Nonperturbative Improvement and Tree-level Correction of the Quark Propagator

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We extend an earlier study of the Landau gauge quark propagator in quenched QCD where we used two forms of the $\mathcal{O}(a)$-improved propagator with the Sheikholeslami-Wohlert quark action. In the present study we use the nonperturbative value for the clover coefficient $c_{\rm sw}$ and mean-field improvement coefficients in our improved quark propagators. We compare this to our earlier results which used the mean-field $c_{\rm sw}$ and tree-level improvement coefficients for the propagator. We also compare three different implementations of tree-level correction: additive, multiplicative, and hybrid. We show that the hybrid approach is the most robust and reliable and can successfully deal even with strong ultraviolet behavior and zero-crossing of the lattice tree-level expression. We find good agreement between our improved quark propagators when using the appropriate nonperturbative improvement coefficients and hybrid tree-level correction. We also present a simple extrapolation of the quark mass function to the chiral limit.

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I. INTRODUCTION

Lattice studies of the quark propagator provide a direct, model-independent window into the mechanism of dynamical chiral symmetry breaking and its momentum dependence. In addition it provides insight into the the nature and location of the transition region of QCD where inherently nonperturbative behavior evolves into the more analytically accessible perturbative form. Furthermore, direct lattice calculations of the quark propagator inform hadron model building from the relativized constituent quark picture to quark models based on Schwinger-Dyson equations [1, 2].

In a recent paper [3] we presented a method for removing the dominant ultraviolet tree-level lattice artifacts in the momentum-space quark propagator. This was a generalization of the concept of tree-level correction, which was first introduced in the study of the gluon propagator [4, 5, 6, 7]. It was shown that, for two different $\mathcal{O}(a)$-improved propagators S_I and S_R, see Eqs. (2) and (3), and using a mean-field improved action, this leads to a dramatic improvement in the ultraviolet behavior of the propagator. However, the remaining ultraviolet artifacts are sufficiently large to make the results unreliable beyond $pa \sim 1.2$. Moreover, the two improved propagators remain discernibly different even in the infrared, yielding different estimates for the infrared quark mass.

Here we will present results using nonperturbatively determined values for the $\mathcal{O}(a)$ improvement coefficients, rather than the tree-level and mean-field improved coefficients used in Ref. [3]. We will also present two alternative techniques for removing tree-level artifacts and will discuss the relative merits of the three methods.

II. IMPROVEMENT

The general scheme for $\mathcal{O}(a)$ improvement of the quark propagator was discussed in Ref. [3]. Here we restrict ourselves to presenting the formulae and definitions which we will be using in this paper. For further details, see Ref. [3] and references therein.

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The SW fermion action,

$$\mathcal{L}(x) = \mathcal{L}^W(x) - \frac{i}{4} c_{\mathrm{sw}} a \overline{\psi}(x) \sigma_{\mu\nu} F_{\mu\nu}(x) \psi(x) \,, \tag{1}$$

combined with appropriate improvements of operators can be shown [8] to remove all $\mathcal{O}(a)$ errors in on-shell matrix elements. For off-shell quantities such as the quark propagator it is not that simple, and no general proof of $\mathcal{O}(a)$ improvement is known. Indeed, to calculate gauge dependent quantities one might expect to have to introduce gauge non-invariant (but BRST invariant) terms in the action. However, at tree level it is possible to proceed by adding all possible dimension-5 operators to the action and eliminating all but the clover (SW) term by a field redefinition [9]. Beyond tree level, one may proceed by adding all possible terms with the correct dimensionality and quantum numbers to the operator in question, and tuning the parameters to eliminate $\mathcal{O}(a)$ terms. Ignoring the gauge non-invariant terms (which are discussed in Refs. [10, 11]) we may write down the following expressions for the $\mathcal{O}(a)$ improved quark propagator,

$$S_I(x,y) \equiv \langle S_I(x,y;U) \rangle \equiv \langle (1 + b_q am) S_0(x,y;U) - a\lambda\delta(x-y) \rangle \,, \tag{2}$$

$$S_R(x,y) \equiv \langle S_R(x,y;U) \rangle \equiv \langle (1 + b'_q am) \left[1 - c'_q \not{D}(x) \right] S_0(x,y;U) \left[1 + c'_q \overleftarrow{\not{D}}(y) \right] \rangle \,. \tag{3}$$

where $S_0(x,y;U)$ for a given configuration U is the inverse of the fermion matrix. Note that if we are only interested in on-shell quantities such as hadronic matrix elements, the δ-function can be ignored, so we only need S_0 together with the improvement coefficients for the various operators. From this, S_I is easily obtained, whereas S_R is computationally somewhat more expensive.

The coefficients b_q, b'_q, λ and c'_q must be tuned in order to eliminate $\mathcal{O}(a)$ errors in the propagator (as far as this is possible). At tree level, their values are $b_q = 1, \lambda = b'_q = \frac{1}{2}, c'_q = \frac{1}{4}$. The values for b_q and λ have recently been calculated at one-loop level [12]. The mean-field improved values for all these coefficients may be obtained by dividing the tree-level values by the mean link u_0. The one-loop mean-field improved values have also been calculated in Ref. [12] but as we will argue, the small changes in values obtained by including the one-loop contribution have very little practical effect, so we will not use these here. It should also be noted that the mean-field improved value for λ is very close to the nonperturbative value reported in Ref. [13]. This indicates that mean-field improvement of the coefficients b_q and λ (or, alternatively, b'_q and c'_q) may be sufficient to remove $\mathcal{O}(a)$ errors to the desired precision.

The bare mass also receives an $\mathcal{O}(a)$ correction, which can be expressed as follows

$$\widetilde{m} = (1 + b_m am) m \qquad , \qquad am = \frac{1}{2\kappa} - \frac{1}{2\kappa_c} \,. \tag{4}$$

The coefficient b_m has been calculated at one-loop order [14],

$$b_m = -\frac{1}{2} - 0.0962 g_0^2 + \mathcal{O}(g_0^4) \,. \tag{5}$$

When evaluating Eq. (5), we will be using the boosted coupling constant $g^2 = g_0^2/u_0^4$.

