Gauge-independent W -boson partial decay widths

Bernd A. Kniehl, Fantina Madricardo, and Matthias Steinhauser

II. Institut für Theoretische Physik, Universität Hamburg, Luruper Chaussee 149, D-22761 Hamburg, Germany

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We calculate the partial decay widths of the W boson at one loop in the standard model using the on-shell renormalization scheme endowed with a gauge-independent definition of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix. We work in the R_ξ gauge and explicitly verify that the final expressions are independent of the gauge parameters. Furthermore, we establish the relationship between the on-shell and $\overline{\text{MS}}$ definitions of the CKM matrix, both in its generic form and in the Wolfenstein parametrization. As a by-product of our analysis, we recover the beta function of the CKM matrix.

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I. INTRODUCTION

The discovery of the W boson at the CERN Super Proton Synchrotron ($Sp\bar{p}S$) collider in 1983 [1] was one of the great successes of the standard model (SM) of the electroweak interactions. The properties of the W boson, including its partial decay widths, have been extensively studied at the $Sp\bar{p}S$ collider and the Fermilab Tevatron, and since 1995 also at the CERN Large Electron-Positron Collider (LEP2). On the theoretical side, the one-loop QED and electroweak radiative corrections to the partial decay widths of the W boson were calculated in the light-fermion approximation in Refs. [2,3], respectively, and for finite fermion masses in Ref. [4]. The QCD corrections, which are present for the hadronic decay modes, were computed at one loop for arbitrary quark masses in Refs. [4,5]. The two- and three-loop QCD corrections for massless quarks may be extracted from Ref. [6], and the respective terms proportional to m_q^2/M_W^2 , where m_q is a generic quark mass, may be found in Ref. [7].

In the calculation of the electroweak corrections, the treatment of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [8], which rotates the weak eigenstates of the quark fields into their mass eigenstates, deserves special attention. In the approximation of neglecting the down-quark masses against the W -boson mass, the CKM matrix can be taken to be unity, so that it does not need to be renormalized. In fact, this avenue was taken in Refs. [3,4]. As a matter of principle, however, the CKM matrix elements must be renormalized because they are parameters of the bare Lagrangian. This was realized for the Cabibbo angle in the SM with two fermion generations in a pioneering paper by Marciano and Sirlin [9]. A compact and plausible on-shell renormalization prescription for the CKM matrix of the three-generation SM was proposed in Ref. [10] on the basis of a detailed inspection of the ultraviolet singularities that are left over in the one-loop expressions for the hadronic decay widths of the W boson if one does not include renormalization constants for the CKM matrix elements appearing in the tree-level formulas. Since the analysis of Ref. [10] was performed in a specific gauge, namely in the 't Hooft-Feynman gauge, it could not be checked if the final one-loop results are gauge independent as required. A recent analysis in the R_ξ gauge has revealed that this is not the case [11,12]. In fact, the finite

parts of the renormalization constants for the CKM matrix elements as defined in Ref. [10] do depend on the gauge parameter ξ_W associated with the W boson, and so do the renormalized CKM matrix elements. In Refs. [11,12], an alternative renormalization prescription for the CKM matrix was proposed which, at one loop, is consistent with the relevant Ward-Takahashi identities and avoids this problem.

In this paper, we explicitly calculate the W -boson partial decay widths at one loop in the on-shell renormalization scheme supplemented by the alternative renormalization prescription for the CKM matrix [11,12]. For simplicity, we henceforth refer to this scheme as the on-shell scheme. As in Ref. [4], we keep the full dependence on the fermion masses. We work in the R_ξ gauge, with arbitrary gauge parameters ξ_W , ξ_Z , and ξ_A , and verify that the final results are indeed independent of them. Using up-to-date information on the input parameters, we present quantitative predictions for the various leptonic and hadronic decay widths of the W boson, which can be readily confronted with precise experimental data from the Tevatron and from LEP2. Furthermore, we establish the relationships between the CKM matrix elements renormalized according to the modified minimal subtraction ($\overline{\text{MS}}$) scheme and their counterparts in the on-shell scheme. We also provide similar relationships appropriate for the Wolfenstein parametrization. From these relationships, we recover the beta functions of the CKM matrix elements [13], which may be relevant for studies within grand unified theories.

This paper is organized as follows. In Sec. II, we establish our formalism and outline our calculation. In Sec. III, we exhibit the relationships between the CKM matrix elements defined in the on-shell and $\overline{\text{MS}}$ schemes and extract their beta functions. In Sec. IV, we present our quantitative predictions. Our conclusions are contained in Sec. V. In the Appendix, we list the fermion two-point functions at one loop in the R_ξ gauge.

