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Two-loop top and bottom Yukawa corrections to the Higgs-boson masses in the complex MSSM

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Results for the two-loop corrections to the Higgs-boson masses of the MSSM with complex parameters of $\mathcal{O}\big(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2\big)$ from the Yukawa sector in the gauge-less limit are presented. The corresponding self-energies and their renormalization have been obtained in the Feynman-diagrammatic approach. The impact of the new contributions on the Higgs spectrum is investigated. Furthermore, a comparison with an existing result in the limit of the MSSM with real parameters is carried out. The new results will be included in the public code `FeynHiggs`.

1 Introduction

After the discovery of a Higgs boson [1, 2] with a mass around 125 GeV, intense studies were performed to reveal its nature. Although within the present experimental uncertainties the measured properties of this new boson are consistent with the expectations for the Higgs boson of the Standard Model (SM) [3, 4], it could be part of an extended model like the theoretically well motivated minimal supersymmetric Standard Model (MSSM). In the MSSM the observed particle could in principle be interpreted as one of the three neutral physical Higgs bosons. At the tree-level, the physical states are given by the neutral CP-even h, H and CP-odd A bosons, together with the charged $H^\pm$ bosons, and can be parametrized in terms of the A-boson mass m_A and the ratio of the two vacuum expectation values, $\tan\beta = v_2/v_1$. An admixture of these CP eigenstates is introduced to the Higgs sector via loop contributions involving complex parameters from other supersymmetric (SUSY) sectors [5–8].

Loop corrections to the masses of the Higgs bosons are sizable and therefore phenomenologically very important. Accordingly, numerous calculations for higher-order corrections to the mass spectrum within the MSSM for the case where CP conservation has been assumed [9–57] as well as for the general case of the MSSM with complex parameters (cMSSM) [5–8, 36, 44, 50, 58–66] have already been performed. The largest one-loop contributions originate from the Yukawa sector due

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to the size of the top-quark Yukawa coupling h_t, where $\alpha_t = h_t^2/(4\pi)$. For large values of $\tan\beta$ contributions of the order $\alpha_b = h_b^2/(4\pi)$, with the bottom Yukawa coupling h_b, can become sizable. At the two-loop level both types of contributions receive further potentially large corrections. The dominant contribution is given by the leading $\mathcal{O}(\alpha_t\alpha_s)$ terms [24, 27, 28, 62] which are known in the MSSM with complex parameters. Additional corrections involving the strong coupling α_s are known in the special case of the CP-conserving MSSM [53–55]. Another important class of two-loop corrections are Yukawa-coupling enhanced contributions of the order $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ which are known in the CP-conserving MSSM as well [35, 41]. A computation of the leading corrections of $\mathcal{O}(\alpha_t^2)$ has been published for the general MSSM [63, 64]. In this article also the other pieces of the two-loop Yukawa terms are obtained for the general case of the MSSM with complex parameters.

The phases of complex parameters in the MSSM are constrained by limits on electric dipole moments (EDMs) [67–72], the impact of meson mixings and decays (see Ref. [73] and references therein), and Higgs-coupling measurements [4].

Following the usual convention, we choose to fix the phase of the mass of the electroweakinos, ϕ_{M_2}, to zero; then the phase of μ from the superpotential, ϕ_μ, needs to be close to zero or π in order to be compatible with the experimental constraints. The other relevant parameters are the phase of the gluino mass parameter, ϕ_{M_3}, and the trilinear soft-breaking parameters of the stops, ϕ_{A_t}, and sbottoms, ϕ_{A_b}. These phases, ϕ_{M_3}, ϕ_{A_t} and ϕ_{A_b}, are less constrained; especially the bounds on the phases of the trilinear soft-breaking parameters are weaker for the third generation than for the second and first generation.

The calculation presented here extends the Yukawa-type contributions of $\mathcal{O}(\alpha_t^2)$ from Refs. [63, 64], and profits from previously developed tools [74]. For this reason the theoretical framework is just briefly outlined and only new aspects are explained in detail in Section 2. The numerical analysis in Section 3 is focussed on the impact of the new contributions on the Higgs masses, showing substantial shifts of 2 GeV and more in certain regions of parameter space. In the limit of vanishing phases of the parameters, our results agree well with previous results in the MSSM for the case where CP conservation has been assumed [41]; differences are shown for the comparison with an interpolation for non-zero phases which so far has been used in `FeynHiggs` [27, 38, 66, 75, 76]. The new results will become part of the public code `FeynHiggs`.

2 Higgs masses at higher orders in the complex MSSM

In this section, we briefly outline the theoretical framework for Higgs-mass predictions at higher orders in the MSSM. We introduce our conventions, explain some details of our chosen renormalization scheme, and comment on the gauge-less limit and the bottom mass resummation.

2.1 Notation and conventions at the tree level

The two scalar $SU(2)$ Higgs doublets are expressed in terms of their components in the following way,

$$\mathcal{H}_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad \mathcal{H}_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}. \tag{2.1}$$

After rotation to mass eigenstates, the Higgs potential reads

$$\begin{aligned} V_H = &-T_h\, h - T_H\, H - T_A\, A - T_G\, G \\ &+ \frac{1}{2}\left(h,\ H,\ A,\ G\right) \mathbf{M}_{hHAG} \begin{pmatrix} h \\ H \\ A \\ G \end{pmatrix} + \left(H^-,\ G^-\right) \mathbf{M}_{H^\pm G^\pm} \begin{pmatrix} H^+ \\ G^+ \end{pmatrix} + \dots\,, \end{aligned} \tag{2.2}$$

with the tadpole coefficients $T_{h,H,A,G}$, and the mass matrices

$$\mathbf{M}_{hHAG} = \begin{pmatrix} m_h^2 & m_{hH}^2 & m_{hA}^2 & m_{hG}^2 \\ m_{hH}^2 & m_H^2 & m_{HA}^2 & m_{HG}^2 \\ m_{hA}^2 & m_{HA}^2 & m_A^2 & m_{AG}^2 \\ m_{hG}^2 & m_{HG}^2 & m_{AG}^2 & m_G^2 \end{pmatrix}, \qquad \mathbf{M}_{H^\pm G^\pm} = \begin{pmatrix} m_{H^\pm}^2 & m_{H^-G^+}^2 \\ m_{G^-H^+}^2 & m_{G^\pm}^2 \end{pmatrix}. \tag{2.3}$$

The matrices $\mathbf{M}_{hHAG}$ and $\mathbf{M}_{H^\pm G^\pm}$ are diagonal at the tree level after minimizing the potential. Explicit expressions for the entries are given in Ref. [66].

