

Spectroscopy and Regge Trajectories of Light Mesons

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Masses of light tetraquarks are obtained in the diquark-antidiquark picture. Such exotic states can be identified with the known light scalar mesons forming an $SU(3)$ nonet (σ meson $f_0(600)$, κ meson $K_0^*(800)$, $f_0(980)$ and $a_0(980)$). Mass spectra of light (u, d, s) quark-antiquark mesons are calculated within the QCD-motivated relativistic quark model. The relativistic treatment of the light quark dynamics results in mass spectra which agree well with available experimental data for the masses of the most well-established states. Their Regge trajectories exhibit linearity and equidistance.

The consistent theoretical understanding of the light meson sector remains an important problem already for many years [1]. An extensive analysis of the data on highly excited light non-strange meson states up to a mass of 2400 MeV collected by Crystal Barrel experiment at LEAR (CERN) has been published. Classification of these new data requires better theoretical description of light meson mass spectra. This is especially important, since light exotic states (such as tetraquarks, glueballs, hybrids) within quantum chromodynamics (QCD) are expected to have masses in this range. Particular interest is focused on scalar mesons, their properties and abundance. A generally accepted consistent picture has not yet emerged. Experimental and theoretical evidence for the existence of $f_0(600)(\sigma)$, $K_0^*(800)(\kappa)$, $f_0(980)$ and $a_0(980)$ indicates that lightest scalars form a complete $SU(3)$ flavour nonet. A peculiar feature of their mass spectrum is the inversion of the mass ordering, which cannot be naturally understood in the $q\bar{q}$ picture. This fact stimulated various alternative interpretations of light scalars as four quark states (tetraquarks) in particular diquark-antidiquark bound states. The proximity of f_0/a_0 to the $K\bar{K}$ threshold led to the $K\bar{K}$ molecular picture.

In the quasipotential approach and diquark-antidiquark picture of tetraquarks the interaction of two quarks in a diquark and the diquark-antidiquark interaction in a tetraquark are described by the diquark wave function (Ψ_d) of the bound quark-quark state and by the tetraquark wave function (Ψ_T) of the bound diquark-antidiquark state, respectively. These wave functions satisfy the quasipotential equation of the Schrödinger type [2]

$$\left(\frac{b^2(M)}{2\mu_R} - \frac{\mathbf{p}^2}{2\mu_R} \right) \Psi_{d,T}(\mathbf{p}) = \int \frac{d^3q}{(2\pi)^3} V(\mathbf{p}, \mathbf{q}; M) \Psi_{d,T}(\mathbf{q}), \quad (1)$$

where the relativistic reduced mass is

$$\mu_R = \frac{E_1 E_2}{E_1 + E_2} = \frac{M^4 - (m_1^2 - m_2^2)^2}{4M^3}, \quad (2)$$

State J^{PC}	Diquark content	Theory mass	Experiment			
			$I = 0$	mass	$I = 1$	mass
$(qq)(\bar{q}\bar{q})$						
0^{++}	$S\bar{S}$	596	$f_0(600) (\sigma)$	400-1200		-
$1^{+\pm}$	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	672				
0^{++}	$A\bar{A}$	1179	$f_0(1370)$	1200-1500		
1^{+-}	$A\bar{A}$	1773				
2^{++}	$A\bar{A}$	1915	$\begin{cases} f_2(1910) \\ f_2(1950) \end{cases}$	$\begin{cases} 1903(9) \\ 1944(12) \end{cases}$		
$(qs)(\bar{q}\bar{s})$						
0^{++}	$S\bar{S}$	992	$f_0(980)$	980(10)	$a_0(980)$	984.7(12)
1^{++}	$(S\bar{A} + \bar{S}A)/\sqrt{2}$	1201	$f_1(1285)$	1281.8(6)	$a_1(1260)$	1230(40)
1^{+-}	$(S\bar{A} - \bar{S}A)/\sqrt{2}$	1201	$h_1(1170)$	1170(20)	$b_1(1235)$	1229.5(32)
0^{++}	$A\bar{A}$	1480	$f_0(1500)$	1505(6)	$a_0(1450)$	1474(19)
1^{+-}	$A\bar{A}$	1942	$h_1(1965)$	1965(45)	$b_1(1960)$	1960(35)
2^{++}	$A\bar{A}$	2097	$\begin{cases} f_2(2010) \\ f_2(2140) \end{cases}$	$\begin{cases} 2011(70) \\ 2141(12) \end{cases}$	$\begin{cases} a_2(1990) \\ a_2(2080) \end{cases}$	$\begin{cases} 2050(45) \\ 2100(20) \end{cases}$
$(ss)(\bar{s}\bar{s})$						
0^{++}	$A\bar{A}$	2203	$f_0(2200)$	2189(13)		-
1^{+-}	$A\bar{A}$	2267	$h_1(2215)$	2215(40)		-
2^{++}	$A\bar{A}$	2357	$f_2(2340)$	2339(60)		-