II. ANALYTICAL RESULTS

We now describe our analytical analysis, largely adopting the notations from Ref. [14]. We consider the two-particle decay of the W^+ boson to generic leptons or quarks,

$$W^+(k) \rightarrow f_i(p_1) \bar{f}'_j(p_2), \quad (2.1)$$

where $f_i = \nu_e, \nu_\mu, \nu_\tau, u, c$, $f'_j = e, \mu, \tau, d, s, b$, the overbar indicates antiparticles, and the four-momenta are specified in parentheses. We wish to calculate the electroweak and QCD one-loop corrections to the partial decay width of process (2.1), with finite fermion masses $m_{f,i}$ and $m_{f',j}$ and general CKM matrix V_{ij} . The result for the charge-conjugate process, $W^- \rightarrow \bar{f}_i f'_j$, will be the same. The relevant standard matrix elements read [14]

$$\begin{aligned}\mathcal{M}_1^\sigma &= \bar{u}(p_1) \not{\epsilon}(k) \omega_\sigma v(p_2), \\ \mathcal{M}_2^\sigma &= \bar{u}(p_1) \omega_\sigma v(p_2) \epsilon(k) \cdot p_1,\end{aligned}\quad (2.2)$$

where $\omega_\pm = (1 \pm \gamma_5)/2$, $\epsilon(k)$ is the polarization four-vector of the W^+ boson, and $\bar{u}(p_1)$ and $v(p_2)$ are the spinors of the fermions f_i and $\bar{f}'_j$, respectively. It is convenient to define [14]

$$\begin{aligned}G_1^- &= \sum_{\text{pol}} \mathcal{M}_1^{-\dagger} \mathcal{M}_1^- = 2M_W^2 - m_{f,i}^2 - m_{f',j}^2 - \frac{(m_{f,i}^2 - m_{f',j}^2)^2}{M_W^2}, \\ G_1^+ &= \sum_{\text{pol}} \mathcal{M}_1^{-\dagger} \mathcal{M}_1^+ = 6m_{f,i} m_{f',j}, \\ G_2^- &= \sum_{\text{pol}} \mathcal{M}_1^{-\dagger} \mathcal{M}_2^- = -\frac{m_{f,i}}{2} \frac{\kappa^2(M_W^2, m_{f,i}^2, m_{f',j}^2)}{M_W^2}, \\ G_2^+ &= \sum_{\text{pol}} \mathcal{M}_1^{-\dagger} \mathcal{M}_2^+ = -\frac{m_{f',j}}{2} \frac{\kappa^2(M_W^2, m_{f,i}^2, m_{f',j}^2)}{M_W^2},\end{aligned}\quad (2.3)$$

where $\kappa(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2(xy + yz + zx)}$ is the Källén function and it is summed over the polarization states of the W^+ boson and the spin states of the fermions f_i and $\bar{f}'_j$. Let us first deal with the quark case, which is more involved. In the Born approximation, the transition (T) matrix element of process (2.1) then reads

$$\mathcal{M}_0^{wfif'_j} = -\frac{eV_{ij}}{\sqrt{2}s_w} \mathcal{M}_1^-, \quad (2.4)$$

where e is the electron charge magnitude and $s_w^2 = 1 - c_w^2 = 1 - M_W^2/M_Z^2$. Thus, the partial decay width is given by

$$\Gamma_0^{wfif'_j} = \frac{N_C^f \alpha^2 |V_{ij}|^2}{24s_w^2 M_W^3} \kappa(M_W^2, m_{f,i}^2, m_{f',j}^2) G_1^-, \quad (2.5)$$

where $N_C^f = 3$ and $\alpha = e^2/(4\pi)$ is Sommerfeld's fine-structure constant.

The one-loop-corrected T -matrix element of process (2.1) emerges from Eq. (2.4) by including the renormalization constants for the parameters e , s_w , and V_{ij} , those for the W^+ , f_i , and $\bar{f}'_j$ fields, and the proper vertex correction. It reads

$$\begin{aligned}\mathcal{M}_1^{wfif'_j} &= -\frac{eV_{ij}}{\sqrt{2}s_w} \left\{ \mathcal{M}_1^- \left[1 + \frac{\delta e}{e} - \frac{\delta s_w}{s_w} + \frac{\delta V_{ij}}{V_{ij}} + \frac{1}{2} \delta Z_W \right. \right. \\ &\quad \left. \left. + \frac{1}{2} \sum_k (\delta Z_{ik}^{f,L\dagger} V_{kj} + V_{ik} \delta Z_{kj}^{f',L}) \right] \right. \\ &\quad \left. + \sum_{a=1}^2 \sum_{\sigma=\pm} \mathcal{M}_a^\sigma \delta F_a^\sigma(M_W, m_{f,i}, m_{f',j}) \right\},\end{aligned}\quad (2.6)$$