2.2 Gauge-less limit

The gauge-less limit in our calculation is defined by neglecting all couplings proportional to g_1 or g_2. As a consequence of this approximation the gauge-boson masses M_W and M_Z are equal to zero in the new two-loop contributions.

Accordingly, the Higgs-boson masses entering the two-loop calculation take on the values

$$m_h = m_G = m_{G^\pm} = 0\,, \qquad\qquad m_H = m_A = m_{H^\pm}\,. \tag{2.4}$$

In this limit, the tree-level mixing angles $\alpha \in [-\pi/2, 0)$ and $\beta \in [0, \pi/2)$ fullfil the relation

$$\alpha = \beta - \frac{\pi}{2}\,. \tag{2.5}$$

2.3 Higgs masses at the two-loop order

The Higgs mass matrix elements at the two-loop order receive contributions from self-energies, leading in general to mixing of all neutral states. In this article the full one-loop corrections are used, while the $\mathcal{O}(\alpha_t\alpha_s)$ and the new $\mathcal{O}(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ terms are evaluated in the gauge-less limit and at zero external momentum. Therefore, the loop-corrected propagator Δ_{hHAG} is given by

$$\Delta_{hHAG}(p^2) = i\left[p^2\mathbf{1} - \mathbf{M}_{hHAG}^{(0)} + \hat{\mathbf{\Sigma}}_{hHAG}^{(1)}(p^2) + \hat{\mathbf{\Sigma}}_{hHAG}^{(2)}(0)\right]^{-1}. \tag{2.6}$$

Therein, $\hat{\mathbf{\Sigma}}_{hHAG}^{(k)}$ denotes the matrix of the renormalized diagonal and non-diagonal self-energies for the h, H, A, G fields at loop order k, and $\mathbf{M}_{hHAG}^{(0)}$ denotes the diagonal tree-level mass matrix.

Mixing of the Goldstone boson (and of the longitudinal Z boson) with the other Higgs bosons yields negligible effects to the propagators of the physical Higgs bosons [77–79]. Therefore, in the following we will only consider the (3×3) submatrix of Δ_{hHAG} involving the physical Higgs bosons. Though, Goldstone–Higgs mixing is taken into account in subloop renormalization terms of the type $(\text{one-loop})^2$ [64].

The neutral Higgs masses are derived from the real parts of the complex poles of the hHA propagator matrix, obtained as the zeroes of the determinant of the renormalized two-point function,

$$\det\hat{\Gamma}_{hHA}(p^2) = 0\,, \qquad \hat{\Gamma}_{hHA}(p^2) = i\left[p^2\mathbf{1} - \mathbf{M}_{hHA}^{(0)} + \hat{\mathbf{\Sigma}}_{hHA}^{(1)}(p^2) + \hat{\mathbf{\Sigma}}_{hHA}^{(2)}(0)\right]. \tag{2.7}$$

2.4 Counterterms

The renormalized two-loop self-energies can be written as

$$\hat{\mathbf{\Sigma}}^{(2)}_{hHA}(p^2) = \mathbf{\Sigma}^{(2)}_{hHA}(p^2) - \delta^{(2)}\mathbf{M}^{\mathbf{Z}}_{hHA}\,, \tag{2.8}$$

with $\mathbf{\Sigma}^{(2)}_{hHA}$ denoting the unrenormalized self-energies at the two-loop order, and $\delta^{(2)}\mathbf{M}^{\mathbf{Z}}_{hHA}$ comprising all two-loop counterterms resulting from parameter and field renormalization. The notation follows [64], where the required expressions for $\delta^{(2)}\mathbf{M}^{\mathbf{Z}}_{hHA}$ can be found.

The Feynman-diagrammatic calculation of the self-energies has been performed with the help of `FeynArts` [80, 81] for the generation of the Feynman diagrams, and `TwoCalc` [82] for the two-loop tensor reduction and trace evaluation. The one-loop renormalization constants have been obtained with the help of `FormCalc` [83].

2.4.1 Genuine two-loop renormalization

The two-loop counterterms for the Higgs self-energies given in Ref. [64] also apply to the corrections described in the present article. However, there is an interesting difference for the cancellation of the divergence in the self-energy $\Sigma^{(2)}_{hH}(0)$. The corresponding counterterm reads

$$\delta^{(2)}m^{\mathbf{Z}}_{hH} = \delta^{(2)}m^2_{hH} + \tfrac{1}{2}m^2_{H^\pm}\delta^{(2)}Z_{hH} + \dots\,, \tag{2.9}$$

where terms with products of two one-loop counterterms have been omitted. In the gauge-less limit $\delta^{(2)}m^2_{hH}$ is the only counterterm which contains $\delta^{(2)}t_\beta$,

$$\delta^{(2)}m^2_{hH} = m^2_{H^\pm}\, c^2_\beta\, \delta^{(2)}t_\beta + \dots\,. \tag{2.10}$$

Here we define $t_\beta \equiv \tan\beta$, $s_\beta \equiv \sin\beta$ and $c_\beta \equiv \cos\beta$. The two-loop field renormalization constant for the same matrix element is given by

$$\delta^{(2)}Z_{hH} = -c_\beta s_\beta \left[\delta^{(2)}Z_{\mathcal{H}_2} - \tfrac{1}{4}\left(\delta^{(1)}Z_{\mathcal{H}_2}\right)^2 - \delta^{(2)}Z_{\mathcal{H}_1} + \tfrac{1}{4}\left(\delta^{(1)}Z_{\mathcal{H}_1}\right)^2\right]. \tag{2.11}$$

In the gauge-less limit, the two-loop counterterm for t_β can be expressed as

$$\delta^{(2)}t_\beta = \frac{t_\beta}{2}\left[\left(\delta^{(2)}Z_{\mathcal{H}_2} - \delta^{(2)}Z_{\mathcal{H}_1}\right) - \tfrac{1}{4}\left(\delta^{(1)}Z_{\mathcal{H}_2} - \delta^{(1)}Z_{\mathcal{H}_1}\right)^2 - \left(\delta^{(1)}Z_{\mathcal{H}_2} - \delta^{(1)}Z_{\mathcal{H}_1}\right)\delta^{(1)}Z_{\mathcal{H}_1}\right]. \tag{2.12}$$

Combining Eqs. (2.9)–(2.12) yields

$$\delta^{(2)}m^{\mathbf{Z}}_{hH} = \frac{c_\beta\, s_\beta\, m^2_{H^\pm}}{4}\left(\delta^{(1)}Z_{\mathcal{H}_2} - \delta^{(1)}Z_{\mathcal{H}_1}\right)\delta^{(1)}Z_{\mathcal{H}_1} + \dots\,. \tag{2.13}$$

The $\overline{\mathrm{DR}}$ field-renormalization constant $\delta^{(1)}Z_{\mathcal{H}_1}$ is a pure UV-divergent term, calculated as the derivative with respect to the external momentum p^2 of the ϕ_1 Higgs self-energy. The only contribution in the gauge-less limit is a bottom loop, *i. e.* in the case of the previously calculated $\mathcal{O}(\alpha_t^2)$ corrections [64], $\delta^{(1)}Z_{\mathcal{H}_1}$ was equal to zero due to the approximation $m_b = 0$. The terms originating from two-loop field-renormalization and two-loop renormalization of t_β cancelled each other exactly.

