Relativistic Description of Baryon Properties

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Properties of heavy and strange baryons are investigated in the framework of the relativistic quark-diquark picture. It is assumed that two quarks in a baryon form a diquark and baryon is considered as the bound quark-diquark system. The relativistic effects and diquark internal structure are consistently taken into account. Calculations are performed up to rather high orbital and radial excitations of heavy and strange baryons. On this basis the Regge trajectories are constructed. The rates of semileptonic decays of heavy baryons are calculated. The obtained results agree well with available experimental data.

The convincing evidence of the existence of diquark correlations in hadrons has been collected. Thus recently several charged charmonium- and bottomonium-like states were discovered. They should be inevitably multiquark, at least four quark — tetraquark, states. One of the most successful pictures of such tetraquark states is the diquark-antidiquark model [1]. In the light meson sector it has been argued for a long time that mesons forming the inverted lightest scalar nonet can be well described as tetraquarks treated as diquark-antidiquark bound states [2]. In the baryon sector it is well known that the number of observed excited states both in the light and heavy sectors is considerably lower than the number of excited states predicted in the three-quark picture [3]. The introduction of diquarks significantly reduces this number since in such a picture some of degrees of freedom are frozen and thus the number of possible excitations is substantially smaller.

Here we study spectroscopy and weak decays of heavy and strange baryons in the relativistic quark-diquark picture in the framework of the quasipotential approach. The interaction of two quarks in a diquark and the quark-diquark interaction in a baryon are described by the diquark wave function Ψ_d of the bound quark-quark state and by the baryon wave function Ψ_B of the bound quark-diquark state respectively, which satisfy the relativistic quasipotential equation of the Schrödinger type [4]

$$\left(\frac{b^2(M)}{2\mu_R}-\frac{\mathbf{p}^2}{2\mu_R}\right)\Psi_{d,B}(\mathbf{p})=\int\frac{d^3q}{(2\pi)^3}V(\mathbf{p},\mathbf{q};M)\Psi_{d,B}(\mathbf{q}), \tag{1}$$

where the relativistic reduced mass and the center-of-mass system relative momentum squared on mass shell are

$$\mu_R=\frac{M^4-(m_1^2-m_2^2)^2}{4M^3},\qquad b^2(M)=\frac{[M^2-(m_1+m_2)^2][M^2-(m_1-m_2)^2]}{4M^2},$$

and M is the bound state mass (diquark or baryon), $m_{1,2}$ are the masses of quarks (q_1 and q_2) which form the diquark or of the diquark (d) and quark (q) which form the baryon (B), and $\mathbf{p}$ is their relative momentum.

The quasipotentials $V(\mathbf{p},\mathbf{q};M)$ of the quark-quark or quark-diquark interaction are constructed with the help of the QCD-motivated off-mass-shell scattering amplitude, projected onto the positive energy states. The effective quark interaction is taken to be the sum of the usual one-gluon exchange term and the mixture of long-range vector and scalar linear confining potentials with the mixing coefficient ε. It is also assumed that the vector confining potential contains not only the Dirac term but the Pauli term, thus introducing the anomalous chromomagnetic quark moment κ. The explicit expressions for the quasipotentials are given in Ref. [5].

In the nonrelativistic limit the usual Cornell-like potential is reproduced

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + Ar + B, \tag{2}$$

where α_s is the QCD coupling constant with freezing [2].

All parameters of the model such as quark masses, parameters of the linear confining potential A and B, the mixing coefficient ε and anomalous chromomagnetic quark moment κ were fixed previously from calculations of meson and baryon properties [4, 5]. The constituent quark masses $m_u = m_d = 0.33$ GeV, $m_s = 0.5$ GeV, $m_c = 1.55$ GeV, $m_b = 4.88$ GeV and the parameters of the linear potential $A = 0.18$ GeV^2 and $B = -0.3$ GeV have the usual values of quark models. The value of the mixing coefficient of vector and scalar confining potentials $\varepsilon = -1$ has been determined from the consideration of the heavy quark expansion for the semileptonic heavy meson decays and charmonium radiative decays [4]. The universal Pauli interaction constant $\kappa = -1$ has been fixed from the analysis of the fine splitting of heavy quarkonia ${}^3P_J$-states [4]. Note that the long-range chromomagnetic contribution to the potential, which is proportional to $(1+\kappa)$, vanishes for the chosen value of $\kappa = -1$.

