Multihadron decays of new mesons

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We discuss the hadronic decays of the new $I=0$ mesons seen in e^+e^- : $J(\psi)$ or ψ' with $G=-$ and $P_c(\chi)$ or X with $G=+$. We present some isospin inequalities for $I=0$ pure pionic final states, and a discussion of $K\bar{K}$ and η, η' fractions. We also present a statistical-model analysis of pion final states, and conclude that a large fraction of hadronic $J(\psi)$ decays contain something besides pions and $K\bar{K}$ —probably η and η' , possibly radiative modes.

I. INTRODUCTION

There is now plenty of evidence that the new resonances $J(\psi)(3.1)^{1,2}$ and $\psi'(3.7)^3$ are hadrons, with $J^{PC}=1^{--}$, odd G parity, and isospin zero. Isospin-violating decays take place via $J \rightarrow 1\gamma \rightarrow$ hadrons. Recently, even- C states have been found at 3.4 and 3.5 GeV, reached via $\psi' \rightarrow P_c(\chi) + \gamma$.^{4,5,6} There is also some evidence for a state at 2.8 GeV, $J \rightarrow X(2.8) + \gamma$, $X \rightarrow \gamma\gamma$ and $p\bar{p}$.^{6,7} The former states have the decay modes (3.4), (3.5) $\rightarrow 2(\pi^+\pi^-)$, $\rightarrow 3(\pi^+\pi^-)$.⁵ This shows that they have even G and even I . Evidence that (3.4) $\rightarrow \pi^+\pi^-$ and $K\bar{K}$ indicates $I=0$ (see Ref. 5); so would a $p\bar{p}$ final state.

In view of the present experimental situation, it is useful to study the hadronic decays of J , ψ' , and χ , using the fact that they have $I=0$, or assuming it. That is what we will do here. First, we present rigorous isospin bounds according to the methods of Llewellyn Smith and Pais,⁸ also extending the old isospin statistical model of Pais⁹ to a larger number of pions. To this we add a few remarks on final states with $K\bar{K}$, η , and η' . The rest of the paper contains a discussion of the present experimental data.

We hope that this material will be useful to experimentalists and offer a partial view of the final hadron states in the decay of the new particles seen in e^+e^- . This may be of importance in view of the very small hadronic width of these mesons. Perhaps a study of final states will help us understand the mechanism which suppresses the hadronic width.

II. PIONS, KAONS, η , and η'

A. Pions

In this sub section we present isospin bounds and a statistical model for pure $I=0$ pionic final states. The method goes back to a very useful paper of Pais.⁹ We review his method briefly. For such bounds to be useful in practice, final

state pions must not arise from η or η' decay. We will discuss this at the end of the next subsection. We will assume here that events containing $K\bar{K}$, η , η' and nucleon-antinucleon pairs can be segregated out and treated separately.

Isospin-0 states with N pions can be labelled by three numbers N_1, N_2, N_3 , where $N_1 \geq N_2 \geq N_3 \geq 0$, $N_1 - N_3$ even, $N_2 - N_3$ even, $N = N_1 + N_2 + N_3$.¹⁰ These numbers designate a Young tableau of the unitary group in three dimensions $U(3)$, and constitute a labeling of all unitary representations of this group. All the consequences of isospin conservation for $I=0$ decay to pions then follow from the fact that the isospin group $SU(2)_I$ is a subgroup of this $U(3)$. Labeling a charge partition of m_c ($\pi^+\pi^-$) pairs and m_0 π^0 mesons by (m_c, m_0) , the probability that this will be found in a state $(N_1N_2N_3)$ is given by a Pais coefficient⁹ $[N_1N_2N_3 | m_cm_0]$, normalized so that $\sum_{m_cm_0} [N_1N_2N_3 | m_cm_0] = 1$ for fixed $N = 2m_c + m_0$. These coefficients can be calculated by combinatoric methods and by establishing nontrivial identities between them. The branching ratio Γ for a state containing only N pions is

$$\Gamma(2m_c\pi^c, m_0\pi^0) = \frac{1}{\Delta} \sum_{N_i} \alpha(N_1N_2N_3) [N_1N_2N_3 | m_cm_0], \quad (1)$$

where

$$\alpha(N_1N_2N_3) = \rho(N_1N_2N_3) K(N_1N_2N_3),$$

$$\rho(N_1N_2N_3) = \frac{N!(N_1 - N_2 + 1)(N_1 - N_3 + 2)(N_2 - N_3 + 1)}{(N_1 + 2)!(N_2 + 1)!(N_3)!}, \quad (2)$$