Now, for the more general case of a non-zero bottom mass, also $\delta^{(1)}Z_{\mathcal{H}_1}$ is non-zero and the cancellation is not complete anymore. The genuine two-loop parts of the field-renormalization constants, $\delta^{(2)}Z_{\mathcal{H}_1}$ and $\delta^{(2)}Z_{\mathcal{H}_2}$, drop out in the gauge-less limit at zero external momentum in Eq. (2.13) because of a cancelation of the contributions in $\delta^{(2)}t_\beta$ and $\delta^{(2)}Z_{hH}$. In principle, $\delta^{(2)}Z_{\mathcal{H}_1}$

and $\delta^{(2)} Z_{\mathcal{H}_2}$ could still appear as field-renormalization constants for the other Higgs-mass counter-terms. However, also there they drop out exactly (see Eq. (2.23) in [64]):

- for $\delta^{(2)} m_h^{2\mathbf{Z}}$ since $m_h^2 = 0$ in the gauge-less limit,
- for $\delta^{(2)} m_H^{2\mathbf{Z}}$ since $m_H^2 = m_{H^\pm}^2$ and $\alpha = \beta - \frac{\pi}{2}$ in the gauge-less limit,
- for $\delta^{(2)} m_A^{2\mathbf{Z}}$ since $m_A^2 = m_{H^\pm}^2$ in the gauge-less limit,
- for $\delta^{(2)} m_{hA}^{2\mathbf{Z}}$ and $\delta^{(2)} m_{HA}^{2\mathbf{Z}}$ since the Higgs sector is CP conserving at the tree level.

2.4.2 Resummation

Radiative corrections to the relation between the bottom-quark mass and the Yukawa coupling of the bottom quark h_b are proportional to t_β. In order to resum the leading t_β-enhanced contributions, an effective bottom Yukawa coupling is used as described in Refs. [78, 84–89], leading to a UV finite and complex correction factor Δm_b. Using a $\overline{\text{DR}}$ renormalization for m_b in the MSSM, the largest contributions of this type are captured through an effective bottom-quark mass which is given by

$$m_b^{\overline{\text{DR}},\text{MSSM}}(m_t^{\text{os}}) \simeq m_{b,\text{eff}} = \frac{m_b^{\overline{\text{DR}},\text{SM}}(m_t^{\text{os}})}{|1 + \Delta m_b|}. \tag{2.14}$$

The symbol $m_b^{\overline{\text{DR}},\text{SM}}(m_t^{\text{os}})$ denotes the bottom mass in the $\overline{\text{DR}}$ renormalization scheme, taking into account SM-type QCD corrections, evaluated at the on-shell top mass.

We use the correction factor Δm_b at the one-loop order which is implemented in `FeynHiggs`. For illustrating the effects seen in our numerical analysis below, we give here the explicit form of the leading contribution:

$$\Delta m_b = \frac{2\,\alpha_s}{3\,\pi}\,\mu^*\,M_3^*\,t_\beta\,\mathcal{I}\Big(m_{\tilde{b}_1}^2, m_{\tilde{b}_2}^2, m_{\tilde{g}}^2\Big)\,, \tag{2.15a}$$

$$\mathcal{I}(a,b,c) = -\frac{b\,a\,\log\frac{b}{a} + c\,b\,\log\frac{c}{b} + a\,c\,\log\frac{a}{c}}{(b-a)\,(c-b)\,(a-c)}. \tag{2.15b}$$

In order to avoid a double counting of contributions from the bottom–sbottom sector to the Higgs-boson self-energies, the bottom-mass is renormalized in the $\overline{\text{DR}}$ scheme as specified in Eq. (2.14).

2.4.3 Subloop renormalization

One-loop counterterms for subloop renormalization enter the self-energies $\mathbf{\Sigma}_{hHA}^{(2)}$ in Eq. (2.8). In contrast to the previously calculated $\mathcal{O}\big(\alpha_t^2\big)$ corrections, the approximation of massless bottom quarks is dropped in the present calculation. Accordingly, new counterterms for the bottom–sbottom sector are induced, which are specified in the following.

The squark mass matrices in the $\big(\tilde{q}_\text{L}, \tilde{q}_\text{R}\big)$ bases, $q = t, b$, in the gauge-less limit are given by

$$\mathbf{M}_{\tilde{q}} = \begin{pmatrix} m_{\tilde{q}_\text{L}}^2 + m_q^2 & m_q\,\big(A_q^* - \mu\,\kappa_q\big) \\ m_q\,(A_q - \mu^*\,\kappa_q) & m_{\tilde{q}_\text{R}}^2 + m_q^2 \end{pmatrix}, \qquad \kappa_t = \frac{1}{t_\beta}, \qquad \kappa_b = t_\beta. \tag{2.16}$$

$SU(2)$-invariance requires $m_{\tilde{t}_\text{L}}^2 = m_{\tilde{b}_\text{L}}^2 \equiv m_{\tilde{Q}_3}^2$. The squark mass eigenvalues can be obtained by performing unitary transformations,

$$\mathbf{U}_{\tilde{q}}\mathbf{M}_{\tilde{q}}\mathbf{U}_{\tilde{q}}^\dagger = \text{diag}\big(m_{\tilde{q}_1}^2,\, m_{\tilde{q}_2}^2\big). \tag{2.17}$$