Table 1: Masses of light unflavored diquark-antidiquark ground state ($\langle \mathbf{L}^2 \rangle = 0$) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

and E_1, E_2 are given by

$$E_1 = \frac{M^2 - m_2^2 + m_1^2}{2M}, \quad E_2 = \frac{M^2 - m_1^2 + m_2^2}{2M}. \quad (3)$$

Here, $M = E_1 + E_2$ is the bound-state mass (diquark or tetraquark), $m_{1,2}$ are the masses of quarks ($q = u, d$ and s) which form the diquark or of the diquark (d) and antidiquark (\bar{d}') which form the light tetraquark (T), and \mathbf{p} is their relative momentum. In the center-of-mass system the relative momentum squared on mass shell reads

$$b^2(M) = \frac{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}{4M^2}. \quad (4)$$

The kernel $V(\mathbf{p}, \mathbf{q}; M)$ in Eq. (1) is the quasipotential operator of the quark-quark or diquark-antidiquark interaction. It is constructed with the help of the off-mass-shell scattering amplitude, projected onto the positive-energy states. For the quark-quark interaction in a diquark we use the relation $V_{qq} = V_{q\bar{q}}/2$ arising under the assumption of an octet structure of the interaction from the difference in the qq and $q\bar{q}$ colour states. An important role in this construction is played by the Lorentz structure of the confining interaction. In our analysis of mesons, while constructing the quasipotential of the quark-antiquark interaction, we assumed that the effective interaction is the sum of the usual one-gluon exchange term and a mixture of long-range vector and scalar linear confining potentials, where the vector confining potential

State J^P	Diquark content	Theory mass	Experiment	
			$I = \frac{1}{2}$	mass
$(qq)(\bar{s}\bar{q})$ or $(sq)(\bar{q}\bar{q})$				
0^+	$S\bar{S}$	730	$K_0^*(800) (\kappa)$	672(40)
1^+	$(S\bar{A} \pm \bar{S}A)/\sqrt{2}$	1057		
0^+	$A\bar{A}$	1332	$K_0^*(1430)$	1425(50)
1^+	$A\bar{A}$	1855		
2^+	$A\bar{A}$	2001	$K_2^*(1980)$	1973(26)

Table 2: Masses of strange diquark-antidiquark ground state ($\langle \mathbf{L}^2 \rangle = 0$) tetraquarks (in MeV) and possible experimental candidates. S and A denote scalar and axial vector diquarks.

contains the Pauli term. We use the same conventions for the construction of the quark-quark and diquark-antidiquark interactions in the tetraquark.

At the first step, we take the masses and form factors of the light diquarks from the previous consideration of light diquarks in heavy baryons. At the second step, we calculate the masses of light tetraquarks considered as the bound states of a light diquark and antidiquark.

The obtained mass spectra of ground state light tetraquarks are presented in Tables 1 and 2. We see that the diquark-antidiquark picture can provide a natural explanation for the inversion of masses of light scalar 0^+ mesons. Indeed all lightest experimentally observed scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be interpreted in our model as light tetraquarks composed from a scalar diquark and antidiquark ($S\bar{S}$). Therefore, the $f_0(980)$ and $a_0(980)$ tetraquarks contain, in comparison to the $q\bar{q}$ picture, an additional pair of strange quarks which gives a natural explanation why their masses are heavier than the strange $K_0^*(800)$ (κ).

