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Improved Test of *CPT* Invariance in Neutral-Kaon Decays

G. V. Dass

Deutsches Elektronen Synchrotron, Hamburg, Germany

and

P. K. Kabir*

University of Virginia, Charlottesville, Virginia 22901

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A measurement by Morse *et al.* is reanalyzed and combined with earlier measurements to improve existing limits on $\text{Re}\delta = \frac{1}{2}(\epsilon_S - \epsilon_L)$, a measure of possible deviation from *CPT* invariance in the neutral-kaon system. Some of the possible methods to further improve the limits on this parameter are reviewed.

Recently, Morse *et al.*¹ reported a measurement of the fraction of K^0 mesons which decay before $6\tau_S$, from which they deduced a certain limit on the possible deviation from *CPT* invariance in the neutral-kaon system. We reexamine the conclusion to be drawn from their measurements and show how, by combining their result with others previously obtained, one can improve the existing limits on possible variances from *CPT* invariance. Finally, we indicate methods which could further improve these limits.

Assuming only that there are two quasidegenerate neutral-kaon states $|K^0\rangle$ and $|\bar{K}^0\rangle$ whose decay is adequately described by the hypothesis that two particular linear superpositions $|K_S\rangle$ and $|K_L\rangle$ decay exponentially, these can be written in general as

$$|K_S\rangle \doteq [(1 + \epsilon_S + \frac{1}{2}i\theta)|K^0\rangle + (1 - \epsilon_S - \frac{1}{2}i\theta)|\bar{K}^0\rangle]/\sqrt{2}, \quad (1a)$$

$$|K_L\rangle \doteq [(1 + \epsilon_L - \frac{1}{2}i\theta)|K^0\rangle - (1 - \epsilon_L + \frac{1}{2}i\theta)|\bar{K}^0\rangle]/\sqrt{2}, \quad (1b)$$

where the *real* parameters² ϵ_S , ϵ_L , and θ can be shown^{3,4} to be small quantities of order 10^{-2} , which have been retained to lowest order only in Eqs. (1); equations valid only to lowest order in these parameters will be denoted by $\doteq$. *CPT* invariance requires $\epsilon_L = \epsilon_S$ and $\theta = 0$, i.e., $\delta = 0$. Inversion of Eqs. (1) yields

$$|K^0\rangle \doteq 2^{-1/2}[(1 - \epsilon_L + \frac{1}{2}i\theta)|K_S\rangle + (1 - \epsilon_S - \frac{1}{2}i\theta)|K_L\rangle], \quad (2a)$$

$$|\bar{K}^0\rangle \doteq 2^{-1/2}[(1 + \epsilon_L - \frac{1}{2}i\theta)|K_S\rangle - (1 + \epsilon_S + \frac{1}{2}i\theta)|K_L\rangle], \quad (2b)$$

so that if one has a number of K^0 mesons initially, the fraction surviving at time τ is

$$F(\tau) \doteq \frac{1}{2}(1 - 2\epsilon_L)e^{-\gamma_S\tau} + \frac{1}{2}(1 - 2\epsilon_S)e^{-\gamma_L\tau} + e^{-(\gamma_S + \gamma_L)\tau/2}[(\epsilon_L + \epsilon_S)\cos(m_L - m_S)\tau - \theta\sin(m_L - m_S)\tau]. \quad (3)$$

Morse *et al.* have measured the fraction that decays before $\tau = 6\tau_S$ and report the value as

$$G(6\tau_S) = 1 - F(6\tau_S) = 0.5037 \pm 0.0072. \quad (4)$$

For this value of τ , $F(\tau)$ receives its largest contribution from the second term in Eq. (3) and is therefore sensitive mainly to the value of ϵ_S . Using the same values for K_L and K_S lifetimes and masses, we find, in agreement with Ref. 1 except

for the sign of the last term, that

$$G(6\tau_s) \doteq 0.5037 + 1.037\epsilon_s + 0.049\epsilon_L + 0.017\theta. \quad (5)$$