The independent parameters entering the two-loop calculation via the quark–squark sector are the quark masses m_q, the soft SUSY-breaking parameters $m_{\tilde{Q}_3}$ and $m_{\tilde{q}_\text{R}}$, $\tilde{q} = \tilde{t}, \tilde{b}$, the complex trilinear couplings $A_q = |A_q|\mathrm{e}^{i\,\phi_{A_q}}$, $q = t, b$, the complex μ parameter from the superpotential, and the ratio of the vacuum expectation values t_β. All of them have to be renormalized at the one-loop level,

$$m_q \to m_q + \delta^{(1)} m_q, \tag{2.18a}$$

$$\mathbf{M}_{\tilde{q}} \to \mathbf{M}_{\tilde{q}} + \delta^{(1)} \mathbf{M}_{\tilde{q}}. \tag{2.18b}$$

The renormalization of the top–stop sector, as well as of μ and t_β is carried out as specified in Ref. [64]. For the renormalization of the bottom–sbottom sector, we refer to Ref. [41, 46, 90] where renormalization of m_b and A_b in the $\overline{\mathrm{DR}}$ scheme has been proposed to avoid numerical instabilities. Also for the applied resummation of Δm_b the $\overline{\mathrm{DR}}$ scheme for m_b is convenient, as explained above. The renormalization scale is chosen to be the on-shell top mass.

- The bottom-quark self-energy in Lorentz decomposition is given by

$$\Sigma_b(p) = \not{p}\,\omega_-\,\Sigma_b^{\mathrm{L}}(p^2) + \not{p}\,\omega_+\,\Sigma_b^{\mathrm{R}}(p^2) + m_b\,\Sigma_b^{\mathrm{S}}(p^2) + m_b\gamma_5\,\Sigma_b^{\mathrm{PS}}(p^2)\,, \tag{2.19}$$

  with the left-vector part Σ_b^{L}, right-vector part Σ_b^{R}, scalar part Σ_b^{S}, and pseudo-scalar part Σ_b^{PS}. The bottom-quark mass renormalization is fixed at the on-shell top-mass scale via

$$\delta^{(1)} m_b = m_b\,\mathfrak{Re}\left[\frac{1}{2}\left(\Sigma_b^{\mathrm{L}}(m_b^2) + \Sigma_b^{\mathrm{R}}(m_b^2)\right) + \Sigma_b^{\mathrm{S}}(m_b^2)\right]_{\overline{\mathrm{DR}}}, \tag{2.20}$$

- With Eqs. (2.17)–(2.18) we define

$$\mathbf{U}_{\tilde{q}}\,\delta\mathbf{M}_{\tilde{q}}\,\mathbf{U}_{\tilde{q}}^\dagger = \begin{pmatrix} \delta^{(1)} m_{\tilde{q}_1}^2 & \delta^{(1)} m_{\tilde{q}_1\tilde{q}_2}^2 \\ \delta^{(1)} m_{\tilde{q}_1\tilde{q}_2}^{2\,*} & \delta^{(1)} m_{\tilde{q}_2}^2 \end{pmatrix}. \tag{2.21}$$

  The renormalization of the soft-breaking parameter A_b follows from Eqs. (2.16) and (2.21) with $q = b$, yielding

$$\begin{aligned} \delta^{(1)} A_b = \Bigg[& \mathbf{U}_{\tilde{b}\,11}\mathbf{U}_{\tilde{b}\,12}^* \frac{\delta^{(1)} m_{\tilde{b}_1}^2 - \delta^{(1)} m_{\tilde{b}_2}^2}{m_b} + \mathbf{U}_{\tilde{b}\,21}\mathbf{U}_{\tilde{b}\,12}^* \frac{\delta^{(1)} m_{\tilde{b}_1\tilde{b}_2}^2}{m_b} \\ & + \mathbf{U}_{\tilde{b}\,22}\mathbf{U}_{\tilde{b}\,11}^* \frac{\delta^{(1)} m_{\tilde{b}_1\tilde{b}_2}^{2\,*}}{m_b} - (A_b - \mu^* \tan\beta)\,\frac{\delta^{(1)} m_b}{m_b} \Bigg]_{\overline{\mathrm{DR}}}. \end{aligned} \tag{2.22}$$

  The counterterms $\delta^{(1)} m_{\tilde{b}_i}^2$, $i = 1, 2$ can be computed from the corresponding sbottom self-energies $\Sigma_{\tilde{b}_i\tilde{b}_i}^{(1)}$, $i = 1, 2$, and the counterterm $\delta^{(1)} m_{\tilde{b}_1\tilde{b}_2}^2$ is given by the sbottom mixing $\Sigma_{\tilde{b}_1\tilde{b}_2}^{(1)}$. Again, the renormalization scale is the on-shell top mass.

- Invariance under $SU(2)$ yields the following relation between the stop and sbottom sector,

$$\begin{aligned} \delta^{(1)} m_{\tilde{Q}_3} &\equiv \sum_{i=1}^{2} |\mathbf{U}_{\tilde{t}\,1i}|^2\,\delta^{(1)} m_{\tilde{t}_i}^2 - 2\mathfrak{Re}\Big[\mathbf{U}_{\tilde{t}\,22}\mathbf{U}_{\tilde{t}\,12}^*\,\delta^{(1)} m_{\tilde{t}_1\tilde{t}_2}^2\Big] - 2\,m_t\,\delta^{(1)} m_t \\ &= \sum_{i=1}^{2} |\mathbf{U}_{\tilde{b}\,1i}|^2\,\delta^{(1)} m_{\tilde{b}_i}^2 - 2\mathfrak{Re}\Big[\mathbf{U}_{\tilde{b}\,22}\mathbf{U}_{\tilde{b}\,12}^*\,\delta^{(1)} m_{\tilde{b}_1\tilde{b}_2}^2\Big] - 2\,m_b\,\delta^{(1)} m_b\,. \end{aligned} \tag{2.23}$$

  We apply on-shell conditions to both stop particles and the stop mixing angle, and choose to make $\delta^{(1)} m_{\tilde{b}_1}^2$ a dependent quantity by this relation. The other diagonal sbottom-mass counterterm is fixed on-shell via

$$\delta^{(1)} m^2_{\tilde{b}_2} = \mathfrak{Re}\left[\Sigma^{(1)}_{\tilde{b}_{22}}\left(m^2_{\tilde{b}_2}\right)\right]. \tag{2.24}$$

The quantity $\delta^{(1)} m^2_{\tilde{b}_1\tilde{b}_2}$ is the off-diagonal entry of Eq. (2.21) for $q = b$. It is already fixed by the renormalization condition of Eq. (2.22).