The other scalar tetraquark states can be composed from an axial vector diquark and antidiquark ($A\bar{A}$). Their masses are predicted to be approximately 600 MeV heavier than the $S\bar{S}$ tetraquarks (S is a scalar diquark). The diquark-antidiquark composition also naturally explains the experimentally observed proximity of masses of the unflavored $a_0(1450)$, $f_0(1500)$ and strange $K_0^*(1430)$ scalars.

The calculated masses of light unflavoured and strange ($q\bar{q}$) mesons are given in Tables 3 and 4 taken from Ref. [3]. As already discussed above, the scalar sector presents a special interest due to its complexity and the abundance of experimentally observed light states. We see from Table 3 that the masses of the lightest $q\bar{q}$ scalar mesons have values about 1200 MeV. This confirms our conclusion [2] that light scalar mesons, $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$, with masses below 1 GeV should be described as light tetraquarks consisting from scalar diquark and antidiquark. Moreover the predicted masses of the scalar tetraquarks composed from axial-vector diquark and antidiquark [2] have masses in the same range as the lowest $q\bar{q}$ scalar mesons. The obtained results for the masses indicate that $a_0(1450)$ should be predominantly a tetraquark state which predicted [2] mass 1480 MeV is within experimental error bars of $M_{a_0(1450)} = 1474 \pm 19$ MeV. The exotic scalar state $X(1420)$ from the ‘‘Further States’’ Section [4] could be its isotensor partner. On the other hand $s\bar{q}(1^3P_0)$ interpretation is favored for $K_0^*(1430)$ (see Table 4). This picture naturally explains the experimentally observed proximity of masses of the unflavored $a_0(1450)$ and $f_0(1500)$ with the strange $K_0^*(1430)$. Therefore one could expect an additional predominantly $q\bar{q}$ isovector state a_0 with the mass about 1200 MeV, though it was not observed in several experiments.

Some of the Regge trajectories both in (M^2, J) and (M^2, n_r) planes (M is the mass, J is the spin and n_r is the radial quantum number of the meson state) are shown in Figs. 1-7.