III. TREE-LEVEL CORRECTION

In the continuum, the spin and Lorentz structure of the quark propagator, together with parity symmetry, determines that the propagator must have the following form,

$$S(\mu;p) = \frac{Z(\mu;p^2)}{i \not{p} + M(p^2)} \equiv \frac{1}{i \not{p} A(\mu;p^2) + B(\mu;p^2)} \,. \tag{6}$$

On the lattice, what we measure is the bare (regularized but unrenormalized) propagator. This differs from the renormalized propagator in Eq. (6) by an overall renormalization constant $Z_2(\mu,a)$, which we will absorb into $Z(p)$, as we did in Ref. [3] to simplify the presentation of our results.

In Ref. [3] we defined a tree-level correction procedure involving an overall multiplicative correction and an additive correction of the mass function, as follows,

$$S^{-1}(pa) = \frac{1}{Z(pa) Z^{(0)}(pa)} \left[ia \not{k} + a M^a(pa) + a \Delta M^{(0)}(pa) \right] , \tag{7}$$

where $k_\mu = \sin(p_\mu a)/a$ and $Z^{(0)}$ and $\Delta M^{(0)}$ are defined by the tree-level quark propagator,

$$\left(S^{(0)}(pa)\right)^{-1} = \frac{1}{Z^{(0)}(pa)}\left[i\not{k}a + am + a\Delta M^{(0)}(pa)\right] . \tag{8}$$

The functions $Z(pa)$ and $M^a(pa)$ should then express the nonperturbative behavior of the quark propagator, with the dominant lattice artifacts removed. We saw that this procedure led to a dramatic improvement in the behavior of these functions, but at large momenta the data could still not be trusted because of large cancellations.

Here we will consider an alternative, purely multiplicative tree-level correction procedure, defined by

$$S^{-1}(pa) = \frac{1}{Z(pa)Z^{(0)}(pa)}\left[ia\not{k} + aM^m(pa)Z_m^{(0)}(pa)\right] , \tag{9}$$

where $Z_m^{(0)}(pa)$ is defined by the tree-level expression,

$$\left(S^{(0)}(pa)\right)^{-1} = \frac{1}{Z^{(0)}(pa)}\left[i\not{k}a + amZ_m^{(0)}(pa)\right] . \tag{10}$$

The tree-level corrected mass function M is thus obtained from the uncorrected function $M^L \equiv \mathrm{tr}\, S^{-1}/4N_c$ via

$$aM^m(pa) = M^L(pa)/Z_m^{(0)}(pa) . \tag{11}$$

This procedure should not suffer from the problem of large cancellations. However, it will encounter problems when either the numerator or denominator of Eq. (11) crosses or is close to zero. In order to remedy this problem, we consider a third, 'hybrid' scheme, where the negative part of the tree-level expression is subtracted, while the remaining positive part is multiplicatively corrected. Specifically, we define $\Delta M^{(+)}, \Delta M^{(-)}$ such that

$$\Delta M^{(+)}(pa) + \Delta M^{(-)}(pa) = \Delta M^{(0)}(pa) \tag{12}$$

$$\Delta M^{(+)}(pa) \geq 0; \quad \Delta M^{(-)}(pa) \leq 0 \quad \forall pa \tag{13}$$

Then we can write

$$M^{(0)}(pa) = am + \Delta M^{(+)}(pa) + \Delta M^{(-)}(pa) \equiv amZ_m^{(+)}(pa) + \Delta M^{(-)}(pa) . \tag{14}$$

The tree-level corrected mass function $M^h(pa)$ is then

$$aM^h(pa) = \left(M^L(pa) - a\Delta M^{(-)}(pa)\right)/Z_m^{(+)}(pa) . \tag{15}$$

The definition of this scheme contains an ambiguity, since it is obvious that we may still satisfy (12), (13) by adding any strictly positive term to $\Delta M^{(+)}$ and subtracting the same term from $\Delta M^{(-)}$; e.g. by taking $\Delta M^{(+)} \to \Delta M^{(+)} + k^2; \Delta M^{(-)} \to \Delta M^{(-)} - k^2$. In order to remove this ambiguity, we add the following criteria:

1. Factoring out a common (positive) denominator, $\Delta M^{(+)}$ and $\Delta M^{(-)}$ should be polynomials in the 4 variables $k^2, \hat{k}^2, \Delta k^2$ and m, where we have defined

$$\hat{k}_\mu \equiv \frac{2}{a}\sin(p_\mu a/2); \qquad a^2\Delta k^2 \equiv \hat{k}^2 - k^2 ; \tag{16}$$

2. The coefficient of each term must be positive for $\Delta M^{(+)}$ and negative for $\Delta M^{(-)}$;

3. Any one monomial in $k^2, \hat{k}^2, \Delta k^2$ and m can only occur in one of $\Delta M^{(+)}$ or $\Delta M^{(-)}$; eg., if there is a term proportional to mk^2 in $\Delta M^{(-)}$ there cannot be a term proportional to mk^2 in $\Delta M^{(+)}$

These criteria ensure that $\Delta M^{(+)}$ and $\Delta M^{(-)}$ are as small as possible, leading to the minimum possible distortion of the data.

Specifically, the expressions we use are

$$Z_{m,I}^{(+)}(p) = \frac{am + (b_q - \lambda)a^2m^2 + \lambda a^4\Delta k^2 + (\frac{1}{2}b_q - \lambda)a^3m\hat{k}^2}{am(1+am)} , \tag{17}$$

$$a\Delta M_I^{(-)}(p) = -\frac{\lambda a^4\hat{k}^4/2 + (2\lambda - 1)a^2\hat{k}^2}{2(1+am)} , \tag{18}$$

$$aZ_{m,R}^{(+)}(p) = \frac{1}{amA'_R(p)}\left(am + \frac{1}{2}a^4\Delta k^2\right) , \tag{19}$$

$$a\Delta M_R^{(-)}(p) = -\frac{1}{16A'_R(p)}\left(a^3mk^2 + \frac{1}{2}a^4k^2\hat{k}^2\right) , \tag{20}$$