3 Numerical results for the Higgs spectrum

In the following numerical analysis the new contributions of $\mathcal{O}\left(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2\right)$ are added to the known Higgs-mass corrections in the general case of the MSSM with complex parameters which are implemented in `FeynHiggs` (version 2.12.0).[1] The improvement by resummation of leading logarithms as described in Refs. [56, 91] is not included. The large impact of the $\mathcal{O}\left(\alpha_t^2\right)$ terms has been investigated in Refs. [63, 64] and is not presented here again. Instead the focus is set on the new corrections induced by the finite bottom mass. If not stated otherwise, we choose the following default setting for the parameters entering through the new contributions:

$$t_\beta = 50,\ m_{H^\pm} = 1.5\,\text{TeV}, \quad m_t = 173.2\,\text{GeV}, \quad m_{\tilde{t}_\text{R}} = m_{\tilde{b}_\text{R}} = 2\,\text{TeV},\ m_{\tilde{Q}_3} = 2.1\,\text{TeV}, \tag{3.1a}$$

$$A_t = \left|1.3\, m_{\tilde{t}_\text{R}} + \frac{\mu}{t_\beta}\right| \mathrm{e}^{i\,\phi_{A_t}},\quad A_b = 2.5\, m_{\tilde{b}_\text{R}}\, \mathrm{e}^{i\,\phi_{A_b}},\quad M_3 = 2.5\,\text{TeV}\,\mathrm{e}^{i\,\phi_{M_3}}, \quad \mu = \text{sgn}[\mu]\, 1\,\text{TeV}. \tag{3.1b}$$

The quantities in Eq. (3.1a) are real parameters. The charged Higgs mass $m_{H^\pm}$ is chosen as an input parameter and its value is set to ensure the compatibilty of scenarios with high t_β with the current experimental constraints from searches for heavy MSSM Higgs bosons [92, 93]. The parameters in Eq. (3.1b) are in general complex. Their respective phases ϕ_{A_t}, ϕ_{A_b} and ϕ_{M_3} are scanned in section 3.2. Thereby the gluino mass parameter M_3 does not occur directly in the new Higgs self-energy contributions, but it appears in the leading term of the bottom-mass resummation. The parameter μ is also complex in general, but its phase is constrained to be very close to zero or π by EDM limits (see above). We remark that the phases ϕ_{M_3}, ϕ_{A_t} and ϕ_{A_b} are also constrained by EDM limits, but scenarios with large phases are possible (see *e.g.* Ref. [94]). We show results for the Higgs mass when varying two phases at the same time.

The absolute value of A_t has been fixed to yield a lightest Higgs-boson mass close to 125 GeV which can then be identified with the Higgs signal discovered at ATLAS and CMS. Together with $m_{\tilde{Q}_3}$ and $m_{\tilde{t}_\text{R}}$ it determines the mass shift which is induced by the stop contributions. We choose different values for $m_{\tilde{Q}_3}$ and $m_{\tilde{t}_\text{R}}$, $m_{\tilde{b}_\text{R}}$ to avoid numerical instabilities due to degeneracies. Different setups for $m_{\tilde{t}_\text{R}}$ and $X_t = A_t - \mu^*/t_\beta$ are possible to yield a lightest Higgs mass of 125 GeV as can be seen in Fig. 1. Therein the gray bar indicates the mass range $125.1 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})$ GeV as measured by ATLAS and CMS [95].

The absolute value of A_b is close to the upper limit

$$|A_b|^2 < 3\left(m^2_{H_d} + |\mu|^2 + m^2_{\tilde{Q}_3} + m^2_{\tilde{b}_\text{R}}\right), \tag{3.2a}$$

$$m^2_{H_d} + |\mu|^2 = \left(m^2_{H^\pm} - m^2_W\right)\sin^2\beta - \frac{1}{2} m_Z^2 \cos 2\beta\ , \tag{3.2b}$$

from the approximate bound from the requirement of vacuum stability to avoid charge- and color-breaking minima [96, 97] (see Refs. [98–104] for more detailed discussions of this issue).

[1] The previously implemented contributions of $\mathcal{O}\left(\alpha_t^2\right)$ are replaced by the new result.

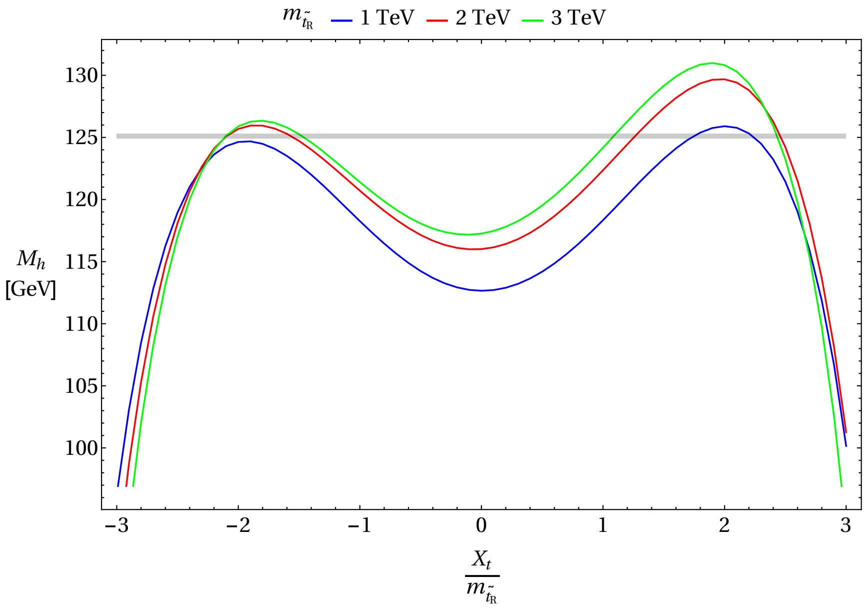

Figure 1. Dependence of the lightest Higgs-mass M_h on $m_{\tilde{t}_R}$ and X_t. The other parameters except A_t and $m_{\tilde{t}_R}$ have been fixed to the values given in Eqs. (3.1) with vanishing phases.

In the following analyses we call ΔM_h the shift of the lightest Higgs-boson mass by the new Yukawa terms of $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$, *i. e.* excluding the previously analyzed contributions of $\mathcal{O}(\alpha_t^2)$. In section 3.1, the impact of different parameters on the lightest Higgs boson mass in the CP-conserving case is investigated.

