}^{(3)}\delta^{\frac{5}{2}}, \tag{54}$$

$$A_5 = A_5^{(1)}\delta^{\frac{1}{2}} + A_5^{(2)}\delta^{\frac{3}{2}} + A_5^{(3)}\delta^{\frac{5}{2}}. \tag{55}$$

where for each observable O, $O^{(n)}$ is the coefficient of the $n^{\rm th}$ term in the expansion. The polynomial fit to data is not based on Heavy Quark Effective Theory (HQET) or any other theoretical assumption. A parametric fit to data is performed, so as to obtain the limiting values of the coefficients to determine the slope and second derivative of the observables at $q^2_{\rm max}$. It should be noted that the polynomial parameterizations are inadequate to describe the q^2 dependent behavior of resonances. However, a very thorough discussion on the systematics of the resonance effects can be found in [16].

The relation in Eq. (51) between form factors is expected to be satisfied in the large q^2 region. Eq. (51) is naturally satisfied if it is valid at each order in the Taylor expansion of the form factors:

$$q^2\frac{\widetilde{\mathcal{G}}_\lambda}{\mathcal{F}_\lambda} = q^2_{\rm max}\frac{\widetilde{\mathcal{G}}_\lambda^{(1)} + \delta\,(\widetilde{\mathcal{G}}_\lambda^{(2)} - \dfrac{\widetilde{\mathcal{G}}_\lambda^{(1)}}{q^2_{\rm max}}) + \mathcal{O}(\delta^2)}{\mathcal{F}_\lambda^{(1)} + \delta\,\mathcal{F}_\lambda^{(2)} + \mathcal{O}(\delta^2)}. \tag{56}$$

We require only that the relation be valid up to order δ. In order for Eq. (56) to have a constant value in the neighborhood of $q^2_{\rm max}$ up to $\mathcal{O}(\delta)$, we must have $\mathcal{F}^{(2)}_\lambda = c\,\mathcal{F}^{(1)}_\lambda$ and $(q^2_{\rm max}\,\widetilde{\mathcal{G}}^{(2)}_\lambda - \widetilde{\mathcal{G}}^{(1)}_\lambda) = c\,q^2_{\rm max}\,\widetilde{\mathcal{G}}^{(1)}_\lambda$ where c is any constant. As discussed earlier, $\mathsf{P}_2 = \sqrt{2}\mathsf{P}_1$ at $q^2_{\rm max}$, hence, we must have $\mathsf{P}^{(1)}_2 = \sqrt{2}\mathsf{P}^{(1)}_1$, where $\mathsf{P}^{(1)}_{1,2}$ are the coefficients of the leading $\mathcal{O}(\sqrt{\delta})$ term in the expansion. However, the above argument implies that at the next order, we must also have $\mathsf{P}^{(2)}_2 = \sqrt{2}\mathsf{P}^{(2)}_1$, since $\mathcal{F}^{(2)}_\lambda = c\,\mathcal{F}^{(1)}_\lambda$. This provides the needed input that together with Eq. (49) determines $\mathsf{P}^{(1)}_1$ purely in terms of observables.

The zeroth order coefficients of the observable expansions are assumed from the constraints arising from Lorentz invariance and decay kinematics derived in Ref. [14], whereas all the higher order coefficients are extracted by fitting the polynomials with 14 bin LHCb data as shown in Fig. 2.

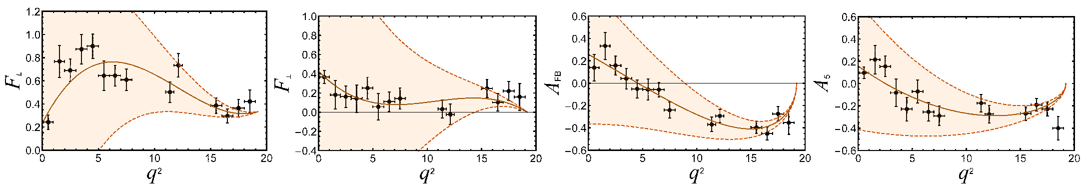

Figure 2: An analytic fit to 14-bin LHCb data using Taylor expansion at $q^2_{\rm max}$ for the observables F_L, $F_\perp$, $A_{\rm FB}$ and A_5 are shown as the brown curves. The $\pm1\sigma$ error bands are indicated by the light brown shaded regions, derived including correlation among all observables. The points with the black error bars are LHCb 14-bin measurements [11].