β	Volume	a^{-1} (GeV)	$c_{\rm sw}$	κ	prop	am	m (MeV)	$\widetilde{m}$ (MeV)	$N_{\rm cfg}$
6.0	$16^3 \times 48$	2.120	MF	0.13700	S_I	0.0579	123	118	499
					S_R				20
				0.13810	S_I	0.0289	61	60	499
6.0	$16^3 \times 48$	2.120	NP	0.13344	S_I	0.0498	105	102	10
					S_R				20
				0.13417	S_I	0.0294	62	61	10
				0.13455	S_I	0.0188	40	39	10
6.2	$24^3 \times 48$	2.907	MF	0.13640	S_I	0.0399	116	113	54
				0.13710	S_I	0.0212	62	61	54

TABLE I: Simulation parameters. 'MF' and 'NP' refer to the mean-field improved and nonperturbatively determined values for $c_{\rm sw}$ respectively. The improved propagators S_I and S_R are defined in Eqs. (2) and (3).

where we have written

$$A'_R(p) = 1 + \frac{1}{2}am + \frac{3}{16}a^2k^2 + \frac{1}{4}a^4\Delta k^2\,. \tag{21}$$

$$\tag{22}$$

It should be remembered that the mass function $M(p)$ must be renormalization-point independent in a renormalizable theory and that the current quark mass at the renormalization point $m(\mu)$ is given by $m(\mu) = M(p = \mu)$. The ultraviolet mass function is of course only constant up to logarithmic corrections. The multiplicative and hybrid tree-level correction ensures that the zeroth-order perturbative behavior of the mass function in the ultraviolet matches that of the continuum. The logarithmic corrections should principle show up in the lattice data, as they did for the gluon propagator in Ref. [5]. However, this is a small effect compared to the tree-level lattice artifacts. It will be a measure of the success of our improvement and correction scheme whether the logarithmic corrections may be extracted from the lattice data.

IV. RESULTS

In addition to the data used in Ref. [3], we have analyzed data at $\beta = 6.0$ using the nonperturbatively determined value for $c_{\rm sw}$ (=1.769), and at $\beta = 6.2$ using the mean-field improved $c_{\rm sw}$ (=1.442). This is a subset of the UKQCD data analyzed in Ref. [15], with the values for $c_{\rm sw}$ taken from Ref. [16]. The simulation parameters are given in Table I. Note that the values of am are different from those given in Ref. [3]; this is because we have here used the determination of κ_c reported in Ref. [15] instead of an earlier, preliminary value. All the data shown have been obtained from the raw data using the cylinder cut described in Ref. [3]. The scale is taken from the hadronic radius r_0 [17] using the interpolating formula of Ref. [18] and the phenomenological value $r_0 = 0.5$ fm. Note that this differs from the scale used in Ref. [3], which was taken from an earlier determination of the string tension. The gauge fixing is identical to that of Ref. [5], which was also used in Ref. [3]. This is a version of lattice Landau gauge that contains Gribov copies; the effects of these have not been studied here.

A. Results with mean-field improved $c_{\rm sw}$

We first consider the effect of employing the multiplicative and hybrid correction schemes on the data analyzed in Ref. [3]. Figure 1 shows the tree-level corrected mass function M evaluated using the three schemes, for both S_I and S_R at $\beta = 6.0, c_{\rm sw} = $ MF. It is clear that the ultraviolet behavior is much improved, but the multiplicatively corrected M from S_I exhibits pathological behavior at intermediate momenta. This is a consequence of a zero crossing in the tree-level mass function, leading to division by near-zero numbers in Eq. (11). In the hybrid scheme, this problem is absent.

It is worth noting that although the mass function approaches the subtracted bare mass m in the ultraviolet, the actual values obtained using the multiplicative and hybrid schemes differ from each other and from the bare mass by up to 20%. It is clear that at this stage this procedure is not good enough to yield a good estimate of current quark masses. We also see that there is no sign in these data of the logarithmic running of the current quark mass.

For S_R, we also see a clear improvement in the ultraviolet behavior, as well as a small but significant difference in the ultraviolet mass between the multiplicative and hybrid schemes. Since the tree-level mass function for S_R does

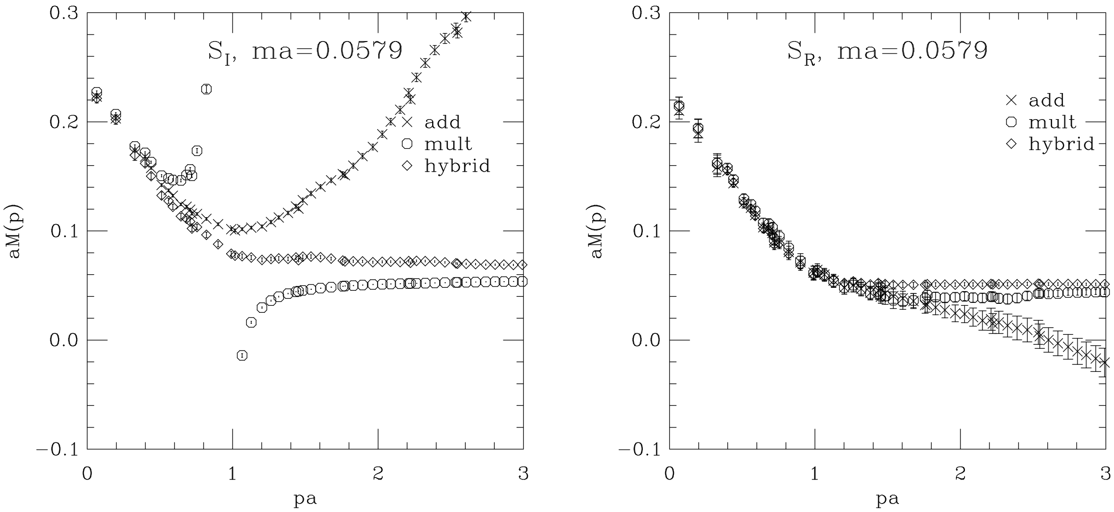

FIG. 1: The tree-level corrected mass function, for S_I (left) and S_R (right) with $c_{\rm sw} = \mathrm{MF}, \kappa = 0.13700$, using additive, multiplicative and hybrid correction. The hybrid scheme is robust even in the presence of a zero crossing of the tree-level mass function for S_I.

not have any zero crossings, the multiplicatively corrected mass does not exhibit the same pathological behavior as for S_I. The tree-level mass function for S_R does not cross zero, but does approach it, and the effects of this may be detected at intermediate momenta. Hence we consider the hybrid correction to be more reliable for S_R.

