The interference effects of multi-channel pion-pion scattering in final states of charmonia and bottomonia decays

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There is presented a unified analysis of all available data on the decays $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2,3,4,5$, $n = 1,2,3$), $J/\psi \to \phi(\pi\pi, K\overline{K})$, and $\psi(2S) \to J/\psi\pi\pi$ and the data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. The multi-channel $\pi\pi$ scattering is described in our model-independent approach based on analyticity and unitarity and using an uniformization procedure. It is shown that the basic shape of dipion mass distributions in the two-pion transitions of both charmonia and bottomonia states are explained by an unified mechanism based on the contribution of the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ coupled channels including their interference.

1 Introduction

The extensive study of the properties of scalar mesons is important for the most profound topics concerning the QCD vacuum, because both sectors affect each other due to possible "direct" transitions between them. Obviously, those transitions influence the f_0-meson parameters, enlarging, in first turn, a total width of the $f_0(500)$ in comparison with its decay widths, i.e. the former contains some information on the QCD vacuum. The problem of a unique structure interpretation of the scalar mesons is far away from being solved completely [1].

In the 3-channel analysis of data on the isoscalar S-wave multi-channel $\pi\pi$ scattering ($\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$), which was performed in our model-independent approach based on analyticity, unitarity and on the use of the uniformization procedure [2], we obtained two solutions for resonance parameters, which distinguish themselves mainly in the $f_0(500)$ width. Expansion of the analysis via the inclusion of the decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ with data from the Mark III and DM2 Collaborations has changed a little the parameters of some resonances but two solutions remained [3]. The further expansion of the above combined analysis with adding the data on $J/\psi \to \phi\pi\pi$ from the BES II Collaboration, which are given in the wider region of the $\pi\pi$ mass spectrum, helped to narrow down the $f_0(500)$ solution to the one with the larger width; other resonance parameters were practically not changed [4]. In Ref.[4] the data on $J/\psi \to \phi\pi\pi$ from the BES II are described to about 1.2 GeV. For expanding the description to higher energies, the amplitude for $\pi\pi \to \eta\eta$ should be taken into account. From experiment only the modulus of this amplitude is known. It is needed to know the phase shift of $\eta\eta$ scattering amplitude. We hope to obtain this description under reasonable assumptions. This is worth to pursue because this will give additional information on the $f_0(1370)$ (there are doubts whether it exists or not) and on the interesting 1500-MeV region.

Generally, a possibility of combined analysis of the isoscalar S-wave multi-channel $\pi\pi$ scattering and the decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ and also the two-pion transitions among bottomonia is related to the expected fact that the is related to the expected fact that pairs of pseudo-scalar mesons are produced in S wave and only they undergo final state interactions, whereas the final quarkonium remains a spectator [4, 5, 6].

The above expansion of the combined analysis, in first turn, aimed at the further study of properties of scalar mesons. On the other hand, it was interesting to explain the unexpected (and even enigmatic) behavior of the dipion spectra in the decays $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n, m = 3, 4, 5, n = 1, 2, 3$) — a bell-shaped form in the near-$\pi\pi$-threshold region [especially for the $\Upsilon(3S) \to \Upsilon(1S)\pi^+\pi^-$ and $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$], smooth dips near a dipion mass of 0.7 GeV in $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$, of 0.6 GeV in $\Upsilon(4S, 5S) \to \Upsilon(1S)\pi^+\pi^-$ and of about 0.44 GeV in $\Upsilon(4S) \to \Upsilon(2S)\pi^+\pi^-$, and also sharp dips of about 1 GeV in the $\Upsilon(4S, 5S) \to \Upsilon(1S)\pi^+\pi^-$ transitions.

We considered practically all available data on the two-pion transitions of Υ mesons from the ARGUS, CLEO, CUSB, Crystal Ball, Belle, and *BaBar* Collaborations – $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n$, $m = 2, 3, 4, 5$, $n = 1, 2, 3$) – and also the data on decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ and $\psi(2S) \to J/\psi\pi\pi$ from the Mark III, DM2 and BES II Collaborations to analyze contributions of multi-channel $\pi\pi$ scattering in the final-state interactions. We have shown that all peculiarities of the $\pi\pi$ mass spectra are explained by the unified way via the interference of contributions of the $\pi\pi$-scattering amplitude and the analytically-continued $\pi\pi \to K\overline{K}$ and $\pi\pi \to \eta\eta$ amplitudes.

These results are based on our previous conclusions on wide resonances [4, 7, 8]: If a wide resonance cannot decay into a channel which opens above its mass, but the resonance is strongly coupled to this channel, then one should consider this resonance as a multi-channel state.

2 The amplitudes for multi-channel $\pi\pi$ scattering

When analysing the 3-channel $\pi\pi$ scattering, we considered the reactions $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, because it was shown [7] that namely these coupled channels needed for obtaining correct values of f_0-resonance parameters.

The 3-channel S-matrix is determined on the 8-sheeted Riemann surface. The matrix elements S_{ij}, where $i, j = 1, 2, 3$ denote channels, have the right-hand cuts along the real axis of the s complex plane (s is the invariant total energy squared), starting with the channel thresholds s_i ($i = 1, 2, 3$), and the left-hand cuts related to the crossed channels.