The limiting analytic expressions for R_λ at $q^2 = q^2_{\rm max}$ are

$$R_\perp(q^2_{\rm max}) = \frac{8A^{(1)}_{\rm FB}(-2A^{(2)}_5 + A^{(2)}_{\rm FB}) + 9(3F^{(1)}_L + F^{(1)}_\perp)F^{(1)}_\perp}{8\,(2A^{(2)}_5 - A^{(2)}_{\rm FB})\sqrt{\frac{3}{2}F^{(1)}_\perp - A^{(1)\,2}_{\rm FB}}} = \frac{\omega_2 - \omega_1}{\omega_2\sqrt{\omega_1 - 1}}, \tag{57}$$

$$R_\parallel(q^2_{\rm max}) = \frac{3(3F^{(1)}_L + F^{(1)}_\perp)\sqrt{\frac{3}{2}F^{(1)}_\perp - A^{(1)\,2}_{\rm FB}}}{-8A^{(2)}_5 + 4A^{(1)}_{\rm FB} + 3A^{(1)}_{\rm FB}(3F^{(1)}_L + F^{(1)}_\perp)} = \frac{\sqrt{\omega_1 - 1}}{\omega_2 - 1} = R_0(q^2_{\rm max}) \tag{58}$$

where

$$\omega_1 = \frac{3}{2}\frac{F^{(1)}_\perp}{A^{(1)\,2}_{\rm FB}} \quad \text{and} \quad \omega_2 = \frac{4\,(2A^{(2)}_5 - A^{(2)}_{\rm FB})}{3\,A^{(1)}_{\rm FB}(3F^{(1)}_L + F^{(1)}_\perp)}. \tag{59}$$

It should be noted that Eqs. (57)–(59) are derived only at $q^2_{\rm max}$. However, even at the endpoint, the expressions depend on polynomial coefficients: $F^{(1)}_L$ and $F^{(1)}_\perp$ as well as $A^{(2)}_{\rm FB}$ and $A^{(2)}_5$ which are not related by HQET. Hence, in our approach, corrections beyond HQET are automatically incorporated through fits to data.

In the absence of RH currents or other NP that treats the "⊥" amplitude differently one would expect $R_\perp(q^2_{\rm max}) = R_\parallel(q^2_{\rm max}) = R_0(q^2_{\rm max})$. It is easily seen that the LHS of Eq. (43) is positive around $q^2_{\rm max}$ and since $\zeta > 0$, we must have $R_\perp = R_\parallel = R_0 > 0$. Since very large contributions from RH currents are not possible, as they would have been seen elsewhere, $R_\lambda(q^2_{\rm max}) > 0$ still holds and restricts ξ and ξ' to reasonably small values.

It can be seen that ω_1, ω_2 contain coefficients which are extracted completely from data and their estimates using LHCb measurements are: $\omega_1 = 1.10 \pm 0.30$ (1.03 ± 0.34) and $\omega_2 = -4.19 \pm 10.48$ (-4.04 ± 10.12), where the first values are determined using $A^{(1)}_{\rm FB}$ and the values in the round brackets use $2A^{(1)}_5$. The variables R_λ's can be estimated using data only and the allowed region is shown in gray bands in Fig. 3 left panel. A significant deviation is seen from a slope of 45° line (red line) which denotes $R_\perp = R_\parallel = R_0$ and thus hints toward the presence of RH currents without *using any estimate of hadronic contributions*. To quantify the RH couplings, we use Eq. (45) and the results are shown in the last two panels of Fig. 3. The middle panel uses the SM estimate of parameter r/C_{10} [15] and the SM prediction for C'_{10}/C_{10} and C'_9/C_{10} (the origin) is at more than 5σ confidence level. We have performed another analysis where the input r/C_{10} is considered as nuisance parameter and the result is shown in the right most panel of Fig. 3. It can be seen that the uncertainties in fitted parameters C'_{10}/C_{10} and C'_9/C_{10} have increased due to the variation of r/C_{10} and the SM prediction still remains on a 3σ level contour providing evidence of RH currents.