The procedure followed by Morse *et al.* is to assume⁵ that $\text{Re}\epsilon = \frac{1}{2}(\epsilon_L + \epsilon_s)$ is determined by the leptonic charge asymmetry in K_L decays through the relationship which obtains when CPT invariance is exactly valid, including the requirement $\text{Re}\delta = 0$. They also neglect the (weak) dependence of Eq. (5) on $\theta = \frac{1}{2}\text{Im}\delta$. This is a perfectly valid procedure as long as one is trying to test CPT invariance, i.e., the value of $\text{Re}\delta = \frac{1}{2}(\epsilon_s - \epsilon_L)$ deduced in this way must vanish if CPT invariance holds. However, their quoted result

$$\text{Re}\delta = -0.0017 \pm 0.0072 \quad (6)$$

represents a measurement of $\text{Re}\delta$ only under the following joint assumptions:

(i) In the sum of K^0 decays to all channels, the only possible departure from CPT invariance could be through a nonvanishing value of the parameter $\text{Re}\delta$.

(ii) In $\pi l \nu$ decays of K_L , assuming the validity of the $\Delta S = \Delta Q$ rule, there is no deviation from the requirements of CPT invariance including $\text{Re}\delta = 0$.

It is clearly preferable to determine the parameter $\text{Re}\delta$, as well as other measures of possible variances from CPT symmetry, in a manner independent of any hypotheses regarding CPT invariance.

A well-known consequence⁶ of unitarity is

$$\begin{aligned} [\tfrac{1}{2}(\gamma_L + \gamma_s) - i(m_L - m_s)]^{-1} \sum_f \langle f | T | K_L \rangle^* \langle f | T | K_S \rangle \\ = \langle K_L | K_S \rangle \\ \doteq (\epsilon_s + \epsilon_L) + i\theta. \end{aligned} \quad (7)$$

A conservative estimate⁷ based on numerical evaluations^{8,9} of the left-hand side without any assumptions about CPT invariance yields

$$2 \times 10^{-3} \leq (\epsilon_s + \epsilon_L) \leq 4 \times 10^{-3} \quad (8)$$

and

$$|\theta| \leq 2 \times 10^{-3}. \quad (9)$$

Using these limits in conjunction with Eqs. (4) and (5), we obtain

$$\epsilon_s = -0.0001 \pm 0.0073. \quad (10)$$

The value of ϵ_s determined by the measurement of Morse *et al.* [Eq. (5)] thus does not depend on the value of θ at the quoted level of accuracy, while the uncertainty in the value of $(\epsilon_s + \epsilon_L)$ [Eq. (8)] introduces at most an uncertainty of $\pm 10^{-4}$.

Crawford³ has reported the ratio of K_S yields in the reactions

$$\bar{p}p \rightarrow K_S K^- \pi^+ (\pi^0) \quad (11a)$$

and

$$\bar{p}p \rightarrow K_S K^+ \pi^- (\pi^0) \quad (11b)$$

to be

$$R = 1.00 \pm 0.032. \quad (12)$$

Under the hypothesis of charge-conjugation invariance of strong interactions, verified¹⁰ to an accuracy of 1% in such reactions, this represents the ratio of probabilities for K_S decays of K^0 and $\bar{K}^0$ mesons, respectively; according to Eqs. (2), this is given by

$$R \doteq 1 - 4\epsilon_L. \quad (13)$$

Crawford's determination [Eq. (12)] therefore yields

$$\epsilon_L = 0.00 \pm 0.008. \quad (14)$$

If ϵ_s and ϵ_L were treated as uncorrelated variables, the combinations of Eqs. (10) and (14) would yield

$$\begin{aligned} \text{Re}\delta &= \tfrac{1}{2}(\epsilon_s - \epsilon_L) \\ &= 0.00 \pm 0.005. \end{aligned}$$

However, we know that ϵ_s and ϵ_L are constrained by the inequalities (8). If the limits of error in Eqs. (10) and (14) are interpreted as the limits between which the corresponding quantities must

FIG. 1. The values of ϵ_L and ϵ_s are constrained to lie within the shaded strip as a consequence of unitarity. The broken line shows the limits of error for ϵ_L found by Crawford (Ref. 3) while the dot-dashed lines are the limits of error for ϵ_s corresponding to the measurement of Morse *et al.* (Ref. 1). CPT invariance requires $\epsilon_L = \epsilon_s$.