and the non-negative $K(N_1N_2N_3)$ take care of the (unknown) dynamics; $\rho(N_1N_2N_3)$ is the dimensionality of the $(N_1N_2N_3)$ representation of S_3 , and simply counts the number of available states (measures the isospin phase space). The normalizations are

$$\Delta = \sum_{N_i} \alpha(N_1 N_2 N_3), \quad (3)$$

$$\sum_{m_c, m_0} \Gamma(2m_c \pi^c, m_0 \pi^0) = 1,$$

where $N = 2m_c + m_0$ and the sum N_i is over all $I=0$ partitions of fixed $N = N_1 + N_2 + N_3$. Because of the linearity of (1), the problem of finding bounds for

Γ reduces to that of finding that $[N_1 N_2 N_3 | m_c m_0]$ which is largest or smallest for a given N . The bound corresponds to $K(N_1 N_2 N_3) = 1$ for that partition and to zero for all others. The bounds for N odd can be applied to $J, \psi' \rightarrow (N - m_0) \pi^c m_0 \pi^0$, $m_0 = 1, 3 \dots$, or to single-photon $e^+ e^-$ annihilation to pions; the bounds for N even can be applied to $\chi_1 \rightarrow (N - m_0) \pi^c m_0 \pi^0$, $m_0 = 0, 2 \dots$, and to $e^+ e^-$. The bounds for N odd are

$$\left. \begin{array}{l} N=3: 1 \\ N=5: \frac{2}{3} \\ N=7: \frac{1}{3} \\ N \geq 9: 0 \end{array} \right\} \leq \Gamma((N-1)\pi^c, 1\pi^0) \leq \frac{2^{N-3} \left[\left(\frac{N-3}{2} \right)! \right]^2}{(N-2)!}, \quad N \geq 3, \quad (4)$$

$$\left. \begin{array}{l} 5 \leq N \leq 13: \frac{2^{N-4} \left[\left(\frac{N-3}{2} \right)! \right]^2}{(N-2)!} \\ N \geq 15: 0 \end{array} \right\} \leq \Gamma((N-3)\pi^c, 3\pi^0) \leq \begin{cases} \frac{1}{3}, & N=5, \\ \frac{2}{3}, & N=7, \\ \frac{2^{N-9} \left[\left(\frac{N-9}{2} \right)! \right]^2}{(N-8)!}, & N \geq 9. \end{cases}$$

The bounds for N even are

$$\left. \begin{array}{l} N=2: \frac{2}{3} \\ N=4: \frac{1}{3} \\ N \geq 6: 0 \end{array} \right\} \leq \Gamma(N\pi^c) \leq \frac{2^N \left[\left(\frac{N}{2} \right)! \right]^2}{(N+1)!}, \quad N \geq 2, \quad (5)$$

$$\left. \begin{array}{l} 2 \leq N \leq 10: \frac{2^{N-1} \left[\left(\frac{N}{2} \right)! \right]^2}{(N+1)!} \\ N \geq 12: 0 \end{array} \right\} \leq \Gamma((N-2)\pi^c, 2\pi^0) \leq \begin{cases} \frac{1}{3}, & N=2, \\ \frac{2}{3}, & N=4, \\ \frac{2^{N-6} \left[\left(\frac{N-6}{2} \right)! \right]^2}{(N-5)!}, & N \geq 6. \end{cases}$$

The existence of nonzero lower bounds for low N may be especially useful.¹¹ Note that in the limit $N - 3m_0 \rightarrow \infty$ the branching ratios fulfill

$$0 \leq \Gamma((N - m_0)\pi^c, m_0\pi^0) \leq \left[\frac{\pi}{2} (N - 3m_0) \right]^{1/2}.$$

We mention that the isospin bounds arise when all but one certain $K(N_1 N_2 N_3)$ are zero. The opposite extreme is to assume that they are all equal. This produces an isospin statistical model for fixed N , with each $[N_1 N_2 N_3 | m_c m_0]$ weighted by the number of states $\rho(N_1 N_2 N_3)$ corresponding to this class.⁹ Dynamical effects will give deviations from this model. Being unable to find a general

formula, we extended the $[N_1 N_2 N_3 | m_c m_0]$ tables of Pais ($N \leq 8$) up to $N=13$ for odd N and to $N=10$ for even N . The results (including $N \leq 8$ for completeness) can be found in Tables I(a) and I(b).