B. Results with nonperturbative $c_{\rm sw}$

We now turn to the effect of using nonperturbatively determined, rather than tree-level or mean-field, improvement coefficients. Figure 2 shows, on the left, the mass function obtained from our two improved propagators using the nonperturbative value for $c_{\rm sw}$ and the mean-field values for b_q, b'_q and λ (we have not been in a position to obtain data using mean-field or nonperturbative values for c'_q, only the tree-level value $c'_q = \frac{1}{4}$). As indicated in Sec. II, the nonperturbative values for the latter coefficients are currently not known, but at least the nonperturbative value for λ reported in Ref. [13] is close to the mean-field value, and it seems reasonable to guess that this is the case for the other coefficients as well. We therefore assume that although it is not entirely consistent, it is not unreasonable to use the mean-field values. On the right of Fig. 2 are shown equivalent data from Ref. [3], along with data using mean-field values for all the improvement coefficients in S_I.

It is immediately clear that using the NP value for $c_{\rm sw}$ removes the large discrepancy between the two improved propagators for $pa \lesssim 2$, even when using additive tree-level correction. For larger pa, the discrepancy remains, which is not unexpected since at these momenta $\mathcal{O}(a^2)$ and higher errors become dominant.

It is instructive to compare this with the effect of reducing the lattice spacing, as shown in Fig. 3. We do not have data for S_R at $\beta = 6.2$, but we see that S_I changes very little with β in the intermediate momentum range where the discrepancy becomes large. Assuming that S_R changes by a similar amount, we may conclude that reducing the lattice spacing does very little to reduce the discrepancy, although the behavior of S_I at large momenta is somewhat improved, as one would expect. We can also see that going from tree-level to mean-field values for the coefficients b_q and λ has only a very small effect. We may also conclude that using one-loop values for these coefficients will have negligible effect, since the difference between tree-level and one-loop coefficients is even smaller than that between the tree-level and mean-field improved values. This also gives us added confidence in the use of mean-field rather than nonperturbative values for these coefficients.

In Fig. 4 we show the tree-level corrected $Z(p)$ function, for both nonperturbative and mean-field $c_{\rm sw}$. The upper figures show the 'unrenormalized' $Z(p)$ — the upper right figure is taken directly from Fig. 5 in Ref. [3]. We see that there is still a very significant discrepancy between S_I and S_R, even with the nonperturbative $c_{\rm sw}$. Much of this discrepancy, however, amounts to an overall renormalization, which may be included in the quark field renormalization

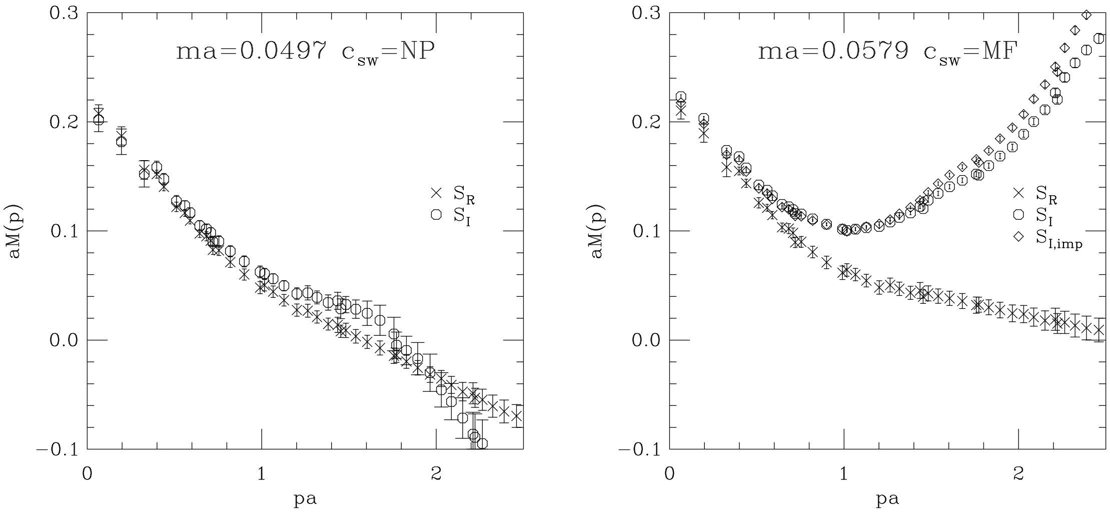

FIG. 2: The additively tree-level corrected mass function, for $c_{\rm sw} = {\rm NP}, \kappa = 0.13344$ (left) and $c_{\rm sw} = {\rm MF}, \kappa = 0.13700$ (right). $S_{I,{\rm imp}}$ in the right-hand figure is obtained using the mean-field improved rather than the tree-level values for b_q and λ in Eq. (2).

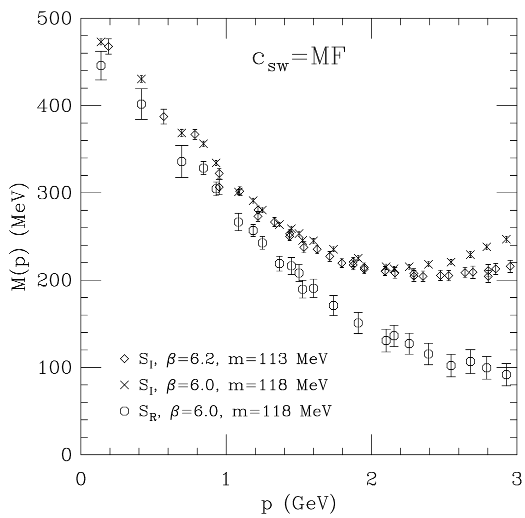

FIG. 3: The additively tree-level corrected mass function, for $c_{\rm sw} = {\rm MF}$, at $\beta = 6.2$ and $\beta = 6.0$.