Resonances are described on the Riemann surface using the formulas of analytic continuations of the S-matrix elements to all sheets. The formulas allow to express the matrix elements on the unphysical sheets by means of the matrix elements on the physical sheet that have only the resonance zeros (aside the real axis), at least, around the physical region [2]. Using these formulas and assuming the resonance zeros on sheet I, we can obtain an arrangement of poles and zeros of the resonance on the whole Riemann surface (pole cluster of resonance).

Let us explain in the 2-channel example how pole cluster describing resonance arises. In the 1-channel consideration of the scattering $1 \to 1$ the main model-independent contribution of resonance is given by a pair of conjugate poles on sheet II and by a pair of conjugate zeros on sheet I at the same points of complex energy in S_{11}. (Conjugate poles and zeros are needed for real analyticity.) In the 2-channel consideration of the processes $1 \to 1$, $1 \to 2$ and $2 \to 2$, we have

$$\begin{aligned}
&S_{11}^{\rm II} = \frac{1}{S_{11}^{\rm I}}, &&S_{11}^{\rm III} = \frac{S_{22}^{\rm I}}{S_{11}^{\rm I}S_{22}^{I} - (S_{12}^{\rm I})^2}, &&S_{11}^{\rm IV} = \frac{S_{11}^{\rm I}S_{22}^{\rm I} - (S_{12}^{\rm I})^2}{S_{22}^{I}},\\
&S_{22}^{\rm II} = \frac{S_{11}^{\rm I}S_{22}^{\rm I} - (S_{12}^{\rm I})^2}{S_{11}^{\rm I}}, &&S_{22}^{\rm III} = \frac{S_{11}^{\rm I}}{S_{11}^{\rm I}S_{22}^{\rm I} - (S_{12}^{\rm I})^2}, &&S_{22}^{\rm IV} = \frac{1}{S_{22}^{\rm I}},\\
&S_{12}^{\rm II} = \frac{iS_{12}^{\rm I}}{S_{11}^{\rm I}}, &&S_{12}^{\rm III} = \frac{-S_{12}^{\rm I}}{S_{11}^{\rm I}S_{22}^{\rm I} - (S_{12}^{\rm I})^2}, &&S_{12}^{\rm IV} = \frac{iS_{12}^{\rm I}}{S_{22}^{\rm I}}.
\end{aligned}$$

In S_{11} a resonance is represented by a pair of conjugate poles on sheet II and by a pair of conjugate zeros on sheet I and also by a pair of conjugate poles on sheet III and by a pair of conjugate zeros on sheet IV at the same points of complex energy if a coupling of channels is absent ($S_{12} = 0$). If the resonance decays into both channels and/or takes part in exchanges in crossing channels, the coupling of channels arises ($S_{12} \neq 0$). Then positions of the poles on sheet III (and of corresponding zeros on sheet IV) turn out to be shifted with respect to positions of zeros on sheet I. Thus we obtain the pole cluster in the 2-channel case.

In the 3-channel case, there are *7 types* of resonances corresponding to 7 possible situations when there are resonance zeros on sheet I only in S_{11} – (**a**); S_{22} – (**b**); S_{33} – (**c**); S_{11} and S_{22} – (**d**); S_{22} and S_{33} – (**e**); S_{11} and S_{33} – (**f**); S_{11}, S_{22} and S_{33} – (**g**).
The resonance of every type which is related to its nature is represented by the pair of complex-conjugate pole clusters.

In order to allow for the Riemann surface structure and the representation of resonances by the pole clusters, we make a conformal map of the 8-sheeted Riemann surface, on which the three-channel S matrix is determined, onto the plane of uniformization of the $\pi\pi$-scattering S-matrix element S_{11}. This is made using the uniformizing variable [8]: $w = (\sqrt{(s-s_2)s_3} + \sqrt{(s-s_3)s_2})/\sqrt{s(s_3-s_2)}$ ($s_2 = 4m_K^2$ and $s_3 = 4m_\eta^2$), in which we have neglected the $\pi\pi$-threshold branch point and allowed for the $K\overline{K}$- and $\eta\eta$-threshold branch points and left-hand branch point at $s = 0$ related to the crossed channels.

The S-matrix elements are taken as the products $S = S_B S_{res}$ where S_{res} represents the main (*model-independent*) contribution of resonances, given by the pole clusters; S_B is the background part which contains possible remaining small (*model-dependent*) contributions of resonances and allows for influence of channels not taken explicitly into account in the uniformizing variable.

