Semi-Leptonic Decays of the F-Mesons and the Ensuing Lepton Spectrum.

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Preliminary evidence for the charmed meson $F^\pm$, through its nonleptonic decay mode $\eta\pi^\pm$, has recently been reported by the DASP collaboration at DESY ⁽¹⁾. The mass of the $F^\pm$ constructed through the $\eta\pi^\pm$ -mode has a value (2.03 ± 0.06) GeV, with the mass of the F^* set at (2.14 ± 0.06) GeV. This establishes the FF and FF* thresholds near 4.1 and 4.2 GeV, respectively. The same group has also presented inclusive electron spectra between 3.99 GeV and 5.2 GeV c.m. energy ⁽²⁾. The events, characterized by a single electron plus at least two charged tracks, are divided into three energy regions, namely, i) $3.99 < E_{\text{c.m.}} < 4.08$ GeV, ii) $4.08 < E_{\text{c.m.}} < 4.52$ GeV and iii) $4.52 < E_{\text{c.m.}} < 5.2$ GeV. The division is made with the view of studying the semi-leptonic decay spectrum coming from i) the D-decays alone, ii) the possible effects due to the F-production and decay, and iii) the effects due to possible charmed-baryon production and decays. The qualitative features of the electron spectra in the three energy regions are not very different, but a more detailed analysis is probably necessary in order to notice any difference. An immediate theoretical question is: How does the lepton spectrum from the $F^\pm$ semi-leptonic decays compare with the one from the D-decays?

We give in this note a quantitative estimate of the semi-leptonic decay modes of the F-meson. The semi-leptonic decays of the D-mesons were previously studied by the present authors ⁽³⁾ and by others ⁽⁴⁾. The qualitative features of the electron spec-

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trum in the interval $3.99 < E_{\text{c.m.}} < 4.08$ GeV are well described by the decay modes $D \rightarrow K\bar{\nu}_e$ and $D \rightarrow K^*\bar{\nu}_e$. This means presumably that the multi-hadronic decay modes do not contribute significantly to the semi-leptonic decays. We shall adopt a similar attitude toward the semi-leptonic decays of the $F^\pm$ meson, namely, it is an equally good approximation to neglect the multi-hadron semi-leptonic decay modes of the F -meson. The semi-leptonic decays of the F -meson are different from that of the D -mesons in the following respect: there are more low lying single-particle modes available to the $F^\pm$ -meson than to the D -mesons. Thus, *a priori*, the lepton spectrum from F -decays could be different from that of the D -meson decays (in details).

We assume the standard $\Delta c = \Delta s$ rule. Since in the standard charm model both the charm-changing charged weak current and the F -meson are isosinglets, the semi-leptonic decays of F involve only final states with $I = Y = 0$. We consider only the following low lying single-particle decay modes:

$$(1) \quad \left\{ \begin{array}{l} F^\pm \rightarrow \eta l^\pm \nu_l, \\ \quad \rightarrow \eta' l^\pm \nu_l, \\ \quad \rightarrow \phi l^\pm \nu_l, \\ \quad \rightarrow \omega l^\pm \nu_l. \end{array} \right.$$

The last decay mode of (1), namely, $\omega l^\pm \nu_l$, is suppressed by the Zweig rule, since ω has very little strange quark content (ideal mixing). But for the pseudoscalars, η and η' , since they are not ideally mixed, the amount of mixing is quite important. This introduces an additional complication which is not present in the decays of D -mesons. To a good approximation, the η and η' mesons are SU_3 octet and singlet respectively,

$$\eta \sim \eta_8 = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s}) \quad \text{and} \quad \eta' \sim \eta_0 = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s}).$$

Dropping the additional mixing angle (between η_8 and η_0), one finds that

$$(2) \quad \frac{\Gamma(F^\pm \rightarrow \eta l^\pm \nu_l)}{\Gamma(F^\pm \rightarrow \eta' l^\pm \nu_l)} \simeq 2 \times \text{phase space},$$

where the factor 2 comes simply from counting the $s\bar{s}$ quark content of η_8 and η_0 . However, the small mixing angle between η_8 and η_0 (around 11 degrees) turns out to make significant modification to (2). If we express η and η' as

$$(3) \quad \left\{ \begin{array}{l} \eta = \cos \theta \eta_8 + \sin \theta \eta_0 = \eta_\sigma \cos \alpha - \eta_s \sin \alpha, \\ \eta' = -\sin \theta \eta_8 + \cos \theta \eta_0 = \eta_\sigma \sin \alpha + \eta_s \cos \alpha \end{array} \right.$$