We have also investigated the mass shifts of the heavier neutral Higgs bosons. In general, the shifts are of the same absolute size as for the lightest Higgs but with opposite sign. However, since the tree-level input value $m_{H^\pm}$ needs to be large for high values of t_β (where the $\mathcal{O}(\alpha_t\alpha_b + \alpha_b^2)$ contributions are relevant) to be in agreement with experimental constraints, the relative mass shift for the heavy Higgs bosons is only $\approx 1‰$. Moreover, both heavy Higgs bosons receive nearly identical corrections; in the investigated scenarios the largest difference was $\approx 0.1\,\text{GeV}$. For this reason we do not present diagrams for the mass shifts of the heavier Higgs bosons here. It should however be noted that even small mass shifts can have an important impact on the resonance-type behaviour that typically occurs between the two heavy neutral Higgs states in CP-violating scenarios, see Refs. [105, 106].

3.1 Scenarios with real parameters

We start to analyze our results by performing a comparison with the previously implemented two-loop corrections in `FeynHiggs`. The two-loop corrections of $\mathcal{O}(\alpha_t\alpha_b)$ were up to now only known for the MSSM with real parameters and m_A being an input. Accordingly, a different Higgs-mass prediction is found when using `FeynHiggs` for the version of the MSSM with real or complex parameters even when using the same input mass m_A.

In Fig. 2 both predictions (dashed: MSSM with real parameters, dotted: MSSM with complex parameters) are compared to our new result (solid) as a function of m_A. The different colors correspond to different values of t_β. The large deviations between the dashed and dotted curves for large values of t_β are induced by the $\mathcal{O}(\alpha_t\alpha_b)$ terms, which are not incorporated in the dotted curve. Our new result significantly reduces that gap, *i. e.* the dashed and solid lines are much closer together. Since our new result additionally contains corrections of $\mathcal{O}(\alpha_b^2)$, which are not

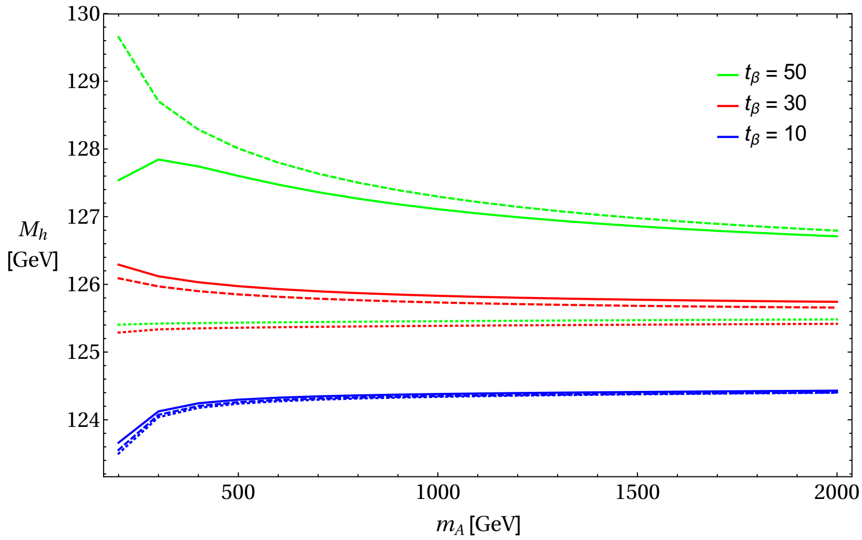

Figure 2. Comparison of the lightest Higgs-boson mass M_h as predicted with our new two-loop corrections (solid), the version of `FeynHiggs` for the MSSM with real parameters, *i. e.* including $\mathcal{O}(\alpha_t\alpha_b)$ corrections (dashed), and the version of `FeynHiggs` for the MSSM with complex parameters, *i. e.* without $\mathcal{O}(\alpha_t\alpha_b)$ corrections (dotted) for the parameters specified in Eqs. (3.1) with vanishing phases.

incorporated in the dashed line, the agreement is not expected to be perfect, and especially for low m_A and large t_β the improved Higgs-mass prediction significantly differs from the previous result.

In our following analyses we choose $m_{H^\pm}$ as an input parameter where also the $\mathcal{O}(\alpha_t\alpha_b)$ terms must be regarded as new contributions. We investigate the dependence of the prediction for M_h on t_β, μ and M_3, whereby all parameters are still kept real. The results are depicted in Figs. 3–6.

As can be seen in Fig. 3 large contributions above 1 GeV are only visible at high values of t_β. In this scenario M_3 is positive, leading to a much bigger ΔM_h if μ is negative, which can be understood from Eqs. (2.14) and (2.15). For later analyses we fix $t_\beta = 50$.

In Fig. 4 we investigate the dependence of ΔM_h on the size of μ. We find very large gradients for the following two cases: positive M_3 and negative $\mu \approx -1.8\,\text{TeV}$, and negative M_3 and positive $\mu \approx 2.6\,\text{TeV}$, which can again be understood from Eqs. (2.14)–(2.15), where for too large values of $|\mu|$ and opposite signs of μ and M_3 the perturbative region of parameter space is left, as $\Delta m_b \to -1$. A further increase of $|\mu|$ in the regions of large gradients leads to a very strong enhancement of the bottom Yukawa coupling and accordingly to very large negative mass shifts, yielding eventually a tachyonic Higgs boson. For the following analyses, we choose to fix $\mu = -1\,\text{TeV}$, *i. e.* below the problematic scale and with $\text{sgn}[\mu] = -1$. However, it should be noted that scenarios with positive μ can lead to large shifts as well, when M_3 is negative, as in both cases the bottom Yukawa coupling is enhanced. Moreover, scenarios with $\text{sgn}[\mu] = 1$ are in better agreement with constraints from the anomalous magnetic moment of the muon [107–109]. Close to $|\mu| = \sqrt{m^2_{\tilde{t}_{1,2}} - m_t^2} \approx 1.8\,\text{TeV}$ one can see kinks which are induced by threshold effects from the higgsino–top–stop system.

In Fig. 5 the impact of the gluino mass parameter is depicted. This effect enters the Higgs self-energies at the investigated order purely via the employed effective bottom mass. We see a rising shift at growing $|M_3|$ for opposite signs of μ and M_3 (yielding the same enhancement in Δm_b, see Eqs. (2.15)). At $|M_3| = \sqrt{m^2_{\tilde{b}_{1,2}} - m_b^2} \approx 2\,\text{TeV}$ (nearly invisible) threshold effects from the gluino–bottom–sbottom system appear. For our following analyses we fix $|M_3|$ above that region at 2.5 TeV.