First we calculate masses and form factors of the diquarks and the vertex $\langle d(P)|J_\mu|d(Q)\rangle$ of the diquark-gluon interaction which is parameterized by the form factor

$$F(r) = 1 - e^{-\xi r - \zeta r^2}, \tag{3}$$

that takes the internal diquark structure into account. The values of diquark masses and parameters ξ and ζ are given in Refs. [5, 6]

Next we calculate the masses of heavy and strange baryons as the bound states of a quark and diquark. The results of calculations can be found in Refs. [5, 6]. Here, as an example, we give in Tables 1,2 the comparison of our predictions for the mass spectra of the Λ_c and Λ with other theoretical estimates and available experimental data. From these tables we see that our diquark model predicts appreciably less states than the three-quark approaches. The differences become apparent with the growth of the orbital and radial excitations in the baryon. Our model reproduces correctly all masses of the well established 4- and 3-star resonances and most of the 2- and 1-star states. Let us emphasize that the experimental mass of the $\frac{1}{2}^-$ 4-star $\Lambda(1405)$ is naturally reproduced if this state is considered as the first orbital excitation $1P$ in the strange quark-light scalar diquark picture of Λ baryons. The rather low mass of this state represents difficulties for most of the three-quark models [7, 8, 9], which predict its mass about 100 MeV higher than experimental value.

We calculated masses of both orbitally and radially excited heavy and strange baryons up to rather high excitation numbers. This makes it possible to construct the heavy baryon Regge trajectories both in the (J, M^2) and in the (n_r, M^2) planes: (a) The (J, M^2) Regge trajectory: $J = \alpha M^2 + \alpha_0$; (b) The (n_r, M^2) Regge trajectory: $n_r = \beta M^2 + \beta_0$, where α, β are the slopes

J^P	Experiment [10]			Theory			
	State	Status	Mass	our	[11]	[3]	[7]
$\frac{1}{2}^+$	Λ_c	***	2286.46	2286	2286	2286	2265
	$\Lambda_c(2765)$	*	2766.6	2769	2766	2791	2775
				3130	3112	3154	3170
				3437	3397		
$\frac{1}{2}^-$	$\Lambda_c(2595)$	***	2592.3	2598	2591	2625	2630
	$\Lambda_c(2940)$	***	2939.3	2983	2989		2780
				3303	3296		2830
$\frac{3}{2}^-$	$\Lambda_c(2625)$	***	2628.1	2627	2629	2636	2640
				3005	3000		2840
				3322	3301		2885
$\frac{3}{2}^+$				2874	2857	2887	2910
				3189	3188	3120	3035
$\frac{5}{2}^+$	$\Lambda_c(2880)$	***	2881.53	2880	2879	2887	2910
				3209	3198	3125	3140
$\frac{5}{2}^-$				3097	3075	2872	2900
$\frac{7}{2}^-$				3078	3092		3125
$\frac{7}{2}^+$				3270	3267		3175
$\frac{9}{2}^+$				3284	3280		
$\frac{9}{2}^-$				3444			
$\frac{11}{2}^-$				3460			

Table 1: Comparison of theoretical predictions for masses of the Λ_c baryons (in MeV).

and α_0, β_0 are intercepts. In Figs. 1,2 we plot the Regge trajectories in the (J, M^2) plane for charmed and strange baryons. Straight lines were obtained by the χ^2 fit of calculated values. The fitted slopes and intercepts of the Regge trajectories can be found in Refs [5, 6]. We see that the calculated heavy and strange baryon masses lie on the linear trajectories, which are parallel and equidistant.

Let us consider weak decays of baryons. The simplest case is the semileptonic decay of the bottom baryon to the charmed baryon, since in this case the model independent considerations based on heavy quark effective theory (HQET) can be applied. To calculate the heavy baryon decay rate it is necessary to determine the corresponding matrix element of the weak current between baryon states, which in the quasipotential approach is given by [14]