(where $\eta_\sigma \equiv (1/\sqrt{2})(u\bar{u} + d\bar{d})$ and $\eta_s \equiv s\bar{s}$), we find that $\theta \simeq 11^\circ$ corresponds to $\alpha \simeq 43.7^\circ$, and thus the probability of finding $s\bar{s}$ pair in the η -meson is about the same as in η' . Consequently we find

$$(4) \quad \frac{\Gamma(F^\pm \rightarrow \eta l^\pm \nu_l)}{\Gamma(F^\pm \rightarrow \eta' l^\pm \nu_l)} \simeq 0.92 \times \text{phase space}.$$

(The number 0.92 becomes 1 if one uses $\alpha = 45^\circ$).

In deriving (4), we have assumed SU_3 symmetry for the ratio of the form factors $f_+(0)$ of the above two processes. Since SU_3 symmetry-breaking correction to the difference of the form factors at zero momentum transfer comes in second order (*), we have reasons to believe that (4) provides a good approximation (up to the uncertainty of the mixing angle). (4) could be checked experimentally.

What remains to be calculated now is the branching ratio of $F \rightarrow \eta l \nu$ and $F \rightarrow \phi l \nu$. We define as in ref. (3) the following form factors:

$$(5) \quad \langle \eta(k) | V_\mu^{\Delta c = \Delta s} | F(p) \rangle = f_+(q^2)(p + k)_\mu + f_-(q^2)q_\mu,$$

where $q = p - k$. Here only the vector current contributes. For $F \rightarrow \phi l \nu$ we have the form factors of the vector and axial-vector currents as given by

$$(6) \quad \begin{cases} \langle \phi(k) | V_\mu | F(p) \rangle = iF_1^V(q^2)\varepsilon_{\mu\nu\lambda\sigma}\varepsilon^\nu k^\lambda q^\sigma, \\ \langle \phi(k) | A_\mu | F(p) \rangle = F_1^A(q^2)\varepsilon_\mu + F_2^A(q^2)(\varepsilon \cdot p)k_\mu + F_3^A(\varepsilon \cdot p)q_\mu, \end{cases}$$

where ε is the polarization vector of the ϕ -meson.

As in ref. (3), we employ the current algebra hard meson technique (5) to calculate the various form factors. However, in the case of $F \rightarrow \eta l \nu$ (or $\eta' l \nu$), the method is not useful, since we do not know the *mixing* among the hitherto poorly understood $I = Y = 0$ axial-vector mesons. But since the form factors $f_+(0)$ for $F \rightarrow \eta l \nu$ and $D \rightarrow K l \nu$ are related by SU_3 , and as remarked above (*) the SU_3 breakings to the difference of them appears in second order, we take the $f_+(0)$ of $F \rightarrow \eta l \nu$ as given by SU_3 symmetry. We assume for simplicity a monopole form factor (**). One finds

$$(7) \quad f_+^\eta(q^2) = f_+^\eta(0) \frac{m_{F^*}^2}{m_{F^*}^2 - q^2} = \frac{1}{\sqrt{2}} \frac{m_{F^*}^2}{m_{F^*}^2 - q^2}.$$

The contribution of the q_μ term in (5), being proportional to m_l in the decay rate, can be neglected.