constant Z_2. To eliminate this possible, unphysical source of disagreement we rescale the data by imposing the 'renormalization condition' $Z(pa = 1) = 1$. The result of this is shown in the lower panel of Fig. 4. We then see that the infrared behavior of the tree-level corrected $Z(p)$ functions agree much better than they did in the previous work in Ref. [3]. For the few most infrared points we see that there is an apparently better agreement for $Z(p)$ between the two forms of the propagator when the nonperturbative $c_{\rm sw}$ is used. The ultraviolet agreement is less than satisfactory even when the nonperturbative $c_{\rm sw}$ is used and we conclude that the ultraviolet behavior of the tree-level form of S_I is too severe to be remedied by our tree-level correction scheme even with nonperturbative and mean-field improved coefficients in the action and propagators respectively. Because of the more reasonable tree-level behavior of S_R and because in this case $Z(p)$ is almost unchanged when using either the nonperturbative or mean-field $c_{\rm sw}$, we take as our best estimate for $Z(p)$ the tree-level corrected result from S_R with nonperturbative $c_{\rm sw}$.

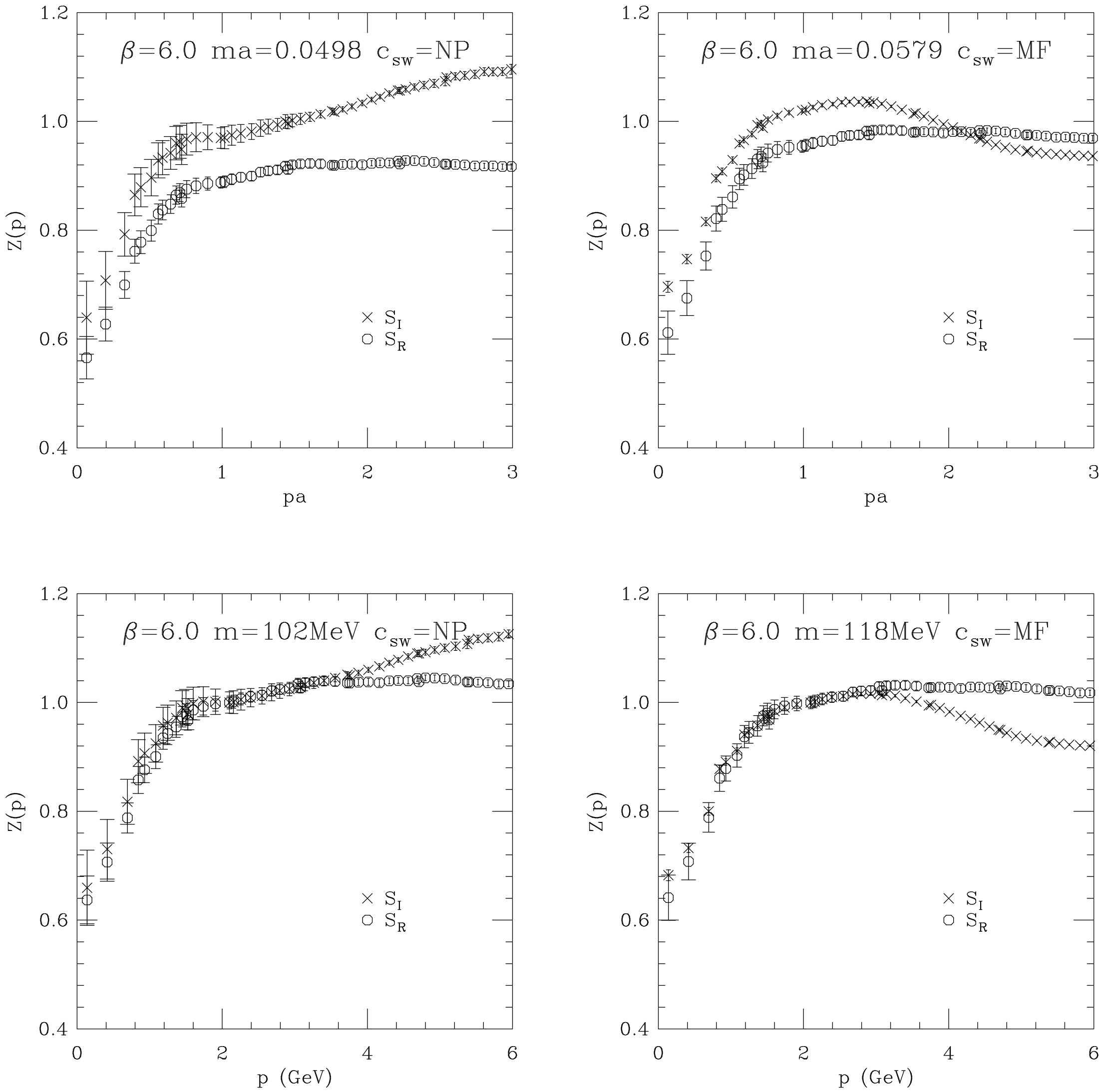

FIG. 4: The tree-level corrected $Z(p)$, for $c_{\rm sw} = \mathrm{NP}, \kappa = 0.137$ (left) and $c_{\rm sw} = \mathrm{MF}, \kappa = 0.13344$ (right). The upper two figures show the data in lattice units, and without rescaling. The lower two figures show $Z(p)$ vs. momentum in physical units, and after rescaling ("renormalizing") so that $Z(2.1\mathrm{GeV}) = 1$. The infrared agreement after rescaling is very good and we take the tree-level corrected $Z(p)$ from S_R with the nonperturbative $c_{\rm sw}$ as the best estimate for this quantity (see text).

Figure 5 shows the mass function for $c_{\rm sw} = \mathrm{NP}$, with the multiplicative and hybrid correction schemes. The multiplicative scheme exhibits the same problems as those we encountered with $c_{\rm sw} = \mathrm{MF}$. In particular, $Z_m^{(0)}(p)$ from S_I crosses zero for $pa \sim 0.5$ and renders the multiplicative scheme meaningless for that case. Even for S_R we find that the uncorrected M^L has small zero-crossings for momentum values in the range $1.5 < pa < 2.3$, which render the multiplicative scheme unsatisfactory. Using the hybrid scheme, however, we avoid these pathologies of the naive multiplicative scheme as can be seen from the two lower figures in Fig. 5. In Fig. 6 we have plotted the mass functions for the two propagators in physical units. We see good agreement between the mass functions using hybrid tree-level correction and the nonperturbative $c_{\rm sw}$ coefficient across the entire range of available momenta. The mass function for S_R dips slightly below that for S_I at intermediate momentum points even though they approach very similar asymptotic values. This residual disagreement implies that we have not succeeded in removing all of the lattice artifacts at intermediate momenta, although hopefully we have gone some significant way toward achieving that end.