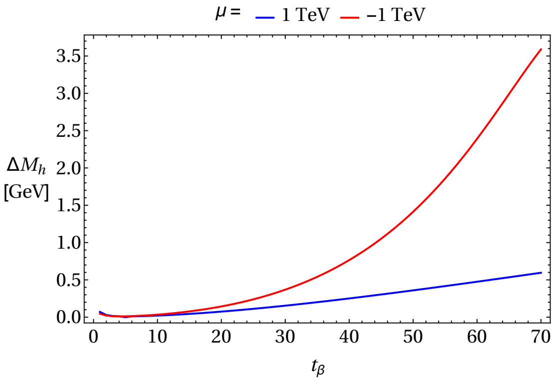

Figure 3. Dependence of the lightest Higgs-mass shift ΔM_h on t_β. The parameter μ is either positive (blue) or negative (red).

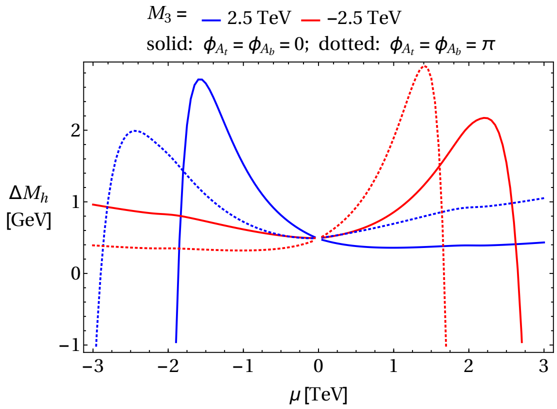

Figure 4. Dependence of the lightest Higgs-mass shift ΔM_h on μ. The parameter M_3 is either positive (blue) or negative (red). The region around $\mu = 0$ is left out because of numerical instabilities.

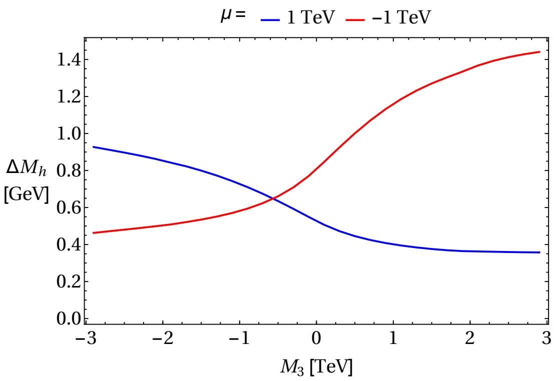

Figure 5. Dependence of the lightest Higgs-mass shift ΔM_h on M_3. The parameter μ is either positive (blue) or negative (red).

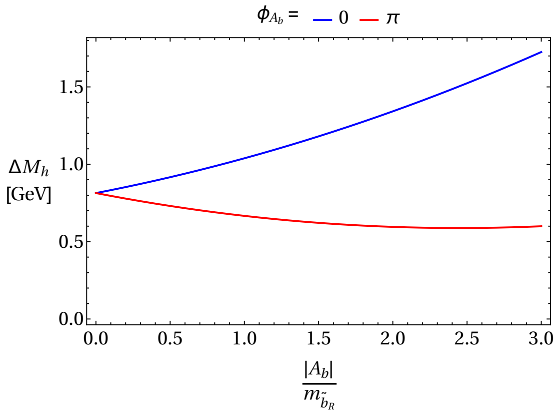

Figure 6. Dependence of the lightest Higgs-mass shift ΔM_h on $|A_b|$. The sign of A_b is either positive (blue) or negative (red), and $\mathrm{sgn}[\mu] = -1$.

Finally, in Fig. 6 the absolute value of A_b is varied, and the resulting mass shift is plotted for positive sign (blue) and negative sign (red) of A_b. The difference between both curves, *i. e.* the impact of the phase ϕ_{A_b}, is enhanced for larger absolute values. However, as too large values of $|A_b|$ lead to instable vacua, we set it to $|A_b| = 2.5\, m_{\tilde{b}_\mathrm{R}}$ which is close to the upper limit of Eq. (3.2).

3.2 Scenarios with complex parameters

Various phases enter the self-energies of the Higgs bosons at $\mathcal{O}\big(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2\big)$. Their impact on the Higgs sector is shown in Figs. 7–9. Here we keep μ negative, *i. e.* $\mathrm{sgn}[\mu] = -1$, and M_3 positive, but we could also have chosen the opposite signs of both parameters to see enhanced effects for the phase dependent terms as has been shown in Fig. 4.

We start with the phases ϕ_{A_t} and ϕ_{A_b}. The results are depicted in Fig. 7, where mass shifts between 0.3 GeV and 1.4 GeV can be seen. For $\phi_{A_b} = 0$ the variation with respect to ϕ_{A_t} is maximal; the larger the phase of A_b, the flatter the dependence on the phase of A_t. Similarly, variation of ϕ_{A_b} yields the largest effects for $\phi_{A_t} = 0$. Also the signs of the phases matter, *e. g.* the mass shifts are different for $\phi_{A_b} = \pm\frac{\pi}{2}$.

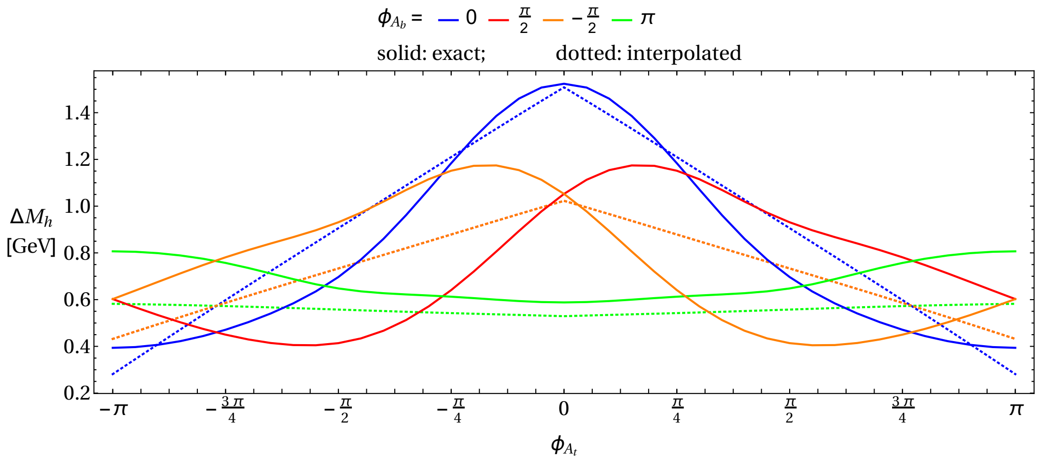

Figure 7. Dependence of the lightest Higgs-mass shift ΔM_h on ϕ_{A_t} and ϕ_{A_b}, $\text{sgn}[\mu] = -1$. solid: exact calculation, dotted: interpolation in `FeynHiggs`, the red-dotted and orange-dotted lines are identical.