where δF_a^σ are electroweak form factors. The various renormalization constants may be expressed in terms of the unrenormalized, one-particle-irreducible two-point functions of the gauge bosons and fermions, as

$$\begin{aligned}\frac{\delta e}{e} &= -\frac{1}{2} \left(\frac{s_w}{c_w} \delta Z_{ZA} + \delta Z_{AA} \right) \\ &= -\frac{s_w}{c_w} \frac{\Pi^{ZA}(0)}{M_Z^2} + \frac{1}{2} \frac{\partial \Pi^{AA}(k^2)}{\partial k^2} \Big|_{k^2=0},\end{aligned}\quad (2.7)$$

$$\begin{aligned}\frac{\delta s_w}{s_w} &= -\frac{c_w^2}{2s_w^2} \left(\frac{\delta M_W^2}{M_W^2} - \frac{\delta M_Z^2}{M_Z^2} \right) \\ &= -\frac{c_w^2}{2s_w^2} \text{Re} \left(\frac{\Pi^{WW}(M_W^2)}{M_W^2} - \frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} \right),\end{aligned}\quad (2.8)$$

$$\begin{aligned}\delta V_{ij} &= \frac{1}{2} \left\{ \sum_{k \neq i} \frac{m_{f,i}^2 + m_{f',k}^2}{m_{f,i}^2 - m_{f',k}^2} [\Sigma_{ik}^{f,L}(0) + 2\Sigma_{ik}^{f,S}(0)] V_{kj} \right. \\ &\quad \left. - \sum_{k \neq j} V_{ik} \frac{m_{f',k}^2 + m_{f',j}^2}{m_{f',k}^2 - m_{f',j}^2} [\Sigma_{kj}^{f',L}(0) + 2\Sigma_{kj}^{f',S}(0)] \right\},\end{aligned}\quad (2.9)$$

$$\delta Z_W = -\text{Re} \frac{\partial \Pi^{WW}(k^2)}{\partial k^2} \Big|_{k^2=M_W^2}, \quad (2.10)$$

$$\begin{aligned}\delta Z_{ij}^{f,L} &= \frac{2}{m_{f,i}^2 - m_{f,j}^2} \text{Re} [m_{f,i}^2 \Sigma_{ij}^{f,L}(m_{f,j}^2) + m_{f,i} m_{f,j} \Sigma_{ij}^{f,R}(m_{f,j}^2) \\ &\quad + (m_{f,i}^2 + m_{f,j}^2) \Sigma_{ij}^{f,S}(m_{f,j}^2)], \quad i \neq j,\end{aligned}\quad (2.11)$$

$$\begin{aligned}\delta Z_{ii}^{f,L} &= -\text{Re} \Sigma_{ii}^{f,L}(m_{f,i}^2) - m_{f,i}^2 \frac{\partial}{\partial p^2} \\ &\quad \times \text{Re} [\Sigma_{ii}^{f,L}(p^2) + \Sigma_{ii}^{f,R}(p^2) + 2\Sigma_{ii}^{f,S}(p^2)]_{p^2=m_{f,i}^2}.\end{aligned}\quad (2.12)$$

Here, $\Pi^{BB'}(k^2)$ is the transverse coefficient of the two-point function, at four-momentum k , of the gauge bosons B and B' , and $\Sigma_{ij}^{f,L/R/S}(p^2)$ are the left-handed, right-handed, and scalar coefficients of the two-point function, at four-momentum p , of the fermions f_i and f_j , respectively. The

latter are listed in Eq. (A2) of the Appendix. The symbol $\overline{\text{Re}}$ takes the dispersive parts of the loop integrals appearing in the two-point functions and commutes with complex-valued parameters, such as V_{ij} . As a consequence, $\delta Z_{ij}^{f,L\ddagger} = \delta Z_{ji}^{f,L*}$ is obtained from $\delta Z_{ji}^{f,L}$ by complex conjugation of the CKM matrix elements contained therein. From Eq. (A2) it hence follows that $\delta Z_{ij}^{f,L\ddagger}$ emerges from $\delta Z_{ij}^{f,L}$ through the interchange of $m_{f,i}$ and $m_{f,j}$. In particular, we have $\delta Z_{ii}^{f,L\ddagger} = \delta Z_{ii}^{f,L}$. Notice that the vertex correction only depends linearly on V_{ij} , which is factored out in Eq. (2.6).¹