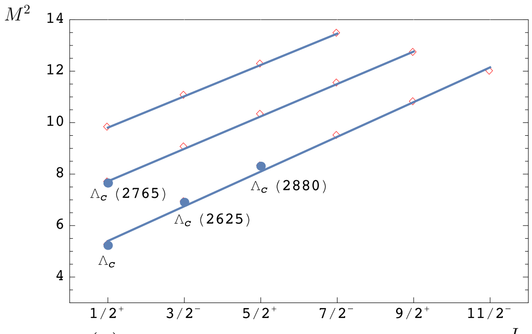
$$\langle \Lambda_{Q'}(p_{Q'})|J^W_\mu|\Lambda_Q(p_Q)\rangle = \int \frac{d^3p\, d^3q}{(2\pi)^6} \bar{\Psi}_{\Lambda_{Q'}\,\mathbf{p}_{Q'}}(\mathbf{p})\Gamma_\mu(\mathbf{p},\mathbf{q})\Psi_{\Lambda_Q\,\mathbf{p}_Q}(\mathbf{q}), \tag{4}$$

where $\Gamma_\mu(\mathbf{p},\mathbf{q})$ is the two-particle vertex function and $\Psi_{\Lambda\,\mathbf{p}_Q}$ are the baryon ($\Lambda = \Lambda_Q, \Lambda_{Q'}$) wave functions projected onto the positive energy states of quarks and boosted to the moving reference frame with momentum $\mathbf{p}_Q$. The wave function of the moving baryon $\Psi_{\Lambda_Q\,\mathbf{p}_Q}$ is connected with the wave function in the rest frame $\Psi_{\Lambda_Q\,\mathbf{0}}$ by the transformation [14]

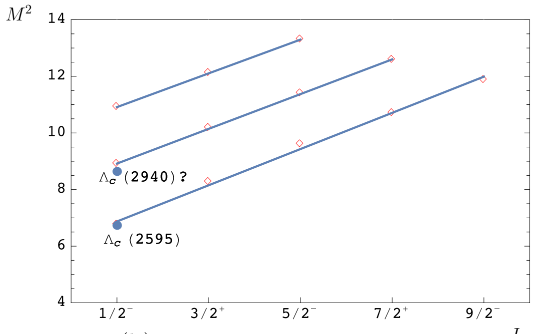
$$\Psi_{\Lambda_Q\,\mathbf{p}_Q}(\mathbf{p}) = D_Q^{1/2}(R^W_{L_{\mathbf{p}_Q}})D_d^{\mathcal{I}}(R^W_{L_{\mathbf{p}_Q}})\Psi_{\Lambda_Q\,\mathbf{0}}(\mathbf{p}), \qquad \mathcal{I} = 0, 1, \tag{5}$$

J^P	Experiment [10]			Theory					
	State	Status	Mass	Our	[7]	[8]	[9]	[12]	[13]
$\frac{1}{2}^+$	Λ	****	1115.683 ± 0.006	1115	1115	1108	1136	1116	1149 ± 18
	$\Lambda(1600)$	***	$1560 - 1600$	1615	1680	1677	1625	1518	1807 ± 94
	$\Lambda(1710)$	*	1713 ± 13						
	$\Lambda(1810)$	***	$1750 - 1810$	1901	1830	1747	1799	1666	2112 ± 54
				1972	1910	1898		1955	2137 ± 69
				1986	2010	2077		1960	
				2042	2105	2099			
				2099	2120	2132			
$\frac{3}{2}^+$	$\Lambda(1890)$	****	$1850 - 1890$	1854	1900	1823		1896	1991 ± 103
				1976	1960	1952			2058 ± 139
				2130	1995	2045			2481 ± 111
				2184	2050	2087			
				2202	2080	2133			
$\frac{5}{2}^+$	$\Lambda(1820)$	****	$1815 - 1820$	1825	1890	1834		1896	
	$\Lambda(2110)$	***	$2090 - 2110$	2098	2035	1999			
				2221	2115	2078			
				2255	2115	2127			
				2258	2180	2150			
$\frac{7}{2}^+$	$\Lambda(2020)$	*	≈ 2020	2251	2120	2130			
				2471		2331			
$\frac{9}{2}^+$	$\Lambda(2350)$	***	$2340 - 2350$	2360		2340			
$\frac{1}{2}^-$	$\Lambda(1405)$	****	$1405.1^{+1.3}_{-1.0}$	1406	1550	1524	1556	1431	1416 ± 81
	$\Lambda(1670)$	****	$1660 - 1670$	1667	1615	1630	1682	1443	1546 ± 110
	$\Lambda(1800)$	***	$1720 - 1800$	1733	1675	1816	1778	1650	1713 ± 116
				1927	2015	2011		1732	2075 ± 249
				2197	2095	2076		1785	
				2218	2160	2117		1854	
$\frac{3}{2}^-$	$\Lambda(1520)$	****	1519.5 ± 1.0	1549	1545	1508	1556	1431	1751 ± 40
	$\Lambda(1690)$	****	$1685 - 1690$	1693	1645	1662	1682	1443	2203 ± 106
				1812	1770	1775		1650	2381 ± 87
	$\Lambda(2050)$	*	2056 ± 22	2035	2030	1987		1732	
				2319	2110	2090		1785	
	$\Lambda(2325)$	*	≈ 2325	2322	2185	2147		1854	
				2392	2230	2259		1928	
				2454	2290	2275		1969	
$\frac{5}{2}^-$	$\Lambda(1830)$	****	$1810 - 1830$	1861	1775	1828	1778	1785	
				2136	2180	2080			
				2350	2250	2179			
$\frac{7}{2}^-$	$\Lambda(2100)$	****	$2090 - 2100$	2097	2150	2090			
				2583	2230	2227			
$\frac{9}{2}^-$				2665		2370			