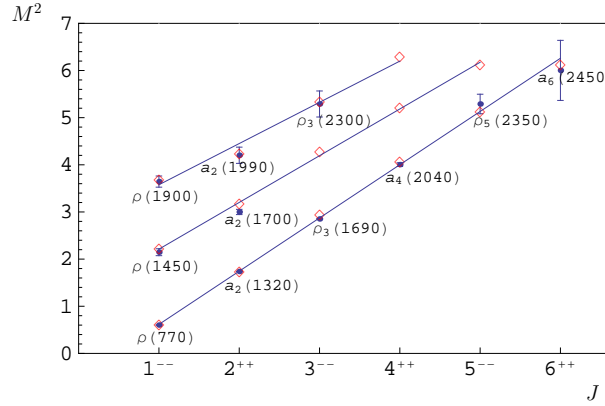
$n^{2S+1}L_J$	J^{PC}	Theory	Experiment				Theory	Experiment	
		$q\bar{q}$	$I = 1$	mass	$I = 0$	mass	$s\bar{s}$	$I = 0$	mass
1^1S_0	0^{-+}	154	π	139.57			743		
1^3S_1	1^{--}	776	ρ	775.49(34)	ω	782.65(12)	1038	φ	1019.455(20)
1^3P_0	0^{++}	1176	a_0	1474(19)	f_0	1200-1500	1420	f_0	1505(6)
1^3P_1	1^{++}	1254	a_1	1230(40)	f_1	1281.8(6)	1464	f_1	1426.4(9)
1^3P_2	2^{++}	1317	a_2	1318.3(6)	f_2	1275.1(12)	1529	f'_2	1525(5)
1^1P_1	1^{+-}	1258	b_1	1229.5(32)	h_1	1170(20)	1485	h_1	1386(19)
2^1S_0	0^{-+}	1292	π	1300(100)	η	1294(4)	1536	η	1476(4)
2^3S_1	1^{--}	1486	ρ	1465(25)	ω	1400-1450	1698	φ	1680(20)
1^3D_1	1^{--}	1557	ρ	1570(70)	ω	1670(30)	1845		
1^3D_2	2^{--}	1661					1908		
1^3D_3	3^{--}	1714	ρ_3	1688.8(21)	ω_3	1667(4)	1950	φ_3	1854(7)
1^1D_2	2^{-+}	1643	π_2	1672.4(32)	η_2	1617(5)	1909	η_2	1842(8)
2^3P_0	0^{++}	1679			f_0	1724(7)	1969		
2^3P_1	1^{++}	1742	a_1	1647(22)			2016	f_1	1971(15)
2^3P_2	2^{++}	1779	a_2	1732(16)	f_2	1755(10)	2030	f_2	2010(70)
2^1P_1	1^{+-}	1721					2024		
3^1S_0	0^{-+}	1788	π	1816(14)	η	1756(9)	2085	η	2103(50)
3^3S_1	1^{--}	1921	ρ	1909(31)	ω	1960(25)	2119	φ	2175(15)
1^3F_2	2^{++}	1797			f_2	1815(12)	2143	f_2	2156(11)
1^3F_3	3^{++}	1910	a_3	1874(105)			2215	f_3	2334(25)
1^3F_4	4^{++}	2018	a_4	2001(10)	f_4	2018(11)	2286		
1^1F_3	3^{+-}	1884					2209	h_3	2275(25)
2^3D_1	1^{--}	1895	ρ	1909(31)			2258	ω	2290(20)
2^3D_2	2^{--}	1983	ρ_2	1940(40)	ω_2	1975(20)	2323		
2^3D_3	3^{--}	2066					2338		
2^1D_2	2^{-+}	1960	π_2	1974(84)	η_2	2030(20)	2321		
3^3P_0	0^{++}	1993	a_0	2025(30)	f_0	1992(16)	2364	f_0	2314(25)
3^3P_1	1^{++}	2039	a_1	2096(123)			2403		
3^3P_2	2^{++}	2048	a_2	2050(42)	f_2	2001(10)	2412	f_2	2339(60)
3^1P_1	1^{+-}	2007	b_1	1960(35)	h_1	1965(45)	2398		
4^1S_0	0^{-+}	2073	π	2070(35)	η	2010(50)	2439		
4^3S_1	1^{--}	2195	ρ	2265(40)	ω	2205(30)	2472		
1^3G_3	3^{--}	2002	ρ_3	1982(14)	ω_3	1945(20)	2403		
1^3G_4	4^{--}	2122	ρ_4	2230(25)	ω_4	2250(30)	2481		
1^3G_5	5^{--}	2264	ρ_5	2300(45)	ω_5	2250(70)	2559		
1^1G_4	4^{-+}	2092					2469		
3^3D_1	1^{--}	2168	ρ	2149(17)			2607		
3^3D_2	2^{--}	2241	ρ_2	2225(35)	ω_2	2195(30)	2667		
3^3D_3	3^{--}	2309	ρ_3	2300(60)	ω_3	2278(28)	2727		
3^1D_2	2^{-+}	2216	π_2	2245(60)	η_2	2248(20)	2662		
2^3F_2	2^{++}	2091	a_2	2100(20)	f_2	2141(12)	2514		

Table 3: Masses of excited light unflavored mesons (in MeV).