It is less obvious how to construct a statistical model for the pion distribution in N . The simplest possibility, which we will use, is to assume a Poisson distribution,

$$P(N) = k^{-1} \frac{\langle N \rangle^N}{N!} e^{-\langle N \rangle}, \quad (6)$$

for N odd or even, $2 \leq N \leq 22$, normalizing by $\sum P(N) = 1$ with the sum over even or odd N as the case requires. We will use this to estimate the total pion branching ratio.

B. Kaons, η , and η'

The situation for final states with $K\bar{K}$ is more obscure. We can obtain a few simple isospin relations for such final states by exploiting the fact that the initial state has $I=0$.¹² There are then

$$[d\sigma(K^0\bar{K}^0X^0) - d\sigma(K^+K^-X^0)]^2 \leq 4d\sigma(K^+\bar{K}^0X^-)[d\sigma(K^0\bar{K}^0X^0) + d\sigma(K^+K^-X^0) - d\sigma(K^+\bar{K}^0X^-)]. \quad (8)$$

These are summed over the internal variables of X but not over $K\bar{K}$ momenta. For pure $I=1K\bar{K}$, (7) is an equality and of course $d\sigma(K^0\bar{K}^0X^0) = d\sigma(K^+K^-X^0)$ for vanishing isoscalar/isovector interference.

It is possible to do better for exclusive final states. For $\bar{K}K\pi$ (or nucleon-antinucleon-pion) we have pure $I=1$ and¹³

$$\begin{aligned} 2\bar{K}^0K^0\pi^0 &= 2K^+K^-\pi^0 \\ &= K^+\bar{K}^0\pi^-. \end{aligned} \quad (9)$$

For $K\bar{K}\pi\pi$ we can get strong relations only after integrating over all $\pi\pi$ momenta. It is also possible to hold the $\pi\pi$ mass fixed (not integrated out), so long as the c.m. $\pi\pi$ angles are integrated over. The isoscalar/isovector interference then vanishes; this is because $I=0$ and $I=1$ correspond to different $\pi\pi$ angular momenta. Then

$$\begin{aligned} K^+\bar{K}^0\pi^-\pi^0 &= K^+K^0\pi^+\pi^0, \\ K^+K^-\pi^+\pi^- &= \bar{K}^0K^0\pi^+\pi^-, \\ K^+K^-\pi^0\pi^0 &= \bar{K}^0K^0\pi^0\pi^0, \\ 2K^+K^-\pi^+\pi^- &= 4K^+K^-\pi^0\pi^0 + K^+\bar{K}^0\pi^-\pi^0. \end{aligned} \quad (10)$$

This last relation enables us to get nearly unmeasurable modes like $K^+K^-\pi^0\pi^0$ and $K^0\bar{K}^0\pi^0\pi^0$ from measurable ones.

Note that if we again appeal to statistical considerations it appears reasonable to weight the isovector:isoscalar ratios by 3:1 and to ignore the interference of the two. Then

$$\begin{aligned} d\sigma(\bar{K}^0K^0X^0) &= d\sigma(K^+K^-X^0) \\ &= d\sigma(K^+\bar{K}^0X^-) \\ &= d\sigma(K^+K^-X^+). \end{aligned} \quad (11)$$

Inclusive isospin relations can also be derived for other combinations of final-state particles using the zero isospin of the initial state. Just as an example, consider inclusive $K\pi$ states with isospin $\frac{1}{2}$ amplitude a and $I=\frac{3}{2}$ amplitude b . Then

two amplitudes for inclusively produced $K\bar{K}$ with $I=0$ and $I=1$. Eliminating the $I=0,1$ interference term we find the inclusive rate $d\sigma(abX)$

$$d\sigma(K^0\bar{K}^0X^0) + d\sigma(K^+K^-X^0) \geq d\sigma(K^+\bar{K}^0X^-), \quad (7)$$

and bounding the interference term,

$$\begin{aligned} d\sigma(K^+\pi^-) &= \frac{1}{3}|a|^2 + \frac{1}{12}|b|^2 + \frac{1}{3}\text{Re}ab^*, \\ d\sigma(K^+\pi^0) &= \frac{1}{6}|a|^2 + \frac{1}{6}|b|^2 + \frac{1}{3}\text{Re}ab^*, \\ d\sigma(K^0\pi^+) &= \frac{1}{3}|a|^2 + \frac{1}{12}|b|^2 - \frac{1}{3}\text{Re}ab^*, \\ d\sigma(K^0\pi^0) &= \frac{1}{6}|a|^2 + \frac{1}{6}|b|^2 - \frac{1}{3}\text{Re}ab^*, \\ d\sigma(K^+\pi^+) &= d\sigma(K^0\pi^-) = \frac{1}{4}|b|^2, \end{aligned} \quad (12)$$

from which we obtain the equalities

$$\begin{aligned} d\sigma(K^+\pi^-) + d\sigma(K^0\pi^0) &= d\sigma(K^+\pi^0) + d\sigma(K^0\pi^+), \\ 2[d\sigma(K^+\pi^0) + d\sigma(K^0\pi^0)] &= 2d\sigma(K^+\pi^+) + d\sigma(K^+\pi^-) + d\sigma(K^0\pi^+). \end{aligned} \quad (13)$$