Table 2: Comparison of theoretical predictions and experimental data for the masses of the Λ states (in MeV).



(a)



(b)

Figure 1: Parent and daughter (J, M^2) Regge trajectories for the Λ_c baryons with natural $(P = (-1)^{J-1/2})$ (a) and unnatural $(P = (-1)^{J+1/2})$ (b) parities. Diamonds are predicted masses. Available experimental data are given by dots with particle names; M^2 is in GeV^2.

where R^W is the Wigner rotation, $L_{\mathbf{p}}$ is the Lorentz boost from the baryon rest frame to a moving one, and $D^{1/2}(R)$, $D^{\mathcal{I}}(R)$ are rotation matrices of the quark and diquark spins.

The hadronic matrix elements for the semileptonic decay $\Lambda_Q \to \Lambda_{Q'}$ are parameterized in terms of six invariant form factors:

$$\langle \Lambda_{Q'}(v')|V^\mu|\Lambda_Q(v)\rangle = \bar{u}_{\Lambda_{Q'}}(v')\Big[F_1(w)\gamma^\mu + F_2(w)v^\mu + F_3(w)v'^\mu\Big]u_{\Lambda_Q}(v),$$
$$\langle \Lambda_{Q'}(v')|A^\mu|\Lambda_Q(v)\rangle = \bar{u}_{\Lambda_{Q'}}(v')\Big[G_1(w)\gamma^\mu + G_2(w)v^\mu + G_3(w)v'^\mu\Big]\gamma_5 u_{\Lambda_Q}(v), \qquad (6)$$

where $u_{\Lambda_Q}(v)$ and $u_{\Lambda_{Q'}}(v')$ are Dirac spinors of the initial and final baryon with four-velocities v and v', respectively; $w = v \cdot v' = (M^2_{\Lambda_Q} + M^2_{\Lambda_{Q'}} - q^2)/(2M_{\Lambda_Q}M_{\Lambda_{Q'}})$. In the heavy quark limit $m_Q \to \infty$ $(Q = b, c)$ the form factors (6) can be expressed through the single Isgur-Wise function $\zeta(w)$

$$F_1(w) = G_1(w) = \zeta(w), \qquad F_2(w) = F_3(w) = G_2(w) = G_3(w) = 0. \qquad (7)$$

At subleading order of the heavy quark expansion the form factors are given by [14]

$$\begin{aligned}
F_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\Big[2\chi(w) + \zeta(w)\Big], \qquad \bar\Lambda = M_{\Lambda_Q} - m_Q,\\
G_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\Big[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\Big],\\
F_2(w) &= G_2(w) = -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w), \qquad F_3(w) = -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w). \qquad (8)
\end{aligned}$$

The resulting expressions for the semileptonic decay $\Lambda_Q \to \Lambda_{Q'}$ form factors up to subleading order in $1/m_Q$ in our model are the following [14]

$$\begin{aligned}
F_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)[2\chi(w) + \zeta(w)]\\
&\quad +4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} - \frac{\bar\Lambda}{2m_Q}(w+1)\right]\chi(w),
\end{aligned}$$