SPECTROSCOPY AND REGGE TRAJECTORIES OF LIGHT MESONS

$n^{2S+1}L_J$	J^{PC}	Theory		Experiment		Theory		Experiment	
		$q\bar{q}$	$I = 1$	mass	$I = 0$	mass	$s\bar{s}$	$I = 0$	mass
2^3F_3	3^{++}	2191	a_3	2070(20)			2585		
2^3F_4	4^{++}	2284			f_4	2320(60)	2657		
2^1F_3	3^{+-}	2164	b_3	2245(50)			2577		
4^3P_0	0^{++}	2250			f_0	2189(13)	2699		
4^3P_1	1^{++}	2286	a_1	2270(50)	f_1	2310(60)	2729		
4^3P_2	2^{++}	2297	a_2	2280(30)	f_2	2297(28)	2734		
4^1P_1	1^{+-}	2264	b_1	2240(35)	h_1	2215(40)	2717		
2^3G_3	3^{--}	2267	ρ_3	2260(20)	ω_3	2255(15)	2743		
2^3G_4	4^{--}	2375					2819		
2^3G_5	5^{--}	2472					2894		
2^1G_4	4^{-+}	2344	π_4	2250(15)	η_4	2328(30)	2806		
5^1S_0	0^{-+}	2385	π	2360(25)	η	2320(15)	2749		
5^3S_1	1^{--}	2491					2782		
1^3H_4	4^{++}	2234	a_4	2237(5)	f_J	2231.1(35)	2634		
1^3H_5	5^{++}	2359					2720		
1^3H_6	6^{++}	2475	a_6	2450(130)	f_6	2465(50)	2809		
1^1H_5	5^{+-}	2328					2706		

Table 3: Masses of excited light unflavored mesons (in MeV)(continued).


 Figure 1: Parent and daughter (J, M^2) Regge trajectories for isovector light mesons with natural parity (ρ). Diamonds are predicted masses. Available experimental data are given by dots with error bars and particle names. M^2 is in GeV^2 .

We see that the calculated light meson masses fit nicely to the linear trajectories. These trajectories are almost parallel and equidistant. It is important to note that the quality of fitting the π meson Regge trajectories is significantly improved if the ground state π is excluded from the fit. In the kaon case omitting the ground state also improves the fit but not so dramatically as for the pion. The corresponding trajectories are shown in Figs. 4 and 5 by dashed lines. This indicates the special role of the pion originating from the chiral symmetry breaking. The fitted

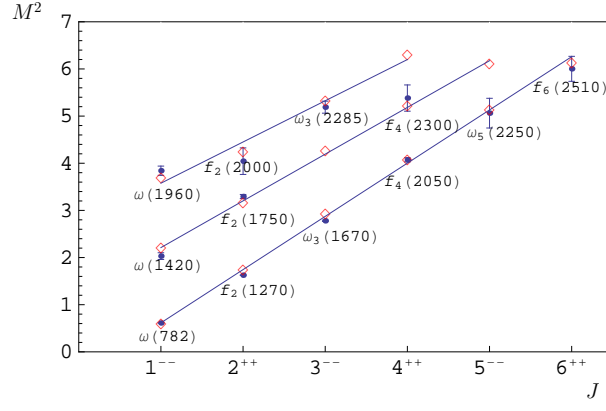


Figure 2: Same as in Fig. 1 for isoscalar light $q\bar{q}$ mesons with natural parity (ω).

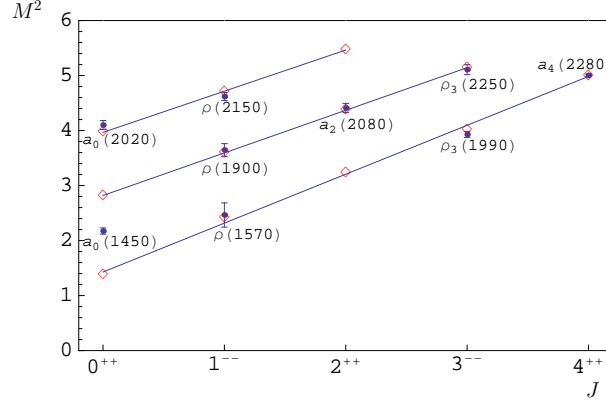


Figure 3: Same as in Fig. 1 for isovector light $q\bar{q}$ mesons with natural parity (a_0).