To obtain the pole clusters describing resonances, it is convenient to use the Le Couteur –

Newton relations [9] which have the following form on the w-plane:

$$S_{11} = \frac{d^*(-w^*)}{d(w)}, \qquad S_{22} = \frac{d(-w^{-1})}{d(w)}, \qquad S_{33} = \frac{d(w^{-1})}{d(w)}, \tag{1}$$
$$S_{11}S_{22} - S_{12}^2 = \frac{d^*(w^{*-1})}{d(w)}, \quad S_{11}S_{33} - S_{13}^2 = \frac{d^*(-w^{*-1})}{d(w)}, \quad S_{22}S_{33} - S_{23}^2 = \frac{d(-w)}{d(w)}.$$

The $d(w)$ function for the resonance part in these relations is $d_{res}(w) = w^{-\frac{M}{2}} \prod_{r=1}^{M}(w + w_r^*)$ with M a number of resonance zeros. For the background part S_B, the d-function has the form

$$d_B = \exp[-i \sum_{n=1}^{3} \frac{\sqrt{s - s_n}}{2m_n}(\alpha_n + i\beta_n \theta(s - s_n))] \tag{2}$$
$$\alpha_n = a_{n1} + a_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + a_{nv}\frac{s - s_v}{s_v}\theta(s - s_v),$$
$$\beta_n = b_{n1} + b_{n\sigma}\frac{s - s_\sigma}{s_\sigma}\theta(s - s_\sigma) + b_{nv}\frac{s - s_v}{s_v}\theta(s - s_v)$$

with s_σ the $\sigma\sigma$ threshold and s_v the effective threshold of three cannels $\eta\eta'$, $\rho\rho$, $\omega\omega$. These thresholds are determined in the analysis.

When performing our combined analysis, data for the multi-channel $\pi\pi$ scattering were taken from many papers (see Refs. in our paper [4]). Satisfactory description of the multi-channel $\pi\pi$ scattering is obtained with the total $\chi^2/\text{ndf} \approx 1.16$. The preferred scenario found is when the $f_0(500)$ is described by the cluster of type (**a**); the $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ with type (**c**); and $f_0'(1500)$ by type (**g**); the $f_0(980)$ is represented only by the pole on sheet II and a shifted pole on sheet III. The obtained pole-clusters for the resonances are shown in Table 1. Generally, the wide multi-channel states are most adequately represented by poles,

Table 1: The pole clusters for resonances on the $\sqrt{s}$-plane. $\sqrt{s_r} = \text{E}_r - i\Gamma_r/2$.

Sheet		$f_0(500)$	$f_0(980)$	$f_0(1370)$	$f_0(1500)$	$f_0'(1500)$	$f_0(1710)$
II	E_r	521.6±12.4	1008.4±3.1			1512.4±4.9	
	$\Gamma_r/2$	467.3±5.9	33.5±1.5			287.2±12.9	
III	E_r	552.5±17.7	976.7±5.8	1387.2±24.4		1506.1±9.0	
	$\Gamma_r/2$	467.3±5.9	53.2±2.6	167.2±41.8		127.8±10.6	
IV	E_r			1387.2±24.4		1512.4±4.9	
	$\Gamma_r/2$			178.2±37.2		215.0±17.6	
V	E_r			1387.2±24.4	1493.9±3.1	1498.8±7.2	1732.8±43.2
	$\Gamma_r/2$			261.0±73.7	72.8±3.9	142.3±6.0	114.8±61.5
VI	E_r	573.4±29.1		1387.2±24.4	1493.9±5.6	1511.5±4.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9		250.0±83.1	58.4±2.8	179.3±4.0	111.2±8.8
VII	E_r	542.5±25.5			1493.9±5.0	1500.4±9.3	1732.8±43.2
	$\Gamma_r/2$	467.3±5.9			47.8±9.3	99.9±18.0	55.2±38.0
VIII	E_r				1493.9±3.2	1512.4±4.9	1732.8±43.2
	$\Gamma_r/2$				62.2±9.2	298.4±14.5	58.8±16.4

because the poles give the main model-independent effect of resonances and are rather stable characteristics for various models, whereas masses and total widths are very model-dependent for wide resonances [10]. The masses, widths, and the coupling constants of resonances should be calculated using the poles on sheets II, IV and VIII, because only on these sheets the analytic continuations have the forms [2]: $\propto 1/S_{11}^{\rm I}$, $\propto 1/S_{22}^{\rm I}$ and $\propto 1/S_{33}^{\rm I}$, respectively, i.e., the pole positions of resonances are at the same points of the complex-energy plane, as the resonance zeros on the physical sheet, and are not shifted due to the coupling of channels.

The obtained background parameters are: $a_{11} = 0.0$, $a_{1\sigma} = 0.0199$, $a_{1v} = 0.0$, $b_{11} = b_{1\sigma} = 0.0$, $b_{1v} = 0.0338$, $a_{21} = -2.4649$, $a_{2\sigma} = -2.3222$, $a_{2v} = -6.611$, $b_{21} = b_{2\sigma} = 0.0$, $b_{2v} = 7.073$; $b_{31} = 0.6421$, $b_{3\sigma} = 0.4851$; $b_{3v} = 0$; $s_\sigma = 1.6338$ GeV^2, $s_v = 2.0857$ GeV^2.