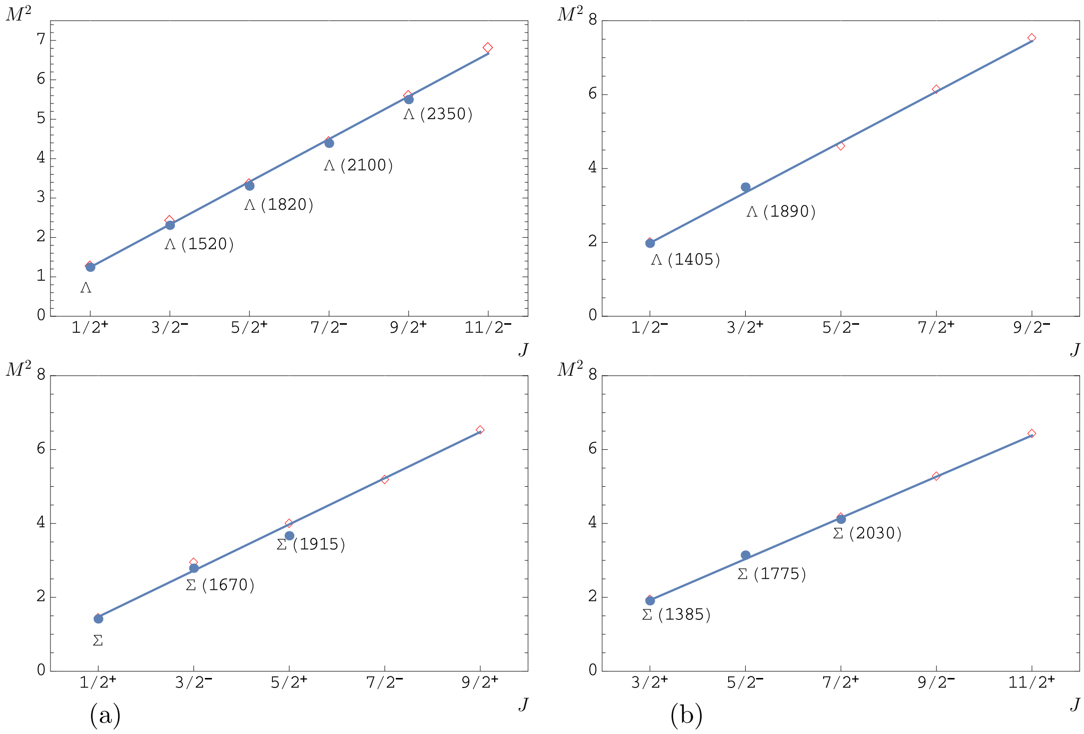


Figure 2: Same as in Fig. 1 but for the Λ and Σ baryons.

$$
\begin{aligned}
G_1(w) &= \zeta(w) + \left(\frac{\bar\Lambda}{2m_Q} + \frac{\bar\Lambda}{2m_{Q'}}\right)\left[2\chi(w) + \frac{w-1}{w+1}\zeta(w)\right] - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q} w\chi(w),\\
F_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\left[\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1} + \frac{\bar\Lambda}{2m_Q} w\right]\chi(w),\\
G_2(w) &= -\frac{\bar\Lambda}{2m_{Q'}}\frac{2}{w+1}\zeta(w) - 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_{Q'}}\frac{1}{w-1}\chi(w),\\
F_3(w) &= -G_3(w) = -\frac{\bar\Lambda}{2m_Q}\frac{2}{w+1}\zeta(w) + 4(1-\varepsilon)(1+\kappa)\frac{\bar\Lambda}{2m_Q}\chi(w),
\end{aligned}
\tag{9}
$$

where the leading order Isgur-Wise function of heavy baryons

$$
\zeta(w) = \lim_{m_Q\to\infty}\int\frac{d^3p}{(2\pi)^3}\Psi_{\Lambda_{Q'}}\!\left(\mathbf{p} + 2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\,\mathbf{e_\Delta}\right)\Psi_{\Lambda_Q}(\mathbf{p}),
\tag{10}
$$

$\mathbf{e_\Delta} = \mathbf{\Delta}/\sqrt{\mathbf{\Delta}^2}$, $\mathbf{\Delta} = M_{\Lambda_{Q'}}\mathbf{v}' - M_{\Lambda_Q}\mathbf{v}$, $\epsilon_d(p) = \sqrt{\mathbf{p}^2 + M_d^2}$ and the subleading function

$$
\chi(w) = -\frac{w-1}{w+1}\lim_{m_Q\to\infty}\int\frac{d^3p}{(2\pi)^3}\frac{\bar\Lambda - \epsilon_d(p)}{2\bar\Lambda}\Psi_{\Lambda_{Q'}}\!\left(\mathbf{p} + 2\epsilon_d(p)\sqrt{\frac{w-1}{w+1}}\,\mathbf{e_\Delta}\right)\Psi_{\Lambda_Q}(\mathbf{p}).
\tag{11}
$$

These functions were calculated using our model wave functions of the Λ_b and Λ_c baryons obtained in their mass spectrum calculations.