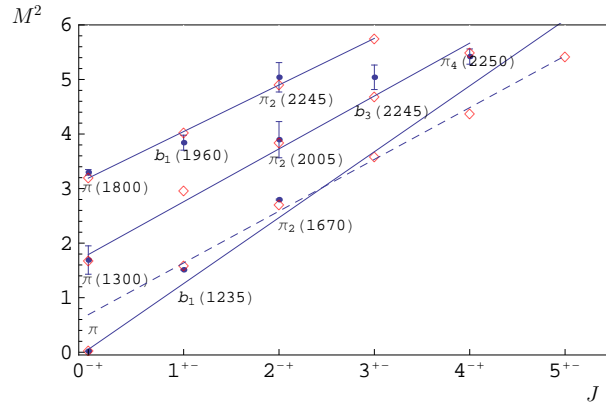


Figure 4: Same as in Fig. 1 for isovector light mesons with unnatural parity (π). Dashed line corresponds to the Regge trajectory, fitted without π .

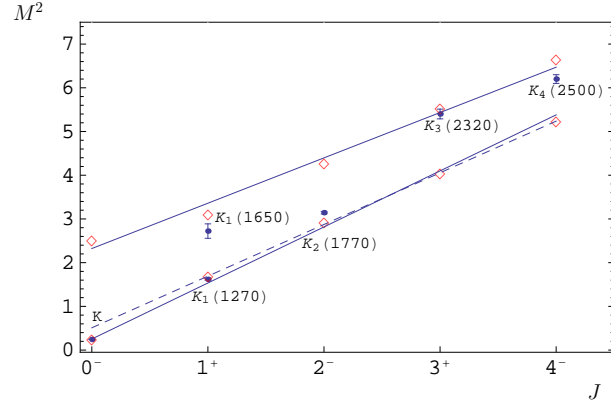


Figure 5: Same as in Fig. 1 for isodoublet light mesons with unnatural parity (K). Dashed line corresponds to the Regge trajectory, fitted without K .

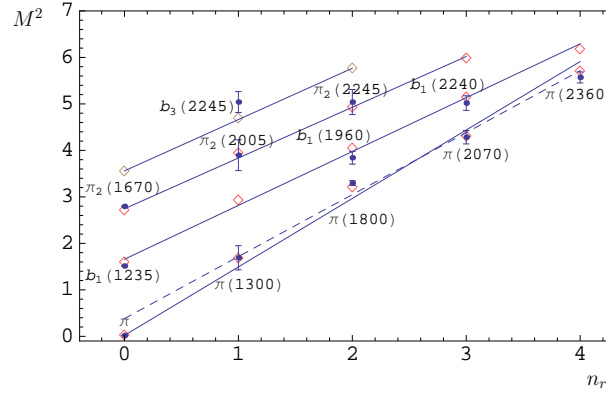


Figure 6: The (n_r, M^2) Regge trajectories for spin-singlet isovector mesons π , b_1 , π_2 and b_3 (from bottom to top). Notations are the same as in Fig. 1. The dashed line corresponds to the Regge trajectory, fitted without π .

$n^{2S+1}L_J$	Theory		Experiment		$n^{2S+1}L_J$	Theory		Experiment	
	J^P	$q\bar{s}$	$I = 1/2$	mass		J^P	$q\bar{s}$	$I = 1/2$	mass
1^1S_0	0^-	482	K	493.677(16)	3^1S_0	0^-	2065		
1^3S_1	1^-	897	K^*	891.66(26)	3^3S_1	1^-	2156		
1^3P_0	0^+	1362	K_0	1425(50)	2^3D_1	1^-	2063		
1^3P_2	2^+	1424	K_2^*	1425.6(15)	2^3D_3	3^-	2182		
$1P_1$	1^+	1412	K_1	1403(7)	$2D_2$	2^-	2163	K_2	2247(17)
$1P_1$	1^+	1294	K_1	1272(7)	$2D_2$	2^-	2066		
2^1S_0	0^-	1538			3^3P_0	0^+	2160		
2^3S_1	1^-	1675	K^*		3^3P_2	2^+	2206		
1^3D_1	1^-	1699	K^*	1717(27)	$3P_1$	1^+	2200		
1^3D_3	3^-	1789	K_3^*	1776(7)	$3P_1$	1^+	2164		
$1D_2$	2^-	1824	K_2	1816(13)	1^3G_3	3^-	2207		
$1D_2$	2^-	1709	K_2	1773(8)	1^3G_5	5^-	2356	K_5^*	2382(24)
2^3P_0	0^+	1791			$1G_4$	4^-	2285		
2^3P_2	2^+	1896			$1G_4$	4^-	2255		
$2P_1$	1^+	1893			2^3F_4	4^+	2436		
$2P_1$	1^+	1757	K_1	1650(50)	$2F_3$	3^+	2348	K_3	2324(24)
1^3F_2	2^+	1964	K_2^*	1973(26)	2^3G_5	5^-	2656		
1^3F_4	4^+	2096	K_4^*	2045(9)	$2G_4$	4^-	2575	K_4	2490(20)
$1F_3$	3^+	2080							
$1F_3$	3^+	2009							