Of course we can go back to (12) and derive inequalities for inclusive rates or for different charge partitions of exclusive channels. In deriving relations for exclusive channels [e.g. Eq. (10')], it is necessary to remember to include combinatorial factors for identical particles.

In order to obtain relations for final states with η or η' , we need to make assumptions beyond isospin invariance. For this reason we will make only a few brief comments, and these are model-dependent.

It is widely believed that the new mesons seen so far in e^+e^- and in hadron collisions are composites of new heavy quarks, and that these states are SU(3) singlets [ignoring SU(3) breaking]. If this is so, and if we can ignore $J \rightarrow 1\gamma \rightarrow$ hadrons in first approximation, we expect all octet mesons to be produced with equal rate, and¹⁴

$$\begin{aligned} d\sigma(K^+X^-) &= d\sigma(K_s^0X^0) \\ &= d\sigma(\eta X^0) \end{aligned} \quad (14)$$

(pure octet η). If in addition η' is an SU(3) singlet built of the familiar u, d, s quarks only,

$$d\sigma(\eta X) = d\sigma(\eta' X). \quad (15)$$

In applying this, remember that $\eta' \rightarrow \pi\pi\pi$ is a prominent decay. Even if SU(3) breaking suppresses production of heavy pseudoscalars K, η, η' relative to pions by a large factor, (14) and (15) may still be good to 20–40% typical for mass splitting

TABLE III. Statistical-model branching ratios (in percent) for $G=+$, N even, $\Gamma((N-m_0)\pi^c, m_0\pi^0)$ normalized to $\sum_N \Gamma_N^{\text{total}} = 100\%$.

N	m_0	0	2	4	6	8	10	Γ_N^{total}
2	2		1					3
4	6		8	1				15
6	5	17	5	0.3				27
8	2	15	9	1	0.03			27
10	0.7	7	8	2	0.1	0.003		18
$\sum_{N \geq 12} \Gamma_N^{\text{total}} = 10$								

directions, we take the fraction of all events with $K\bar{K}$ to be simply $2 \times (14\%) \times 0.86 = 24\%$. The actual fraction could be a bit larger. This includes decays $J \rightarrow 1\gamma \rightarrow \text{hadrons}$ with a $K\bar{K}$ pair. The smallest identifiable branching ratio is for events with $N\bar{N}$. If we take twice the fraction containing $\bar{p}$ with momentum $p < 1$ GeV we have $\approx 6\%$ for this. We estimate the fraction $J \rightarrow N\pi$ (N even) to be 6% , based on the modes $J \rightarrow 4\pi^c, 6\pi^c$ and the same statistical model as before, but now for N even (keeping $\langle N \rangle = 7$). In this way we can obtain

$$\begin{aligned} &\Gamma(J \rightarrow \text{pions}) + \Gamma(J \rightarrow \text{pions} + \geq 1K\bar{K}) \\ &\quad + \Gamma(J \rightarrow \text{pions} + N\bar{N}) \\ &\quad = 29\% + 24\% + 6\% = 59\%, \quad (18) \end{aligned}$$

excluding production of η, η' and also excluding possible radiative decays [e.g. $J \rightarrow \gamma\eta', \gamma X(2.8)$]. The error in (18) is surely large ($\pm 10\%$, say), and hard to estimate. The decays $\gamma\eta, \gamma\eta'$ are negligible^{6,7} but $\gamma X(2.8)$ probably is not. The most likely candidates for most of the marginally "missing" 27% are decays of the type $J \rightarrow (\eta \text{ or } \eta') + \text{pions} \rightarrow \text{more pions}$, plus possible photons from η or η' decays. We have already remarked that the fraction of events with $\eta + \eta'$ could be as large as that with $K\bar{K}$. Notice that G -parity-conserving decays of

J lead to final states $(N\text{pions}) + (\eta \text{ or } \eta')$ with N odd. Thus, since η or η' decays always contain ≥ 1 neutral π^0 or γ , the final pion state in this case has always ≥ 2 neutrals. On the other hand, the presence of η and η' can only lead to final states with charged pions and one neutral through the 1γ decay of J . This source of contamination is surely small, relative to G -conserving decays of J .