For $(1-\varepsilon)(1+\kappa)=0$ the HQET results (8) are reproduced. This can be achieved either setting $\varepsilon=1$ (pure scalar confinement) or $\kappa=-1$. In our model we need vector confining contribution and therefore use the latter option. This consideration gives us additional, based on the HQET, justification for fixing one of the main parameters of the model κ.

The $1/m_Q$ corrections turn out to be rather large, significantly larger than for the heavy-to-heavy mesons transitions. This is the consequence of the larger value of the expansion parameter $\bar{\Lambda}$. Indeed in the case of baryon decays the parameter $\bar{\Lambda}$ is determined by the light diquark energies, while for meson decays it is determined by light quark energies. Therefore consideration of such decays without the heavy quark expansion can significantly improve the precision of predictions. Also such an expansion cannot be applied for the heavy-to-light $\Lambda_b \to p$ weak transitions, since the final baryon contains only light u and d quarks.

It is important to note that both $\Lambda_b \to \Lambda_c l \nu_l$ and $\Lambda_b \to p l \nu_l$ decays have a broad kinematical range. The square of the momentum transfer to the lepton pair q^2 varies from 0 to $q^2_{\max} \approx 12$ GeV^2 for decays to Λ_c and from 0 to $q^2_{\max} \approx 22$ GeV^2 for decays to p. As a result the recoil momentum of the final baryon $|\boldsymbol{\Delta}|$ is almost always significantly larger than the relative quark momentum in the baryon. Thus one can neglect small relative momentum $|\mathbf{p}|$ with respect to the recoil momentum $|\boldsymbol{\Delta}|$ in the energies of quarks in energetic final baryons and replace $\epsilon_q(p+\Delta) \equiv \sqrt{m_q^2+(\mathbf{p}+\boldsymbol{\Delta})^2}$ by $\epsilon_q(\Delta) \equiv \sqrt{m_q^2+\boldsymbol{\Delta}^2}$. It is important to point out that we keep the quark mass in the energies $\epsilon_q(\Delta)$. Thus the resulting expressions are valid both for the heavy-to-light and heavy-to-heavy Λ_b baryon decays. Such replacement is made only in the subleading vertex function $\Gamma^{(2)}_\mu(\mathbf{p},\mathbf{q})$, which takes into account contributions of the intermediate negative-energy states [15], and permits us to perform one of the integrations using the quasipotential equation. As a result, the weak decay matrix elements are expressed through the usual overlap integral of initial and final baryon wave functions. The subleading contributions are proportional to the ratios of baryon binding energies, which are small, to the quark energies and thus turn out to be also small numerically. Therefore we obtain reliable expressions for the form factors in the whole accessible kinematical range [15]. The largest uncertainty, which turns out to be small numerically, occurs for the heavy-to-light transitions in the narrow region near zero recoil of the final light baryon, where the above discussed replacement is less justified. Let us emphasize that we consistently take into account all relativistic corrections including boosts of the baryon wave functions from the rest frame to the moving one. For numerical calculations of the form factors we use the quasipotential wave functions of the Λ_b, Λ_c and p baryons obtained in their mass spectra calculations. We plot the baryon decay form factors in Figs. 3 and 4.

The total decay rates and branching ratios are given in Table 3 in comparison with the results of other theoretical approaches [16, 17, 18, 19, 20, 21, 22] and available experimental data [10]. For absolute values we use the following CKM values $|V_{cb}|=(3.90\pm0.15)\times10^{-2}$, $|V_{ub}|=(4.05\pm0.20)\times10^{-3}$ extracted from our previous analysis of the heavy B and B_s meson decays [23]. In this table we also give our predictions for the average values of the forward-backward asymmetry of the charged lepton $\langle A_{FB}\rangle$, the convexity parameter $\langle C_F\rangle$ and the longitudinal polarization of the final baryon $\langle P_L\rangle$. Note that lattice calculations [22] give somewhat lower predictions for the branching ratios normalized by the square of the corresponding CKM matrix element for $\Lambda_b \to \Lambda_c$ transitions but give higher results for $\Lambda_b \to p$ transitions than other approaches. The lattice determination of form factors is done in the region of small recoils of the final baryon $q^2 \sim q^2_{\max}$ and then their values are extrapolated to the whole kinematical region, which is broad especially for the heavy-to-light $\Lambda_b \to p l \nu_l$ decay. In our model we explicitly determine the form factor q^2 dependence in the whole kinematical range without