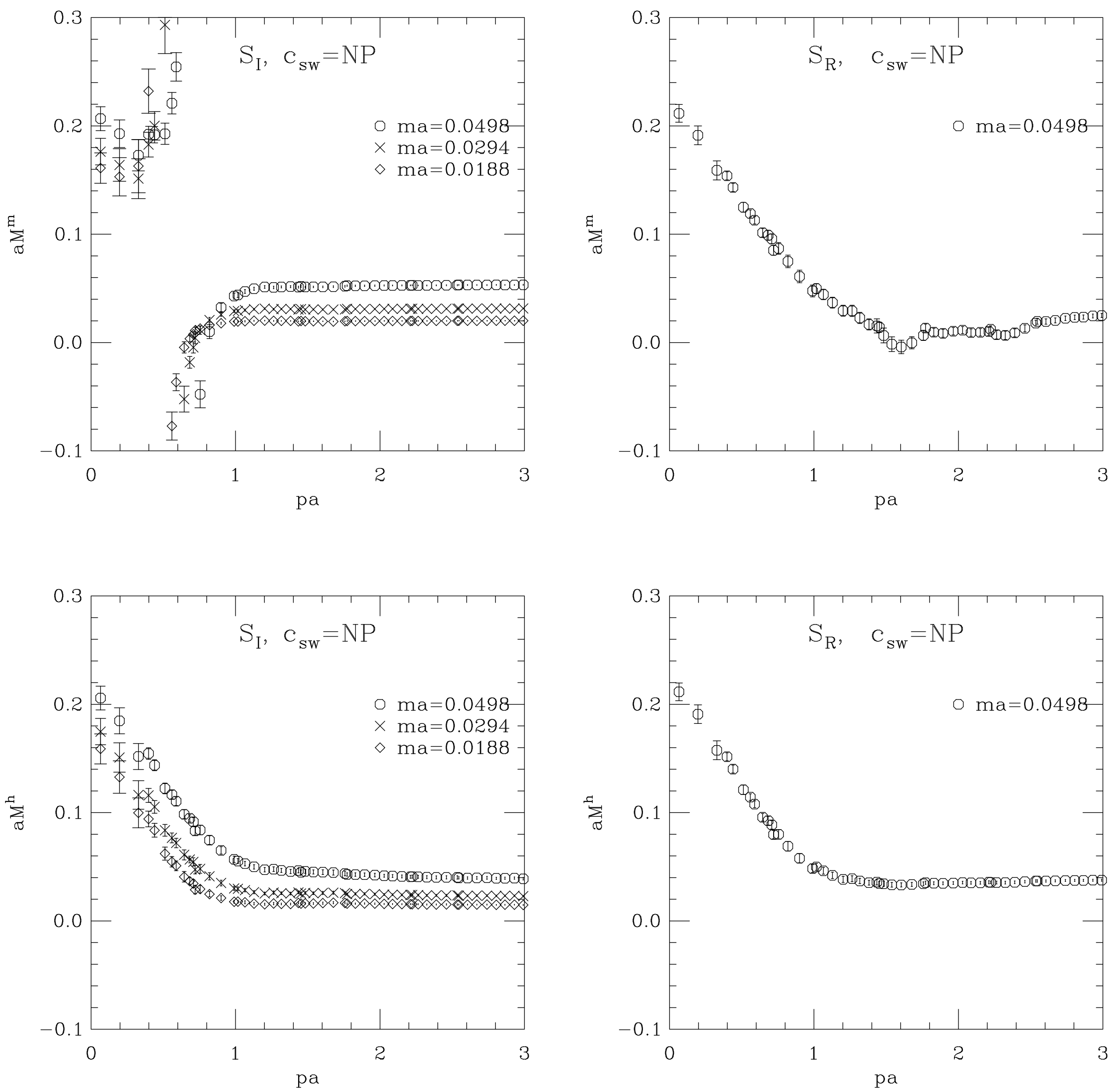

FIG. 5: The tree-level corrected $M(p)$, for $c_{\rm sw}$ = NP, using S_I (left) and S_R (right), and with the multiplicative (top) and hybrid (bottom) correction schemes. The multiplicative scheme clearly fails for S_I, and also performs poorly for S_R; while the hybrid scheme performs well for both and leads to good agreement between the two propagators.

C. Chiral extrapolations

We have available data for three quark masses for S_I with the nonperturbative $c_{\rm sw}$. Having seen the very plausible behavior of the mass function for S_I after hybrid tree-level correction and the good agreement with that for S_R, we are given the confidence to attempt a simple extrapolation of the quark mass function to the chiral limit. The first chiral extrapolation we performed was a linear extrapolation of the ultraviolet mass (obtained by fitting $M^h(p)$ to a constant in the range $2 < pa < 3$) as a function of $\widetilde{m}$. This is shown in Fig. 7, where we see that the ultraviolet mass vanishes in the chiral limit to a very good approximation as it should. Indeed, the extrapolated value of -1 MeV is much smaller than the systematic uncertainties arising from the different tree-level correction schemes discussed above. We also see that for the ultraviolet mass a linear extrapolation does very well. As we noted in Sec. III, the ultraviolet mass function is only constant up to logarithmic corrections, but our ultraviolet behavior is not sufficiently under control that it would be meaningful to attempt to extract those from Fig. 5.