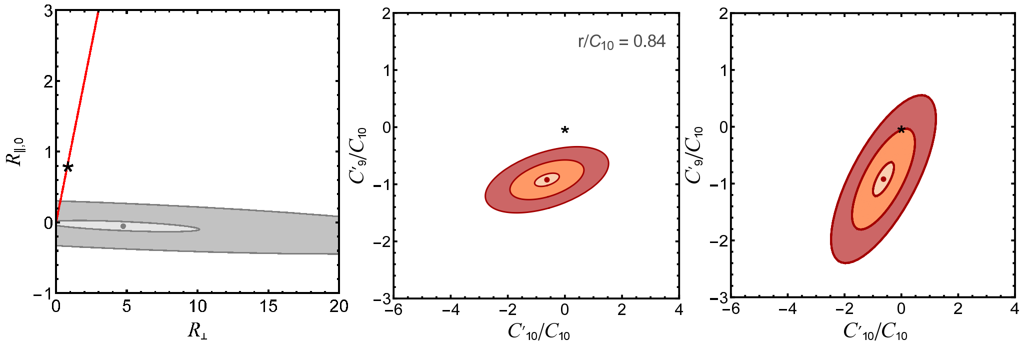

Figure 3: (left panel) Allowed regions in $R_\perp - R_{\parallel,0}$ plane are shown in light and dark gray bands at 1σ and 5σ confidence level, respectively. The red straight line corresponds to the case $R_\perp = R_{\parallel,0}$ i.e. the absence of RH couplings. (middle panel) In $C'_{10}/C_{10} - C'_9/C_{10}$ plane, the yellow, orange and red regions correspond to 1σ, 3σ and 5σ significance level, respectively, where SM input for r/C_{10} [15] is used. The best fit values of C'_{10}/C_{10} and C'_9/C_{10}, with $\pm1\sigma$ errors are -0.63 ± 0.43 and -0.92 ± 0.10, respectively. (right panel) Same color code as the middle panel figure. The input r/C_{10} is varied as a nuisance parameter and hence the obtained uncertainties in C'_{10}/C_{10} and C'_9/C_{10} are increased. The SM predictions for all the three plots are indicated by the stars. Strong evidence of RH current is pronounced from the plots.

Conclusions

- The $B \to K^*\ell^+\ell^-$ mode is an excellent mode to study. It is sensitive to NP and indirect evidences have been found.
- Theoretical issues are well understood. However, the resonance effects are very important to be studied thoroughly.
- Non-local contributions are difficult to estimate but can be handled by eliminating them in terms of observables.
- More efforts should be put to estimate them. One should look for experimental hints to estimate how large they are.
- Observable relations, which are free from hadronic uncertainties, can be used as a probe for New Physics. Discrepancies are found with SM.
- A strong evidence of Right-Handed currents is also found.
- The possibility of some other kinds of NP are also suggested.

Acknowledgment

We acknowledge our coauthors T. E. Browder and Anirban Karan for the work done on Right-Handed currents.

References

[1] C. Bobeth, M. Misiak and J. Urban, Nucl. Phys. B **574**, 291 (2000), [arXiv:hep-ph/9910220].

[2] W. Altmannshofer, P. Ball, A. Bharucha *et al.*, JHEP **01**,(2009) 019, [arXiv:0811.1214].

[3] G. Buchalla, A. J. Buras, M. E. Lautenbacher, Rev. Mod. Phys. **68**, 1125-1144 (1996), [arXiv:hep-ph/9512380].

[4] A. Khodjamirian, Th.Mannel, A. A. Pivovarov and Y. -M. Wang, JHEP **09**, (2010) 089, [arXiv:1006.4945];

A. Khodjamirian, arXiv:1312.6480.

[5] B. Grinstein and Dan Prijol, Phys. Rev. D **70** 114005 (2004), [arXiv:hep-ph/0404250v3].

[6] R. Mandal, R. Sinha and D. Das, Phys. Rev. D **90**, no. 9, 096006 (2014)

[7] F. Kruger, L. M. Sehgal, N. Sinha and R. Sinha, Phys. Rev. **D 61**, 114028 (2000). [arXiv:hep-ph/9907386].

[8] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. **B612**, 25 (2001), [arXiv:hep-ph/0106067].

[9] B. Grinstein, M. J. Savage and M. B. Wise, Nucl. Phys. B**319**, 271 (1989).

[10] R. Mandal and R. Sinha, Phys. Rev. D **95**, no. 1, 014026 (2017) doi:10.1103/PhysRevD.95.014026 [arXiv:1506.04535 [hep-ph]].

[11] R. Aaij *et al.* [LHCb Collaboration], arXiv:1606.04731 [hep-ex].

[12] W. Altmannshofer and D. M. Straub, Eur. Phys. J. C **75**, no. 8, 382 (2015); M. Ciuchini *et al.* JHEP **1606**, 116 (2016). S. Jäger and J. Martin Camalich, Phys. Rev. D **93** (2016) 1, 014028; S. Descotes-Genon, J. Matias and J. Virto, Phys. Rev. D **88**, 074002 (2013) [arXiv:1307.5683 [hep-ph]]. S. Descotes-Genon, L. Hofer, J. Matias and J. Virto, JHEP **1606**, 092 (2016) and references therein.

[13] C. Bobeth, G. Hiller and D. van Dyk, Phys. Rev. D **87** (2013) 3, 034016

[14] G. Hiller and Roman Ziwcky, JHEP **1403**,(2014) 042, [arxiv:1312.1923].

[15] C. Bobeth, G. Hiller and D. van Dyk, JHEP **1007**, 098 (2010).

[16] A. Karan, R. Mandal, A. K. Nayak, R. Sinha and T. E. Browder, arXiv:1603.04355 [hep-ph].