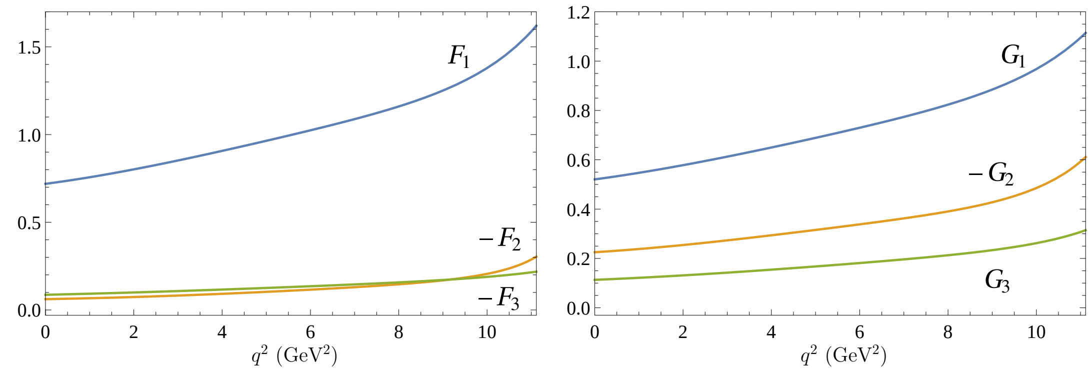


Figure 3: Form factors of the weak $\Lambda_b \to \Lambda_c$ transition.

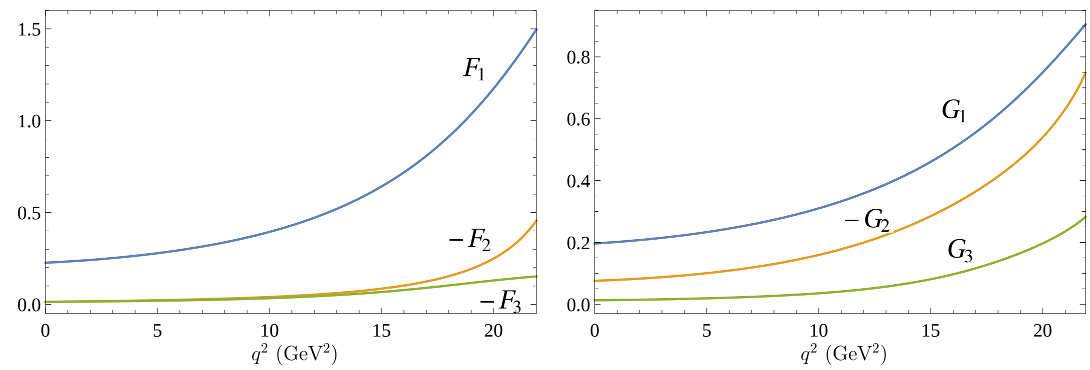


Figure 4: Form factors of the weak $\Lambda_b \to p$ transition.

extrapolations.

Recently the LHCb collaboration [24] measured the ratio of the heavy-to-heavy and heavy-to-light semileptonic Λ_b decays in the limited interval of q^2

$$R_{\Lambda_c p} = \frac{\int_{15\,\mathrm{GeV}^2}^{q^2_{max}} \frac{d\Gamma(\Lambda_b \to p\mu\nu_\mu)}{dq^2} dq^2}{\int_{7\,\mathrm{GeV}^2}^{q^2_{max}} \frac{d\Gamma(\Lambda_b \to \Lambda_c\mu\nu_\mu)}{dq^2} dq^2}. \tag{12}$$

Such a measurement is very important since it allows us for the first time to extract the ratio of CKM matrix elements $|V_{ub}|/|V_{cb}|$ from the Λ_b baryon decays and compare it to the corresponding ratio determined from B and B_s meson decays. Our prediction for the ratio $R_{\Lambda_c p}$ in comparison with the lattice result [22] and experimental value is given in Table 4. From this table we see that our value of the coefficient in front of $|V_{ub}|^2/|V_{cb}|^2$ is significantly lower than the lattice one. This is the result of the deviation of our calculation (and other quark model calculations) from lattice predictions for normalized by the square of CKM matrix element value for heavy-to-heavy and heavy-to-light semileptonic Λ_b decays (see Table 3). This deviation even increases in the ratio.