The small (zero for the elastic region) values of the $\pi\pi$ scattering background parameters (obtained after allowing for the left-hand branch-point at $s = 0$) confirms our assumption $S = S_B S_{res}$ and also that the representation of multi-channel resonances by the pole clusters on the uniformization plane is good and quite sufficient. This also shows that the consideration of the left-hand branch-point at $s = 0$ in the uniformizing variable partly solves a problem of some approaches (see, e.g., Ref. [11]) where the wide-resonance parameters are strongly controlled by the non-resonant background. Another important conclusion related to a practically zero background in $\pi\pi$ scattering and to the fact that the contribution to the $\pi\pi$ scattering amplitude from the crossed channels is given by allowing for the left-hand branch-point at $s = 0$ in the uniformizing variable and the meson-exchange contributions in the left-hand cuts: The zero background in the elastic-scattering region is obtained only when taking into account the left-hand branch-point in the uniformizing variables (both in the 2-channel analysis of processes $\pi\pi \to \pi\pi, K\overline{K}$ [10] and in the 3-channel analysis of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$). This indicates that the ρ- and $f_0(500)$-meson exchange contributions in the left-hand cut practically cancel each other due to gauge invariance.

Let us explain how this is related to the gauge invariance. The propagator of vector particle is $D_{\mu\nu} = -(g_{\mu\nu} - k_\mu k_\nu/m_v^2)/(k^2 - m_v^2)$, of scalar particle $D = 1/(k^2 - m_s^2)$. In the particle exchange processes $k^2 < 0$. Therefore, the scalar-particle exchange leads to an attraction. In the case of the vector exchanged particle one can choose that gauge when in the numerator of propagator "works" only g_{00}. This leads to an repulsion of the scattering particles in the given gauge. Due to gauge invariance this effect extends to arbitrary gauges. To the point, the fact, that the mass terms in the corresponding Hamiltonians are $-\frac{1}{2}m_v^2 V^2$ for the vector meson and $\frac{1}{2}m_s^2 S^2$ for the scalar one, is a reflection of opposite nature of interactions due to the vector- and scalar-meson exchanges. One can show allowing for gauge invariance that the vector- and scalar-meson exchanges contribute with opposite signs. Therefore, the practically zero background in $\pi\pi$ scattering is an additional confirmation that the $f_0(500)$, observed in the analysis as the pole cluster of type **(a)**, is indeed a particle (though very wide), not some dynamically formed resonance. Therefore, one must consider at least in the background the coupled $\sigma\sigma$ channel which is not taken into account explicitly in the uniformizing variable w. In this connection it is reasonable to interpret the effective threshold at $s_\sigma = 1.6338$ GeV^2 in the background phase-shift of the $\pi\pi$ scattering amplitude as related to the $\sigma\sigma$ channel. Only in this channel we have obtained a non-zero background phase-shift in $\pi\pi$ scattering ($a_{1\sigma} = 0.0199$).

Further, since studying the decays of charmonia and bottomonia, we investigated the role of the individual f_0 resonances in contributing to the shape of the di-pion mass distributions in these decays, firstly we studied their role in forming the energy dependence of amplitudes of reactions $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$. In this case we switched off only those resonances [$f_0(500)$, $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$], removal of which can be somehow compensated by correcting the background (maybe, with elements of the pseudo-background) to have the more-or-less acceptable description of the multi-channel $\pi\pi$ scattering. Below we therefore considered description of the multi-channel $\pi\pi$ scattering for two more cases: 1) first, when leaving out a minimal set of the f_0 mesons consisting of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$, which is sufficient to achieve a description of the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ with a total $\chi^2/\text{ndf} \approx 1.20$. 2) Second, from the above-indicated three mesons only the $f_0(500)$ can be omitted while still obtaining a reasonable description of multi-channel $\pi\pi$ scattering (though with appearance of a pseudo-background) with the total $\chi^2/\text{ndf} \approx 1.43$. In Fig. 1 the obtained description of processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ is shown. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$. One can see