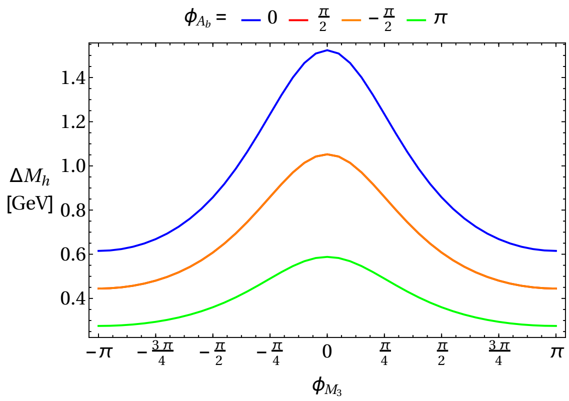

Figure 8. Dependence of the lightest Higgs-mass shift ΔM_h on ϕ_{M_3} and ϕ_{A_b}, $\text{sgn}[\mu] = -1$.

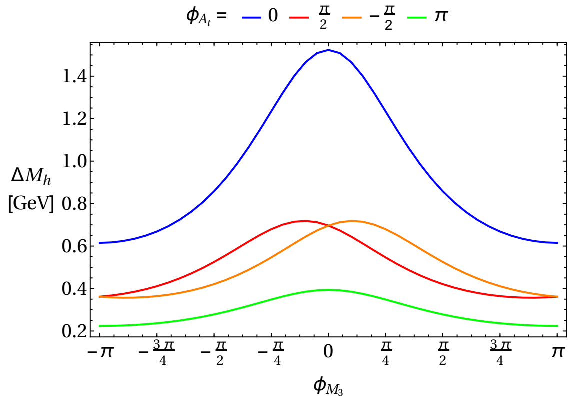

Figure 9. Dependence of the lightest Higgs-mass shift ΔM_h on ϕ_{M_3} and ϕ_{A_t}, $\text{sgn}[\mu] = -1$.

In addition to the exact calculation (solid lines), `FeynHiggs` offers an implemented interpolation of the self-energy corrections that have been known up to now for the case of real parameters but not for the complex case. Since the $\mathcal{O}(\alpha_t\alpha_b)$ terms were only available for real parameters, and the $\mathcal{O}(\alpha_b^2)$ terms were neither in the real nor the complex case incorporated in `FeynHiggs`, deviations from the new mass shifts can be expected even for real parameters. Besides these relatively small differences, the linear interpolation can differ by $\approx 0.5\,\text{GeV}$ from the full result in the investigated scenario. Also the asymmetric behaviour for the change of two phases at the same time was not described correctly by the interpolation.

Figs. 8 and 9 show the influence of varying the gluino phase ϕ_{M_3} and in addition either ϕ_{A_b} or ϕ_{A_t}. These terms are induced by the correction factor Δm_b as the investigated class of two-loop corrections does not contain the parameter M_3. Also here the largest phase dependence is found when one phase is equal to zero. In Fig. 8 the mass shift is nearly symmetric in $\pm\phi_{M_3}$ and $\pm\phi_{A_b}$, *i. e.* the red and yellow curves are lying on top of each other. Nevertheless, there are small asymmetries in the renormalized two-loop self-energies $\hat{\Sigma}_{hA}$ and $\hat{\Sigma}_{HA}$. On the contrary the mass shift ΔM_h in Fig. 9 shows a clear asymmetry similar to Fig. 7.

In summary, phase dependent contributions of $\mathcal{O}\big(\alpha_t\alpha_b + \alpha_b^2\big)$ lead to mass shifts of the lightest Higgs boson of $\approx 1\,\text{GeV}$ in the investigated scenarios. The sign of μ has been chosen to be negative in the considered scenarios, but similar effects can be found at positive large μ (and opposite sign of M_3).

4 Conclusions

The two-loop corrections of $\mathcal{O}\big(\alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2\big)$ to the Higgs-boson masses in the MSSM with complex parameters have been computed in the gauge-less limit at vanishing external momentum. The terms of $\mathcal{O}\big(\alpha_t\alpha_b + \alpha_b^2\big)$ have only been known in the special case of real parameters before, but only the contributions of $\mathcal{O}(\alpha_t\alpha_b)$ are incorporated in `FeynHiggs` at the moment. The specific aspects related to the renormalization of these new contributions have been discussed, and their numerical impact on the Higgs spectrum has been investigated.

For the lightest Higgs boson mass at $\approx 125\,\text{GeV}$ we have found shifts above $1\,\text{GeV}$ at $t_\beta > 40$ for different scenarios: moderate $|\mu| = 1\,\text{TeV}$ with negative sign and positive M_3, A_t, A_b, or with positive sign and negative M_3, A_t, A_b. The reason for that enhancement can be found in the large correction factor Δm_b yielding an enhancement of the bottom Yukawa coupling. The effect of varying the phases ϕ_{M_3}, ϕ_{A_t} and ϕ_{A_b} can be as large as $1\,\text{GeV}$. If one phase is set close to π, the dependence on the other phases is typically weakened; the largest effects are found when only one phase is varied with all others being zero. In `FeynHiggs` an interpolation of the corrections of $\mathcal{O}(\alpha_t\alpha_b)$ obtained for the case of real parameters was implemented for the case of complex parameters. We have found deviations with a size of $\approx 0.5\,\text{GeV}$ from this approximation, especially when several phases are different from zero at the same time.

Mass shifts for the heavier neutral Higgs bosons have not been depicted. They are similar to the ones of the lightest Higgs boson, however with opposite sign. To justify a large value of t_β we have chosen the input mass $m_{H^\pm} = 1.5\,\text{TeV}$. Therefore, the relative size of the mass shifts is small. Moreover, both heavy Higgs bosons receive similar corrections with a maximal difference of $\approx 0.1\,\text{GeV}$ in the investigated scenarios. Nevertheless, such small mass shifts can be important to correctly describe the resonance-type behaviour of nearly mass-degenerate mixed particles like the two heavy Higgs bosons in the MSSM with complex parameters.

The new results will be implemented in the public code `FeynHiggs`.

Acknowledgments

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