Comparing our result for $R_{\Lambda_c p}$ with experimental data [24] we can extract the ratio of the

Parameter	our	[16, 17]	[18]	[19]	[20]	[21]	[22]	Exp. [10]
$\Lambda_b \to \Lambda_c l \nu_l$								
Γ (ns^{-1})	44.2		53.9					
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	29.1						$21.5 \pm 0.8 \pm 1.1$	
Br (%)	6.48	6.9		4.83	6.3			$6.2^{+1.4}_{-1.2}$
$\langle A_{FB} \rangle$	0.195	0.18						
$\langle C_F \rangle$	-0.57	-0.63						
$\langle P_L \rangle$	-0.80	-0.82						
$\Lambda_b \to \Lambda_c \tau \nu_\tau$								
Γ (ns^{-1})	13.9		20.9					
$\Gamma/\lvert V_{cb}\rvert^2$ (ps^{-1})	9.11						$7.15 \pm 0.15 \pm 0.27$	
Br (%)	2.03	2.0		1.63				
$\langle A_{FB} \rangle$	-0.021	-0.0385						
$\langle C_F \rangle$	-0.09	-0.10						
$\langle P_L \rangle$	-0.71	-0.72						
$\Lambda_b \to p l \nu_l$								
$\Gamma/\lvert V_{ub}\rvert^2$ (ps^{-1})	18.7	13.3	7.55				$25.7 \pm 2.6 \pm 4.6$	
Br (10^{-4})	4.5	2.9		3.89	2.54	$4.0^{+2.3}_{-2.0}$		
$\langle A_{FB} \rangle$	0.346	0.388						
$\Lambda_b \to p \tau \nu_\tau$								
$\Gamma/\lvert V_{ub}\rvert^2$ (ps^{-1})	12.1	9.6	6.55				$17.7 \pm 1.3 \pm 1.6$	
Br (10^{-4})	2.9	2.1		2.75				
$\langle A_{FB} \rangle$	0.185	0.220						

Table 3: Comparison of theoretical predictions for the Λ_b semileptonic decay parameters with available experimental data.

Ratio	our	[19]	[22]	Experiment (LHCb) [24]
$R_{\Lambda_c p}$	$(0.78 \pm 0.08)\frac{\lvert V_{ub}\rvert^2}{\lvert V_{cb}\rvert^2}$	0.0101	$(1.471 \pm 0.095 \pm 0.109)\frac{\lvert V_{ub}\rvert^2}{\lvert V_{cb}\rvert^2}$	$(1.00 \pm 0.04 \pm 0.08) \times 10^{-2}$

Table 4: Predictions for the ratio of baryon decay rates.

CKM matrix elements. Using our model value we find

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.113 \pm 0.011|_{\rm theor} \pm 0.006|_{\rm exp}, \tag{13}$$

which is in good agreement with the experimental ratio of these matrix elements extracted from inclusive decays [10]

$$\frac{|V_{ub}|_{\rm incl}}{|V_{cb}|_{\rm incl}} = 0.105 \pm 0.006, \tag{14}$$

and with the corresponding ratio found in our previous analysis of exclusive semileptonic B and B_s meson decays [$|V_{cb}| = (3.90 \pm 0.15) \times 10^{-2}$, $|V_{ub}| = (4.05 \pm 0.20) \times 10^{-3}$] [23]

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.104 \pm 0.012. \tag{15}$$

On the other hand, the lattice value for the ratio $R_{\Lambda_c p}$ gives

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.004 \pm 0.004, \tag{16}$$

which is in agreement with the corresponding CKM matrix element ratio extracted from the comparison of lattice predictions with data on exclusive B meson decays, but more than 3σ lower than the ratio extracted from inclusive B meson decays (14).

In summary, we demonstrated that spectroscopy and weak decays of heavy and strange baryons can be well described in the relativistic quark model based on the quark-diquark picture of baryons.

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