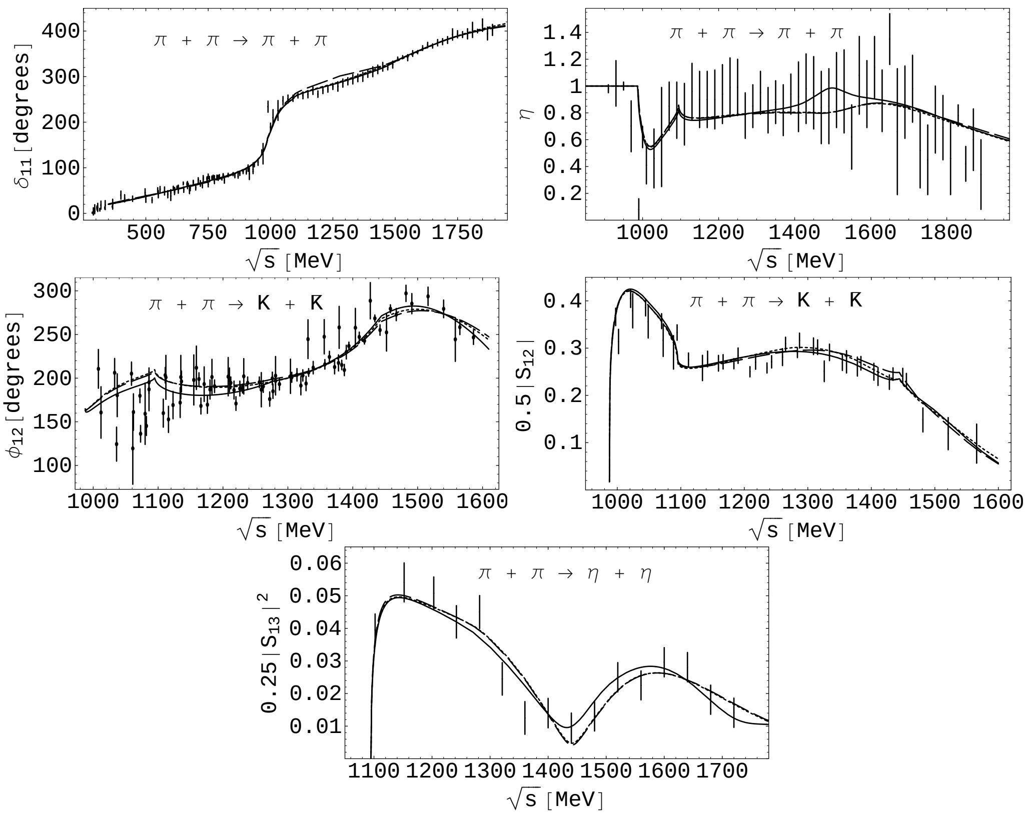

Figure 1: The phase shifts and moduli of the S-matrix element in the S-wave $\pi\pi$-scattering (upper panel), in $\pi\pi \to K\overline{K}$ (middle panel), and the squared modulus of the $\pi\pi \to \eta\eta$ S-matrix element (lower figure).

that the curves are quite similar in all three cases.

3 The contribution of multi-channel $\pi\pi$ scattering in the final states of decays of Ψ- and Υ-meson families

In the combined analysis, for the decay $J/\psi \to \phi\pi^+\pi^-$ the data were taken from Mark III, DM2 and BES II Collaborations; for $\psi(2S) \to J/\psi(\pi^+\pi^-$ and $\pi^0\pi^0)$ — from Mark II and

Crystal Ball(80) (see Refs. in [4]). For $\Upsilon(2S) \to \Upsilon(1S)(\pi^+\pi^-$ and $\pi^0\pi^0)$ data were used from ARGUS, CLEO, CUSB and Crystal Ball Collaborations; for $\Upsilon(3S) \to \Upsilon(1S)(\pi^+\pi^-, \pi^0\pi^0)$ and $\Upsilon(3S) \to \Upsilon(2S)(\pi^+\pi^-, \pi^0\pi^0)$ — from CLEO; for $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ — from *BaBar* and Belle; for $\Upsilon(5S) \to \Upsilon(1S, 2S, 3S)\pi^+\pi^-$ — from Belle Collaboration [12].

The di-meson mass distributions in the quarkonia decays are calculated using a formalism analogous to that proposed in Ref. [5] for the decays $J/\psi \to \phi(\pi\pi, K\overline{K})$ and $V' \to V\pi\pi$ $(V = \psi, \Upsilon)$ which is extended with allowing for amplitudes of transitions between the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ channels in decay formulas. The decay amplitudes are related with the scattering amplitudes T_{ij} $(i, j = 1 - \pi\pi, 2 - K\overline{K}, 3 - \eta\eta)$ as follows

$$F\big(J/\psi \to \phi\pi\pi\big) = c_1(s)T_{11} + \Big(\frac{\alpha_2}{s - \beta_2} + c_2(s)\Big)T_{21} + c_3(s)T_{31}, \tag{3}$$

$$F\big(\psi(2S) \to \psi(1S)\pi\pi\big) = d_1(s)T_{11} + d_2(s)T_{21} + d_3(s)T_{31}, \tag{4}$$

$$F\big(\Upsilon(mS) \to \Upsilon(nS)\pi\pi\big) = e_1^{(mn)}T_{11} + e_2^{(mn)}T_{21} + e_3^{(mn)}T_{31}, \quad m > n,\ m = 2, 3, 4, 5,\ n = 1, 2, 3 \tag{5}$$

where $c_i = \gamma_{i0} + \gamma_{i1}s$, $d_i = \delta_{i0} + \delta_{i1}s$ and $e_i^{(mn)} = \rho_{i0}^{(mn)} + \rho_{i1}^{(mn)}s$; indices m and n correspond to $\Upsilon(mS)$ and $\Upsilon(nS)$, respectively. The free parameters α_2, β_2, γ_{i0}, γ_{i1}, δ_{i0}, δ_{i1}, $\rho_{i0}^{(mn)}$ and $\rho_{i1}^{(mn)}$ depend on the couplings of J/ψ, $\psi(2S)$, and $\Upsilon(mS)$ to the channels $\pi\pi$, $K\overline{K}$ and $\eta\eta$. The pole term in the first equation in front of T_{21} is an approximation of possible ϕK states, not forbidden by OZI rules. The amplitudes T_{ij} are expressed through the S-matrix elements $S_{ij} = \delta_{ij} + 2i\sqrt{\rho_i\rho_j}T_{ij}$ where $\rho_i = \sqrt{1 - s_i/s}$ and s_i is the reaction threshold.