Also in Fig. 7, we show the result of a linear extrapolation of the infrared quark mass $M(p \to 0)$, together with our

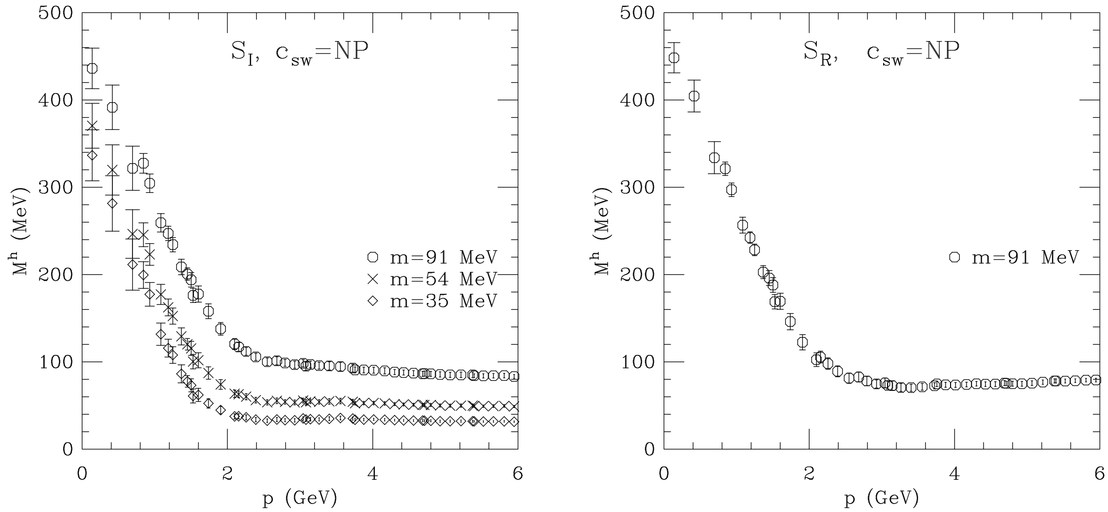

FIG. 6: The hybrid tree-level corrected $M^h(p)$, for $c_{\rm sw}$ = NP, using S_I (left) and S_R (right). For $\widetilde{m} = 91$ MeV we find good agreement between the two data sets, both in the infrared and ultraviolet. The residual disagreement at intermediate momenta is a pointer to lattice artifacts that we have not brought under full control, even with nonperturbative improvement and hybrid tree-level correction.

previous results from Ref. [3]. Due to the small statistics, the error bars for the nonperturbative $c_{\rm sw}$ data are quite large. We still observe that the extrapolated mass value for the nonperturbative $c_{\rm sw}$ is systematically lower than for the mean-field $c_{\rm sw}$, but the results for S_I and S_R are now fully consistent, and also agree with the value obtained from S_R with the mean-field $c_{\rm sw}$. Note that the value in MeV for $M(p \to 0)$ from the mean-field $c_{\rm sw}$ differs from that reported in Ref. [3]. This is due to the different values for the lattice spacing in the two papers. The uncertainty in the lattice spacing adds an additional uncertainty of about 10% to all numbers in physical units. This uncertainty is an intrinsic feature of the quenched approximation.

We also show in Fig. 8 the result of a simple quadratic chiral extrapolation in $\widetilde{m}$ for the entire mass function (for S_I with hybrid tree-level correction and nonperturbative $c_{\rm sw}$). The result has a plausible form and is presumably a good estimate of $M(p)$ in the chiral limit. The very small dip at $p \sim 1.4$ GeV is within two standard deviations and, while not statistically significant, it is a again a hint that we have not completely removed lattice artifacts at intermediate momenta. A linear chiral extrapolation, while adequate in the ultraviolet and infrared, does not fit the data in the intermediate momentum regime. We see that the quadratic extrapolation to the chiral limit is consistent with a vanishing current quark mass and a rapid falling off of $M(p)$ such that it appears to essentially vanish by approximately 1.5 GeV. This suggests that the effects of dynamical chiral symmetry breaking become negligible at a scale p_χ which we estimate to be $p_\chi = 1.45^{+10}_{-13}$ GeV, where the errors are purely statistical.

We have also studied the systematic uncertainties arising from the specific choice of tree-level correction scheme. In order to quantify this, we have modified the hybrid scheme defined in Sec. III by taking

$$\Delta M^{(+)}(pa) \to \Delta M^{(+)}(pa) + \epsilon \Delta M^{(-)}(pa) \qquad \Delta M^{(-)}(pa) \to (1-\epsilon)\Delta M^{(-)}(pa) \tag{23}$$

where ϵ is a free parameter. Considering small variations in ϵ, $-0.1 \lesssim \epsilon \lesssim 0.1$, we find that the correction scheme dependence gives rise to uncertainties in p_χ of about 100 MeV. Using the mean-field $c_{\rm sw}$ at $\beta = 6.0$ and 6.2, we get values for p_χ that are slightly higher, but still consistent withint two standard deviations.

We take as our best estimate for the chiral symmetry scale the value from S_I with the nonperturbative $c_{\rm sw}$ at $\beta = 6.0$: $p_\chi = 1.45^{+10+6}_{-13-8}(14)$ GeV, where the first set of errors are statistical, the second are the systematic uncertainties due to the tree-level correction scheme, and the third is the uncertainty in the lattice spacing. This value is roughly consistent with the chiral symmetry breaking scale $\Lambda_{\chi{\rm SB}}$ arising in low-energy effective theories and in instanton models (see e.g. Ref. [19]). An understanding of the relationship between this result and the recent analysis of the pseudoscalar vertex [20] is an interesting topic for future investigation.