Table 4: Masses of excited strange mesons (in MeV).

slopes and intercepts of the Regge trajectories are given in Tables 5 and 6.

It can be seen in Figs. 1 and 3 that $\rho(1700)$ and $a_0(1450)$ do not lie on the corresponding Regge trajectories. This further confirms our previous conclusion that $a_0(1450)$ should be predominantly a tetraquark state and suggests the possible exotic nature of $\rho(1700)$.

From the comparison of the slopes in Tables 5, 6 we see that the α values are systematically larger than the β ones. The ratio of their mean values is about 1.3 both for the light $q\bar{q}$ isovector and $s\bar{s}$ mesons. Such ratio is lower than predictions of the QCD string model $\alpha/\beta = 2$ and massless Salpeter equation $\alpha/\beta = \pi/2$ [3]. The mean value of the slope $\beta \sim 0.85 \text{ GeV}^{-2}$ for isovector mesons is about two times larger than the result of the above mentioned models, namely, $\beta = 1/(4\pi A) \approx 0.44 \text{ GeV}^{-2}$, for $A = 0.18 \text{ GeV}^2$ used in our approach.

The assignment of the experimentally observed states to the corresponding Regge trajectories in our model based on their masses and J^{PC} values (see Figs. 1-7) is slightly different from the previous phenomenological analysis [5, 6] based on the equal values for the slopes α and β . However the number of states, for which such correspondence is found, is approximately the same. Future experimental data will shed further light on this issue.

In summary, we presented the masses of the ground state light tetraquarks in the diquark-antidiquark picture, the mass spectra and Regge trajectories of light $q\bar{q}$ mesons. In distinction with previous phenomenological treatments, we used the dynamical approach based on the relativistic quark model. Both diquark and tetraquark masses were obtained by numerical solution of the quasipotential wave equations. The diquark structure was taken into account by using diquark-gluon form factors in terms of diquark wave functions. It is important to

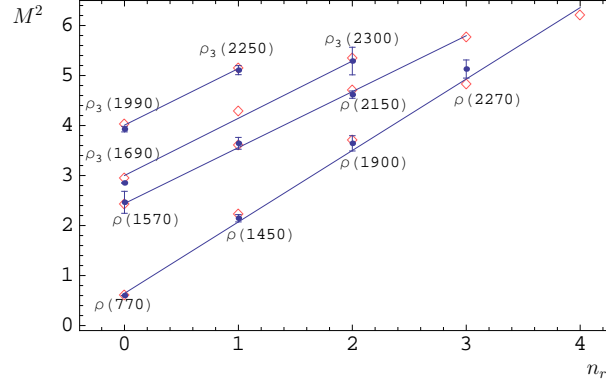


Figure 7: The (n_r, M^2) Regge trajectories for spin-triplet isovector mesons $\rho(^3S_1)$, $\rho(^3D_1)$, $\rho_3(^3D_3)$ and $\rho_3(^3G_3)$ (from bottom to top). Notations are the same as in Fig. 1.