For the decay of $F \rightarrow \phi l \nu$, the axial-vector form factors are given by current-algebra hard-meson technique as previously applied to the $D \rightarrow K^* l \nu$ decay, we find similarly

$$(8) \quad \begin{cases} F_1^A(q^2) = \sqrt{2}m_\phi \left[1 + \frac{1}{2(q^2 - m_{FA}^2)} \left\{ (m_\phi^2 - q^2) + \frac{\delta}{2}(m_F^2 + m_\phi^2 - q^2) \right\} \right], \\ F_2^A(q^2) = -\frac{\delta m_\phi}{\sqrt{2}(q^2 - m_{FA}^2)}, \end{cases}$$

where δ is a parameter as in ref. (3). The above result (8) already takes into account the SU_3 symmetry between $D \rightarrow K^* l \nu$ and $F \rightarrow \phi l \nu$. As in ref. (3), we have assumed $F_D = F_F = F_K = F_\pi$, where the F 's are the (axial-vector current) decay constants of

(*) This is a corollary of the Ademollo-Gatto theorem. Let $\lambda H'$ be the SU_4 symmetry-breaking term. Under SU_3 transformation, $\lambda H' = \lambda_3 u_3 + \lambda_8 u_8 + \lambda_{15} u_{15}$ with obvious notations. By Ademollo-Gatto theorem, the SU_4 corrections to $f_+(0)$ come in second order of λ . But the difference of the two $f_+(0)$ does not depend on λ_{15} , therefore the SU_3 correction to the difference of the two $f_+(0)$ comes in second order of λ_3, λ_8 .

(*) H. J. SCHNITZER and S. WEINBERG: *Phys. Rev.*, **464**, 1828 (1967).

(**) This corresponds to $\delta = -1$ for the $D \rightarrow K^* l \nu$ form factor as given in ref. (3).

the pseudoscalar mesons. The vector form factor of $F \rightarrow \phi l \bar{\nu}$ is not determined by the current algebra, but is related to $D \rightarrow K^* l \bar{\nu}$ by SU_3 . We have, in the case of $D \rightarrow K^* l \bar{\nu}$, considered two approaches: 1) exact SU_4 relation for the three-point vertices, 2) broken SU_4 corrections via the Gell-Mann–Okubo ansatz⁽⁶⁾. This gives for $F \rightarrow \phi l \bar{\nu}$, in pole-dominated form

$$(9) \quad F_1^V(q^2) = \frac{\sqrt{2} g_{F^*} g_{F^* \phi F}}{q^2 - m_{F^*}^2} = \frac{\sqrt{2} (m_{F^*}/m_\rho) g_\rho (g_{\omega \rho \pi}/\sqrt{2})}{q^2 - m_{F^*}^2} = \frac{m_{F^*} m_\rho}{\sqrt{2}} \left[\frac{3\Gamma(\omega \rightarrow \pi \gamma)}{\alpha k^3} \right]^{\frac{1}{2}},$$

where we assumed exact SU_4 symmetry for $g_{F^* \phi F}$ and $g_{\omega \rho \pi}$. g_{F^*} is defined by $\langle 0 | V_\mu | F^* \rangle = g_{F^*} \varepsilon_\mu$. If we take into consideration SU_4 symmetry breaking à la Gell-Mann–Okubo, $g_{F^* \phi F}$ decreases by about 30% compared with (8)⁽³⁾. Since the vector form factor does not give the dominant contribution, the SU_4 symmetry-breaking corrections will not appreciably affect the decay rates.

TABLE I. – Decay rates and the relative rates for the semi-leptonic decays of the $F^\pm$ -mesons with $m_F = 2.03$ GeV and SU_4 symmetry assumed for F_1^V (see text).

Mode	Decay Widths		
	Units (s^{-1})		
$F \rightarrow \eta e \bar{\nu}_e$	$1.2 \cdot 10^{11}$		
$F \rightarrow \eta' e \bar{\nu}_e$	$0.35 \cdot 10^{11}$		
$F \rightarrow \phi e \bar{\nu}_e$	$\delta = 0$	$\delta = -0.5$	$\delta = -1.0$
	$0.49 \cdot 10^{11}$	$0.57 \cdot 10^{11}$	$0.66 \cdot 10^{11}$
$\Gamma(F \rightarrow \eta' e \bar{\nu}_e)/\Gamma(\eta e \bar{\nu}_e) =$	0.29		
$\Gamma(F \rightarrow \phi e \bar{\nu}_e)/\Gamma(F \rightarrow \eta e \bar{\nu}_e) =$	$\delta = 0$	$\delta = -0.5$	$\delta = -1$
	0.4	0.48	0.55

TABLE II. – The same as for table I, with the SU_4 symmetry breaking for F_1^V calculated à la Gell-Mann–Okubo.