lie, the combination of (8) and (14) yields

$$-6 \times 10^{-3} \leq \epsilon_S \leq 12 \times 10^{-3},$$

while the combination of (8) and (10) yields

$$-5 \times 10^{-3} \leq \epsilon_L \leq 11 \times 10^{-3}.$$

Better limits are obtained by combining the unitarity constraint (8) with (14) and (10), yielding

$$-6 \times 10^{-3} \leq \epsilon_S \leq 7 \times 10^{-3} \quad (15)$$

and

$$-5 \times 10^{-3} \leq \epsilon_L \leq 8 \times 10^{-3}. \quad (16)$$

These limits are shown in Fig. 1. The corresponding limits for $\text{Re}\delta = \frac{1}{2}(\epsilon_S - \epsilon_L)$ are

$$-7 \times 10^{-3} \leq \text{Re}\delta \leq 6 \times 10^{-3}, \quad (17)$$

which are better than the existing limits⁷ $-7 \times 10^{-3} \leq \text{Re}\delta \leq 10 \times 10^{-3}$ derived from (8) and (14), or the quoted result (6).

It will be seen that the limits on $\text{Re}\delta$ are appreciably looser than those for $|\text{Im}\delta| \leq 10^{-3}$; in fact they are significantly larger than the maximum possible value of $\text{Re}\epsilon$, the established measure of T noninvariance in neutral-kaon decays. It would obviously be desirable to determine $\text{Re}\delta$ with greater accuracy.

It is apparent from Fig. 1 that an improved measurement of either ϵ_S or ϵ_L would lead to better knowledge of the other parameter and therefore also of $\text{Re}\delta = \frac{1}{2}(\epsilon_S - \epsilon_L)$. Possible methods to determine ϵ_S and ϵ_L have been discussed previously^{3,2} and also recently⁷; they all seem to require measurements with relative accuracy comparable to the desired limit for $\text{Re}\delta$. Besides measurements of the kind reported by Morse *et al.*, which are sensitive mainly to ϵ_S , one may also determine ϵ_L directly by measuring the strangeness of a K_L beam. The probabilities that a K_L meson will behave like a kaon of positive or negative strangeness, respectively, are

$$P_L^\pm \doteq \frac{1}{2}(1 \pm 2\epsilon_L). \quad (18)$$

A measurement of the relative probability to an accuracy of better than 2% would suffice to improve the present limits (16).

A measurement which yields $(\epsilon_S - \epsilon_L)$ directly is a comparison of K_S and K_L yields from K^0 and $\bar{K}^0$ mesons, respectively. If the partial-decay rates into any channel from K^0 and $\bar{K}^0$ mesons, respectively, are given by

$$r_\pm(\tau) = a_\pm e^{-\gamma_S \tau} + b_\pm e^{-\gamma_L \tau} + e^{-(\gamma_L + \gamma_S)\tau/2} \text{Re}(c_\pm e^{i(m_L - m_S)\tau}), \quad (19)$$

we have

$$\frac{(a_+/a_-)}{(b_+/b_-)} \doteq 1 + 4(\epsilon_S - \epsilon_L). \quad (20)$$

Frank *et al.*¹¹ have reported an experiment where such quantities are measured for the $\pi^+\pi^-$ channel. Again, an accuracy of 2% or better in the determination of the ratio (20) would improve our present knowledge of ϵ_S and ϵ_L . Another possible method of measuring $(\epsilon_S - \epsilon_L)$ directly is through accurate measurement of total decay rates at $\tau \ll \tau_S$. The total decay rate, at $\tau=0$, for initial K^0 and $\bar{K}^0$ mesons is given respectively by

$$R_\pm(0) \doteq \frac{1}{2}\gamma_S \left\{ 1 + \frac{\gamma_L}{\gamma_S} \pm \left[(\epsilon_S - \epsilon_L) + 2\theta \left(\frac{m_L - m_S}{\gamma_S} \right) \right] \right\} \quad (21)$$

to lowest order in (γ_L/γ_S) . A precise measurement of $R_+(0)$ or $R_-(0)$ can therefore determine $(\epsilon_S - \epsilon_L)$ to the same accuracy as θ provided that γ_S is known with comparable accuracy. If γ_S is not so well determined, the ratio $R_+(0)/R_-(0)$ can be used.

We hope that improved experiments will allow the CPT -noninvariance parameter $\text{Re}\delta = \frac{1}{2}(\epsilon_S - \epsilon_L)$ to be determined to a level of accuracy at least equal to the magnitude $[(1-2) \times 10^{-3}]$ of the T -noninvariance parameter $\text{Re}\epsilon = \frac{1}{2}(\epsilon_L + \epsilon_S)$.