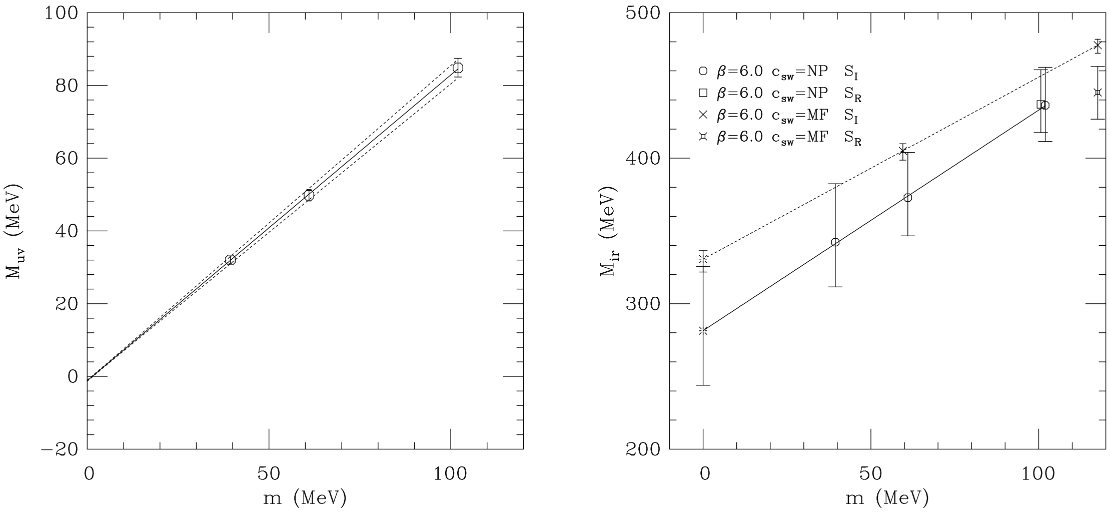

FIG. 7: Left: The ultraviolet quark mass $M_{\rm uv}$ as a function of the bare quark mass $\widetilde{m}$, for S_I with $c_{\rm sw} = {\rm NP}$ and using hybrid tree-level correction. The values are obtained by fitting $M^h(p)$ to a constant for $2 < pa < 3$. Right: The infrared quark mass $M_{\rm ir} \equiv M(p = 0)$, obtained by extrapolating $M(p)$ to $pa = 0$, as a function of $\widetilde{m}$. The bursts indicate the chirally extrapolated values of $M_{\rm ir}$ obtained by a simple straight line fit for each action. The solid line represents the fit for $c_{\rm sw} = {\rm NP}$, while the dotted line is the fit for $c_{\rm sw} = {\rm MF}$. We see that the values of $M_{\rm ir}$ from S_I and S_R agree for $c_{\rm sw} = {\rm NP}$, giving us further indication of the superiority of nonperturbative to mean-field improvement, despite the large statistical uncertainties in the NP data.

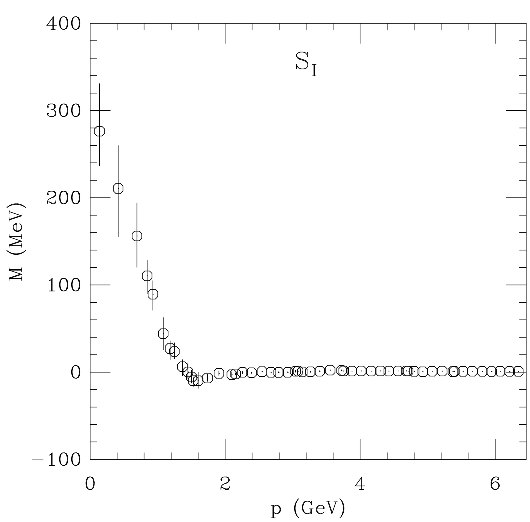

FIG. 8: The hybrid-corrected mass function from S_I with $c_{\rm sw} = {\rm NP}$, with the bare mass $\widetilde{m}$ extrapolated to zero using a quadratic fit. The small dip at $p \sim 1.6$ GeV is not statistically significant and may be due to residual lattice artifacts. The non-zero values for $M(p)$ in the chiral limit are entirely due to dynamical chiral symmetry breaking and provide a direct measure of this effect.

V. DISCUSSION

We have made use of asymptotic freedom to factor out the dominant (tree-level) lattice artifacts in the quark propagator at high momenta. We have discussed several different schemes for applying this idea, referred to as tree-level correction, to the mass function, which is the scalar part of the inverse quark propagator. A purely multiplicative scheme is seen to encounter problems because the value of the tree-level mass function approaches or crosses zero, leading to ill-defined behavior at intermediate momenta. The purely additive scheme defined in Ref. [3], although leading to a dramatic improvement on the uncorrected data, did not give reliable results for the mass function above $pa \sim 1$. We have defined a hybrid tree-level correction scheme which combines the additive and multiplicative schemes in such a way that the mass function becomes well-behaved at all momentum values. Ambiguities in the correction scheme should show up most clearly at intermediate momenta. We find that the uncertainties due to such ambiguities are of comparable magnitude to the statistical uncertainties.

We have also studied the effect of using the nonperturbative value for $c_{\rm sw}$. This was seen to improve the data considerably, as demonstrated most dramatically by the very good agreement between the two definitions S_I and S_R of the improved quark propagator. In contrast, reducing the lattice spacing by going from $\beta = 6.0$ to $\beta = 6.2$ with a mean-field $c_{\rm sw}$ only gave a slight improvement.

Finite volume effects may be estimated by studying the spread of points with momenta in different directions in the infrared, as was done in Refs. [3, 5]. We do not find any significant anisotropy at low momenta either in $Z(p)$ or in $M(p)$, so we conclude that finite volume effects here are small.

We have not here considered the possible effect of Gribov copies. This remains an interesting subject for future study, which is currently being pursued.

In summary, the key results of this study are that one should use the most appropriate (nonperturbative) determinations of improvement coefficients wherever possible. Where tree-level behavior is severe with zero-crossings or near zero-crossings the hybrid tree-level correction scheme can be used in place of the multiplicative one. In the infrared and intermediate momentum regimes we appear to have reasonable control over lattice artifacts. The chiral behavior resulting from a simple chiral extrapolation appears reasonable. We believe that the best estimate of the continuum $Z(p)$ function corresponds to the nonperturbative $c_{\rm sw}$ result for the S_R propagator as shown in Fig. 4. All of the hybrid corrected mass functions with nonperturbative $c_{\rm sw}$ appear reasonable and agree with each other. The chiral extrapolation of these is shown in Fig. 8. We find that the effects of dynamical chiral symmetry breaking become negligible above a momentum scale p_χ. Our best estimate for this scale is $p_\chi = 1.45^{+10+6}_{-13-8}(14)$ GeV.

We emphasize that the real test of these conclusions will be to implement these methods on finer lattice spacings, with further improved actions, and ideally with actions which respect chiral symmetry on the lattice. These subjects are being pursued.

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