Mode	Decay Widths		
	Units (s^{-1})		
$F \rightarrow \eta e \bar{\nu}_e$	$1.2 \cdot 10^{11}$		
$F \rightarrow \eta' e \bar{\nu}_e$	$0.35 \cdot 10^{11}$		
$F \rightarrow \phi e \bar{\nu}_e$	$\delta = 0$	$\delta = -0.5$	$\delta = -1.0$
	$0.45 \cdot 10^{11}$	$0.54 \cdot 10^{11}$	$0.63 \cdot 10^{11}$
$\Gamma(F \rightarrow \eta' e \bar{\nu}_e)/\Gamma(F \rightarrow \eta e \bar{\nu}_e) =$	0.29		
$\Gamma(F \rightarrow \phi e \bar{\nu}_e)/\Gamma(F \rightarrow \eta e \bar{\nu}_e) =$	$\delta = 0$	$\delta = -0.5$	$\delta = -1.0$
	0.38	0.45	0.53

⁽⁶⁾ A. KAZI, G. KRAMER and D. H. SCHILLER: DESY 76¹⁸ (1976).

Taking the F and F* masses from ref. (1), *i.e.* $m_F = 2.03$ GeV, $m_{F^*} = 2.14$ GeV, our numerical results using eqs. (3)-(8) are given in table I for F_1^V given by (9) and table II for F_1^V including the SU_4 symmetry breaking. We take δ to vary from 0

Fig. 1. — Electron spectra from the decays $F^\pm \rightarrow \eta e^\pm \nu_e$, $F^\pm \rightarrow \eta' e^\pm \nu_e$ and $F^\pm \rightarrow \phi e^\pm \nu_e$, all normalized to unit area. The effect of the various values of δ on $F \rightarrow \phi e \bar{\nu}_e$ is negligible. $\delta = 0$ — — —, $\delta = -1$ - - - -.

to -1 . The electron spectrum for each individual decay modes are shown in fig. 1. Up to the q^2 -dependence, the peaking of the electron energy distribution reflects essentially the kinematics of phase space. In fig. 2 we plot the inclusive electron spectrum after summing over the decay modes, with normalization as determined in table I.

Fig. 2. — Inclusive electron spectrum for the semi-leptonic decays of the $F^\pm$ -mesons, normalized with $\Gamma(F \rightarrow \eta' e \bar{\nu}_e)/\Gamma(F \rightarrow \eta e \bar{\nu}_e) = 0.3$, $\Gamma(F \rightarrow \phi e \bar{\nu}_e)/\Gamma(F \rightarrow \eta e \bar{\nu}_e) = 0.55$.

Comparing the electron spectrum from the F-decay with that of the D-decays, we note that the decay modes $F \rightarrow \eta e \bar{\nu}_e$ and $F \rightarrow \phi e \bar{\nu}_e$ are very similar to $D \rightarrow K e \bar{\nu}_e$ and $D \rightarrow K^* e \bar{\nu}_e$, respectively. The semi-leptonic decays of F have an additional decay mode $F \rightarrow \eta' e \bar{\nu}_e$, which tends to make the electron spectrum softer (by phase space). If $\eta' e \bar{\nu}_e$ were an important mode of the semi-leptonic decays, the difference between the electron spectrum from F- and D-decays would be noticeable. Our calculation indicates that only 15% of the semi-leptonic F-decays goes into $\eta' e \bar{\nu}_e$ and thus we do not anticipate any major change in the electron spectrum when the F-meson is produced in e^+e^- annihilation. The data ⁽²⁾ do not indicate any major change in the electron spectrum as $E_{c.m.}$ moves from 4 to 5 GeV.

To conclude, we note that the nonleptonic decays of the F-meson are very different from those of the D-mesons. If we could use the signatures of the nonleptonic decays of the F to separate the electron spectrum of the F-mesons from that of the D-mesons (*e.g.* the electron spectrum from inclusive $e\eta + \dots$ *vs.* $eK + \dots$), we could test the semi-leptonic decays of the F and D mesons in a more quantitative way.

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