The di-meson mass distributions in the decay analysis were calculated using the relation $N|F|^2\sqrt{(s - s_1)[m_\psi^2 - (\sqrt{s} - m_\phi)^2][m_\psi^2 - (\sqrt{s} + m_\phi)^2]}$ for the decay $J/\psi \to \phi\pi\pi$ and with analogous relations for $\psi(2S) \to \psi(1S)\pi\pi$ and $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$. The normalization to the experiment, N, is determined in the analysis.

A satisfactory description of all considered processes (including $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$) was obtained with the total $\chi^2/\text{ndf} \approx 1.24$; for the $\pi\pi$ scattering, $\chi^2/\text{ndf} \approx 1.15$.

Results for the distributions are shown in Figs. 2-4 with the same notation as in Fig. 1. Here the effects of omitting some resonance are more apparent than in Fig. 1.

4 Conclusions and Discussion

The combined analysis was performed for the data on isoscalar S-wave processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$ and on the decays of the charmonia — $J/\psi \to \phi\pi\pi$, $\psi(2S) \to J/\psi\,\pi\pi$ — and of the bottomonia — $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ $(m > n,\ m = 2, 3, 4, 5,\ n = 1, 2, 3)$ from the ARGUS, Crystal Ball, CLEO, CUSB, DM2, Mark II, Mark III, BES II, *BaBar*, and Belle Collaborations.

It is shown that the di-pion mass spectra in the above-indicated decays of charmonia and bottomonia are explained by the unified mechanism which is based on our previous conclusions on wide resonances [4, 7] and is related to contributions of the $\pi\pi$, $K\overline{K}$ and $\eta\eta$ coupled channels including their interference. It is shown that in the final states of these decays (except $\pi\pi$ scattering) the contribution of coupled processes, e.g., $K\overline{K}, \eta\eta \to \pi\pi$, is important even if these processes are energetically forbidden.

It was also very useful to consider the role of individual f_0 resonances in contributions to the di-pion mass distributions in the indicated decays. For example, it is seen that the sharp dips

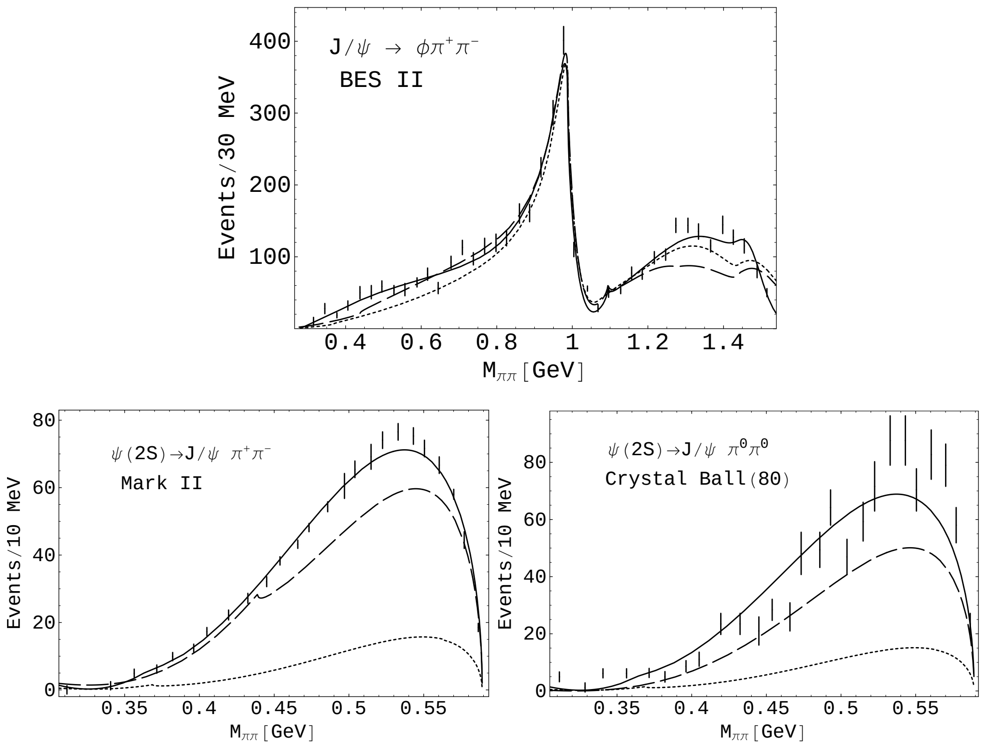

Figure 2: The decays $J/\psi \to \phi\pi\pi$ and $\psi(2S) \to J/\psi\pi\pi$. The solid lines correspond to contribution of all relevant f_0-resonances; the dotted, of the $f_0(500)$, $f_0(980)$, and $f_0'(1500)$; the dashed, of the $f_0(980)$ and $f_0'(1500)$.

near 1 GeV in the $\Upsilon(4S, 5S) \to \Upsilon(1S)\pi^+\pi^-$ decays are related with the $f_0(500)$ contribution to the interfering amplitudes of $\pi\pi$ scattering and $K\overline{K}, \eta\eta \to \pi\pi$ processes. Namely consideration of this role of the $f_0(500)$ allows us to make a conclusion on existence of the sharp dip at about 1 GeV in the di-pion mass spectrum of the $\Upsilon(4S) \to \Upsilon(1S)\pi^+\pi^-$ decay where, unlike $\Upsilon(5S) \to \Upsilon(1S)\pi^+\pi^-$, the scarce data do not permit to draw such conclusions yet.