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in Eqs. (1) by $2\epsilon = \epsilon_S + \epsilon_L$ and $2\delta = (\epsilon_S - \epsilon_L) + i\theta$.

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Sum Rule for Photon Structure Function νW_2^γ

R. A. Buchl and B. P. Nigam

Department of Physics, Arizona State University, Tempe, Arizona 85281

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It is assumed that a sum rule of the Bloom and Gilman type applies to the photon structure function $\nu W_2^\gamma(q^2, \nu)$ and the scaling limit function $\nu W_2^\gamma(\omega)$, where $\omega = -2\nu/q^2$. We consider a dominating resonance for photon-photon scattering and find from the sum rule that the form factor depends only on q^2 .

The deep-inelastic process $e^- + \gamma \rightarrow e^- + \text{anything}$ involving the scattering of electrons on a photon target has recently been investigated by Brodsky, Kinoshita, and Terazawa.¹ The differential cross section for inelastic scattering of electrons on a bremsstrahlung photon contains the hadronic structure functions of the photon, $W_1^\gamma(q^2, \nu)$ and $W_2^\gamma(q^2, \nu)$.¹ The process $\gamma + \gamma \rightarrow \text{anything}$ is also of interest in colliding e - e beams since the two annihilating photons may produce any neutral $C = +1$ hadronic state. Photon-photon scattering has been studied by a number of authors^{2,3} and a total of four form factors are measurable in $e + \gamma \rightarrow e + \text{anything}$,³ although these four form factors may be combined in pairs and expressed in terms of the two photon structure functions.

In this paper we consider the possibility of a resonance of mass M_R formed in photon-photon scattering as illustrated in Fig. 1, and we assume a sum rule of the Bloom and Gilman⁴ type for the structure function νW_2^γ . We therefore consider the amplitude $T_2^\gamma(q^2, \nu)$, and in analogy with virtual Compton scattering,⁵ we write

$$T_2^\gamma(q^2, \nu) = \frac{2|A(q^2, \nu)|^2}{s - M_R^2 - i\epsilon}, \quad (1)$$

where $\nu = p \cdot q$, $p^2 = 0$, and $s = (p + q)^2$. We call $A(q^2, \nu)$ the $\gamma\gamma$ -resonance vertex form factor.

From unitarity, the forward scattering amplitude $T_2^\gamma(q^2, \nu)$ for photons scattering off photons may be related to the photon structure function

$W_2^\gamma(q^2, \nu)$ for deep-inelastic electroproduction by

$$\text{Im} T_2^\gamma(q^2, \nu) = \pi W_2^\gamma(q^2, \nu). \quad (2)$$

It follows from Eqs. (1) and (2) that $W_2^\gamma(q^2, \nu)$ may be expressed as

$$W_2^\gamma(q^2, \nu) = |A(q^2, \nu)|^2 \delta(\nu - \frac{1}{2}(M_R - q^2)). \quad (3)$$

We assume that a Bloom and Gilman⁵ sum rule applies to photon-photon scattering, and express it in the form

$$\frac{2}{-q^2} \int_{-q^2/2}^{(W_R^2 - q^2)/2} d\nu \nu W_2^\gamma(q^2, \nu) = \int_1^{1+W_R^2/(-q^2)} d\omega \nu W_2^\gamma(\omega) \quad (-q^2 \text{ large}), \quad (4)$$

where $W_R = (q^2 + \nu_R)^{1/2}$ and the deep-inelastic scaling variable ω is defined by $\omega = -2\nu/q^2$. The sum rule, Eq. (4), is assumed to be valid provided $-q^2$ and ν_R are chosen in the region where $\nu W_2^\gamma(q^2, \nu)$ scales. This implies that for $\nu < \nu_R$, $\nu W_2^\gamma(\omega)$ is a smooth function which averages $\nu W_2^\gamma(q^2, \nu)$ in the finite-energy sum-rule sense. We find from the sum rule and with $W_2^\gamma(q^2, \nu)$ given by Eq. (3) that

$$\left(1 - \frac{M_R^2}{q^2}\right) |A(q^2, \nu_R)|^2 = \int_1^{1+W_R^2/(-q^2)} d\omega \nu W_2^\gamma(\omega). \quad (5)$$