Trajectory	natural parity		unnatural parity	
	α (GeV^{-2})	α_0	α (GeV^{-2})	α_0
$q\bar{q}$	ρ		π	
parent	0.887 ± 0.008	0.456 ± 0.018	$0.828 \pm 0.057^*$	$-0.025 \pm 0.034^*$
daughter 1	1.009 ± 0.019	-1.232 ± 0.074	1.031 ± 0.063	-1.846 ± 0.217
daughter 2	1.144 ± 0.113	-3.092 ± 0.540	1.171 ± 0.009	-3.737 ± 0.042
$q\bar{q}$	a_0		a_1	
parent	1.125 ± 0.035	-1.607 ± 0.104	1.014 ± 0.036	-0.658 ± 0.120
daughter 1	1.291 ± 0.003	-3.640 ± 0.011	1.148 ± 0.012	-2.497 ± 0.050
daughter 2	1.336 ± 0.022	-5.300 ± 0.102	1.154 ± 0.014	-3.798 ± 0.007
$q\bar{s}$	K^*		K	
parent	0.839 ± 0.004	0.318 ± 0.012	$0.780 \pm 0.022^\dagger$	$-0.197 \pm 0.036^\dagger$
daughter	0.942 ± 0.046	-1.532 ± 0.209	0.964 ± 0.072	-2.240 ± 0.296
$s\bar{s}$	φ		$\eta_{s\bar{s}}$	
parent	0.728 ± 0.011	0.234 ± 0.034	0.715 ± 0.023	-0.444 ± 0.068
daughter 1	0.721 ± 0.089	-1.072 ± 0.047	0.718 ± 0.032	-1.786 ± 0.157
daughter 2	0.684 ± 0.039	-2.047 ± 0.226	0.729 ± 0.010	-3.174 ± 0.057

* fit without π : $\alpha = (1.053 \pm 0.059) \text{ GeV}^{-2}$, $\alpha_0 = -0.725 \pm 0.170$

† fit without K : $\alpha = (0.846 \pm 0.013) \text{ GeV}^{-2}$, $\alpha_0 = -0.431 \pm 0.042$

Table 5: Fitted parameters of the (J, M^2) parent and daughter Regge trajectories for light mesons with natural and unnatural parity ($q = u, d$).

Meson	β (GeV ⁻²)	β_0	Meson	β (GeV ⁻²)	β_0
$q\bar{q}$			$s\bar{s}$		
π	$0.679 \pm 0.023^*$	$-0.018 \pm 0.014^*$	$\eta_{s\bar{s}}$	0.559 ± 0.009	-0.315 ± 0.026
$\rho(^3S_1)$	0.700 ± 0.023	-0.451 ± 0.060	φ	0.597 ± 0.009	-0.662 ± 0.031
a_0	0.830 ± 0.032	-1.214 ± 0.109	f_0	0.566 ± 0.009	-1.156 ± 0.039
a_1	0.840 ± 0.037	-1.401 ± 0.134	f_1	0.561 ± 0.013	-1.224 ± 0.058
b_1	0.863 ± 0.030	-1.431 ± 0.106	h_1	0.575 ± 0.015	-1.292 ± 0.066
$a_2(^3P_2)$	0.867 ± 0.036	-1.585 ± 0.134	f_2	0.581 ± 0.007	-1.370 ± 0.031
$\rho(^3D_1)$	0.894 ± 0.013	-2.182 ± 0.050			
π_2	0.916 ± 0.032	-2.514 ± 0.134			
$\rho_3(^3D_3)$	0.874 ± 0.041	-2.623 ± 0.189			
$a_2(^3F_2)$	0.891 ± 0.010	-2.881 ± 0.043			
a_3	0.890 ± 0.014	-3.254 ± 0.066			
b_3	0.906 ± 0.015	-3.225 ± 0.071			
a_4	0.899 ± 0.016	-3.672 ± 0.084			

* fit without π : $\beta = (0.750 \pm 0.032)$ GeV⁻², $\beta_0 = -0.287 \pm 0.109$

Table 6: Fitted parameters of the (n_r, M^2) Regge trajectories for light mesons.

emphasize that, in our analysis, we did not introduce any free adjustable parameters but used their values fixed from our previous considerations of hadron properties. It was found that the lightest scalar mesons $f_0(600)$ (σ), $K_0^*(800)$ (κ), $f_0(980)$ and $a_0(980)$ can be naturally described in our model as diquark-antidiquark bound systems, while the lightest scalar (1^3P_0) $q\bar{q}$ states have masses above 1 GeV.

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