Also, a manifestation of the $f_0(1370)$ turned out to be interesting and unexpected. First, in the satisfactory description of the $\pi\pi$ spectrum of decay $J/\psi \to \phi\pi\pi$, the second large peak in the 1.4-GeV region can be naively explained as the contribution of the $f_0(1370)$. We have shown that this is not right – the constructive interference between the contributions of the $\eta\eta$ and $\pi\pi$ and $K\overline{K}$ channels plays the main role in formation of the 1.4-GeV peak. This is quite in agreement with our earlier conclusion that the $f_0(1370)$ has a dominant $s\bar{s}$ component [7].

On the other hand, it turned out that the $f_0(1370)$ contributes considerably in the near-$\pi\pi$-threshold region of many di-pion mass distributions, especially making the threshold bell-shaped form of the di-pion spectra in the decays $\Upsilon(mS) \to \Upsilon(nS)\pi\pi$ ($m > n, m = 3, 4, 5, n = 1, 2, 3$). This fact confirms, first, the existence of the $f_0(1370)$ (up to now there is no firm conviction if it exists or not). Second, that the exciting role of this meson in making the threshold bell-shaped form of the di-pion spectra can be explained as follows: the $f_0(1370)$, being predominantly the $s\bar{s}$ state [4] and practically not contributing to the $\pi\pi$-scattering amplitude, influences noticeably

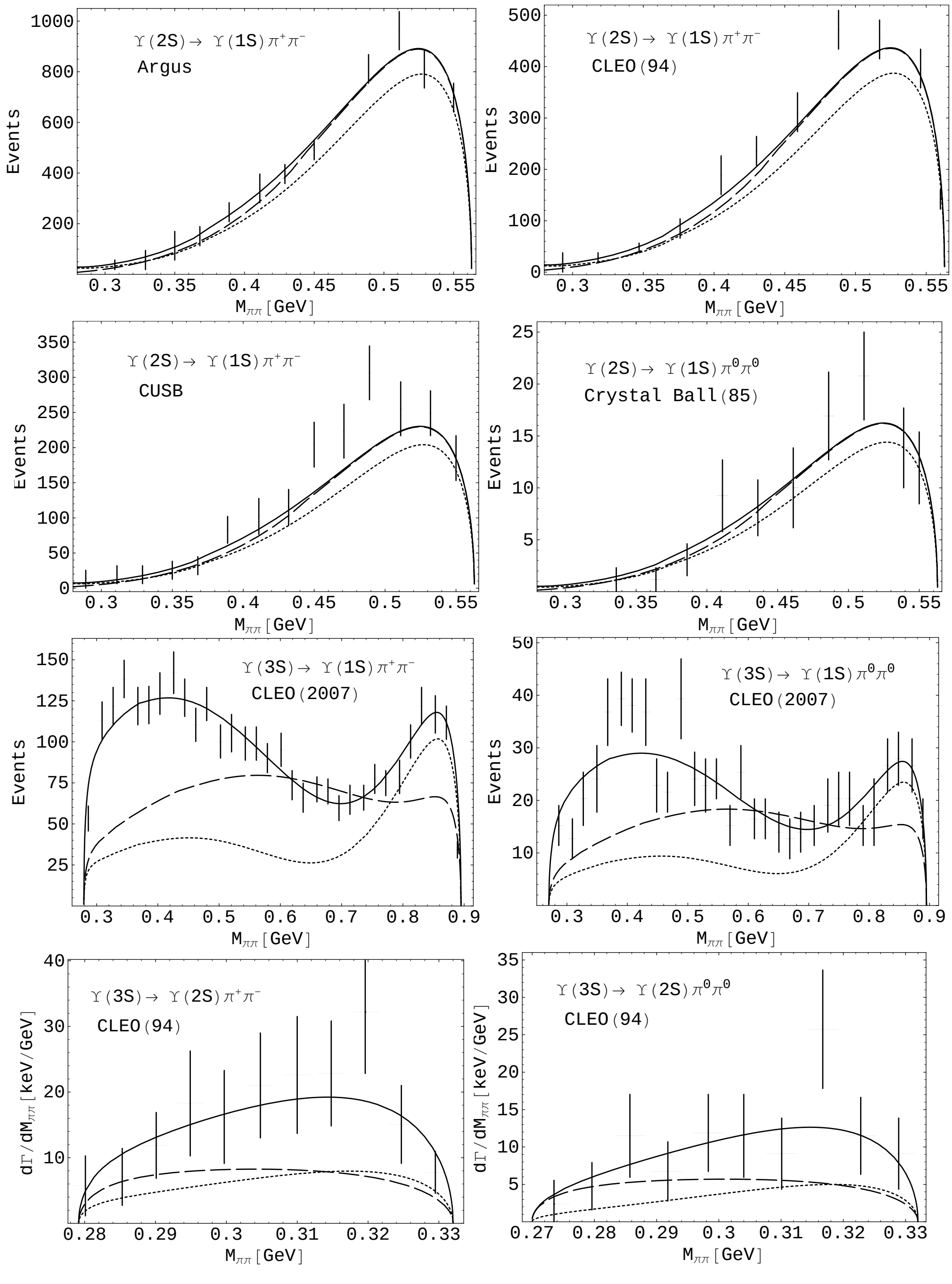

Figure 3: The decays $\Upsilon(2S) \to \Upsilon(1S)\pi\pi$ (two upper panels), $\Upsilon(3S) \to \Upsilon(1S)\pi\pi$ (middle panel) and $\Upsilon(3S) \to \Upsilon(2S)\pi\pi$ (lower panel).