If it is possible to separate out the direct decay of $\psi' \rightarrow (N-1)\pi^c 1\pi^0$, subtracting those that arise from $\psi' \rightarrow J\pi\pi$, it should be possible to use these channels to guess the direct decay branching ratio for $\psi' \rightarrow \text{hadrons}$.

It is difficult to make any definite statements concerning the $C=+$ states at 3.4 and 3.5 GeV at present. We only observe here that for the meson at 3.4 GeV, the branching ratios for ψ' into a photon plus $\pi^+\pi^- + K^+K^-$, $4\pi^+$, or $6\pi^+$, quoted to be⁵ $(0.13 \pm 0.05)\%$, $(0.14 \pm 0.07)\%$, and 0.1% , are consistent with Table III (assuming $\pi^+\pi^- = K^+K^-$). We can use this to guess at a total pion branching ratio $\psi' \rightarrow \gamma + \chi(3.4) \rightarrow \gamma + \text{pions}$ of $(2 \pm 1)\%$. Since kaon channels have been seen, we feel safe in multiplying this by $\approx 86\%/23\% = 3.7$ (the ratio of all hadrons to all pions at the J) so as to estimate a branching ratio $\psi' \rightarrow \gamma + \chi(3.4) \rightarrow \gamma + \text{hadrons}$ of $(7 \pm 3)\%$. The error is solely experimental. A similar number emerges for $P_c(3.5)$. Then the quoted branching ratio $\psi' \rightarrow P_c \gamma \rightarrow J + \gamma\gamma = (4 \pm 2)\%$,⁶ and the fact that $(P_c' \rightarrow J + \gamma)/(P_c \rightarrow J + \gamma)$ is roughly 2 events/6 events = $\frac{1}{3}$,⁶ indicates that the hadronic decays of $P_c'(3.4)$ are 70–90% of the total $P_c'(3.4)$ decay rate, if $P_c' = \chi(3.4)$. For the $P_c(3.5)$, the estimated range is 50–70%. Of course, this all assumes that only G -conserving hadronic or $J + \gamma$ decays occur.

Note added in proof. We wish to thank P. Jasselette for calling our attention to his extensive tables of Pais coefficients.¹⁸

¹J. J. Aubert *et al.*, Phys. Rev. Lett. **33**, 1404 (1974).
²J.-E. Augustin *et al.*, Phys. Rev. Lett. **33**, 1406 (1974).
³G. S. Abrams *et al.*, Phys. Rev. Lett. **33**, 1453 (1974).
⁴DASP Collaboration, Phys. Lett. **57B**, 407 (1975).
⁵G. J. Feldman *et al.*, Phys. Rev. Lett. **35**, 821 (1975).
⁶B. Wiik, DESY Report No. DESY 75/37, 1975 (unpublished).
⁷J. Heintze, in *Proceedings of the 1975 International Symposium on Lepton and Photon Interactions at High Energies, Stanford, California*, edited by W. T. Kirk (SLAC, Stanford, 1976), p. 97.
⁸C. H. Llewellyn Smith and A. Pais, Phys. Rev. D **6**, 2625 (1972).
⁹A. Pais, Ann. Phys. (N.Y.) **9**, 548 (1960).
¹⁰This method is more general; we only need it for $I=0$.
¹¹For corresponding inequalities for prong branching

ratios, see A. Pais, Phys. Rev. Lett. **32**, 1081 (1974).
¹²We ignore the $J \rightarrow 1\gamma \rightarrow I=1$ hadrons decay here.
¹³Many of the following have also been derived by F. Gilman, SLAC Report No. SLAC-PUB-1600, 1975 (unpublished).
¹⁴S. Kitakado and T. F. Walsh, Lett. Nuovo Cimento **12**, 547 (1975).
¹⁵This has also been emphasized by H. Harari, Phys. Lett. **60B**, 172 (1976). He infers a large η' fraction solely from the heavy-quark-antiquark content of the η' .
¹⁶A. M. Boyarski *et al.*, SLAC Report No. SLAC-PUB-1599/LBL-3897 (unpublished).
¹⁷C. Moorhouse, talk given at the SLAC Summer Symposium, 1975 (unpublished).
¹⁸P. Jasselette, Nucl. Phys. **B23**, 481 (1970).