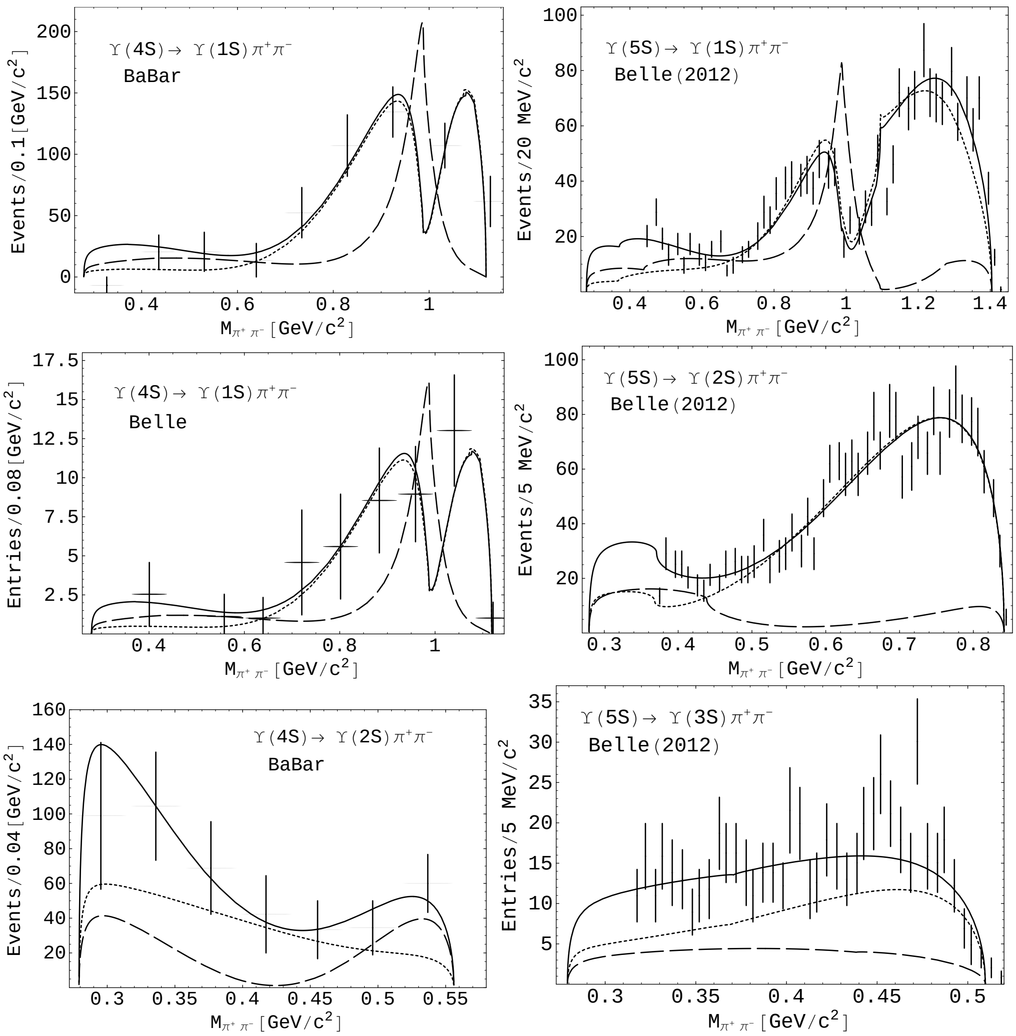

Figure 4: The decays $\Upsilon(4S) \to \Upsilon(1S, 2S)\pi^+\pi^-$ (left-hand) and $\Upsilon(5S) \to \Upsilon(ns)\pi^+\pi^-$ ($n = 1, 2, 3$) (right-hand).

the $K\overline{K}$ scattering; e.g., it was shown that the $K\overline{K}$-scattering length is very sensitive to whether this state does exist or not [10]. The interference of contributions of the $\pi\pi$-scattering amplitude and the analytically-continued $\pi\pi \to K\overline{K}$ and $\pi\pi \to \eta\eta$ amplitudes lead to the observed results.

It is important that we have performed a combined analysis of available data on the processes $\pi\pi \to \pi\pi, K\overline{K}, \eta\eta$, on decays of charmonia and of bottomonia. The convincing description of practically all available data on two-pion transitions of the Ψ and the Υ mesons confirmed all our previous conclusions on the scalar mesons [4].

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