All the renormalization constants appearing in Eq. (2.6) are ultraviolet (UV) divergent. If the renormalized parameters e , s_w , V_{ij} are to represent physical observables, they must be gauge independent, and so must the respective renormalization constants. This is well established for $\delta e/e$ and $\delta s_w/s_w$ given by Eqs. (2.7) and (2.8), respectively, while an appropriate definition of $\delta V_{ij}/V_{ij}$, namely Eq. (2.9), was proposed only recently [11,12]. On the other hand, the field renormalization constants in Eq. (2.6) are gauge dependent and, with the exception of $\delta Z_{ij}^{f,L}$ with $i \neq j$, also infrared (IR) divergent. Since $\mathcal{M}_1^+$ and $\mathcal{M}_2^\pm$ do not yet appear in Eq. (2.4), the respective form factors are finite and gauge independent. However, δF_1^- is IR and UV divergent and gauge dependent. The right-hand side of Eq. (2.6) is UV finite and gauge independent [11,12], but it is IR divergent. This IR divergence is canceled in the one-loop expression for the partial decay width by the real bremsstrahlung correction δ_b^{ew} . Also including the one-loop QCD correction δ^{QCD} , we have

$$\Gamma_1^{Wfif'} = \Gamma_0^{Wfif'} (1 + \delta^{\text{ew}} + \delta^{\text{QCD}}), \quad (2.13)$$

where

$$\delta^{\text{ew}} = \delta_{\text{virt}}^{\text{ew}} + \delta_b^{\text{ew}}, \quad (2.14)$$

with the virtual electroweak correction

$$\begin{aligned} \delta_{\text{virt}}^{\text{ew}} = & 2 \frac{\delta e}{e} - 2 \frac{\delta s_w}{s_w} + 2 \frac{\delta V_{ij}}{V_{ij}} + \delta Z_W \\ & + \sum_k (\delta Z_{ik}^{f,L\ddagger} V_{kj} + V_{ik} \delta Z_{kj}^{f,L}) \\ & + \frac{2}{G_1^-} \sum_{a=1}^2 \sum_{\sigma=\pm} G_a^\sigma \delta F_a^\sigma(M_W, m_{f,i}, m_{f',j}). \end{aligned} \quad (2.15)$$

Note that δ_b^{ew} is gauge independent because $\Gamma_0^{Wfif'} \delta_b^{\text{ew}}$ represents the partial decay width of the physical process $W^+ \rightarrow f_i \bar{f}'_j \gamma$ in the Born approximation. Thus, δ^{ew} is finite and gauge independent, as it must be because it exhausts the

leading-order electroweak correction to the physical process $W^+ \rightarrow f_i \bar{f}'_j (\gamma)$. The QCD correction δ^{QCD} may be obtained from δ^{ew} by retaining only the terms containing Q_f^2 , $Q_{f'}^2$, or $Q_f Q_{f'}$, setting $Q_f = Q_{f'} = 1$, replacing α with the strong-coupling constant $\alpha_s^{(n_f)}(\mu)$, and including the overall color factor $C_F = 4/3$. Also δ^{QCD} is IR and UV finite and gauge independent, as it must be because it exhausts the leading-order QCD correction to the physical process $W^+ \rightarrow f_i \bar{f}'_j (g)$. In the limit $m_{f,i} = m_{f',j} = 0$, the well-known correction factor $\delta^{\text{QCD}} = 1 + \alpha_s^{(n_f)}(\mu)/\pi$ is recovered.

In the lepton case, we have $N_C^f = 1$ and $\delta^{\text{QCD}} = 0$. Furthermore, we have $V_{ij} = \delta_{ij}$ and $\delta V_{ij} = 0$ if we assume the neutrinos to be massless.

We computed all ingredients of Eq. (2.13) in the R_ξ gauge, with arbitrary gauge parameters ξ_W , ξ_Z , and ξ_A , for finite fermion masses $m_{f,i}$ and $m_{f',j}$ and general CKM matrix V_{ij} . We regularized the UV divergences by means of dimensional regularization (DR), in $D = 4 - 2\epsilon$ space-time dimensions, and the IR ones by introducing an infinitesimal photon mass λ . To guarantee the correctness of our results, we chose two independent approaches. The first approach was based on the program packages FEYNARTS [15] and FEYNALC [16], which are written in MATHEMATICA. FEYNARTS generates the relevant Feynman diagrams and translates them into T -matrix elements, in a format which is readable by FEYNALC. FEYNALC then simplifies the expressions and decomposes them into the standard one-loop scalar integrals A_0 , B_0 , and C_0 . The second approach was to essentially perform all the calculations by hand using well-tested custom-made programs, written in FORM [17], in the intermediate steps. Both approaches led to identical results. Our results for the gauge-boson self-energies fully agree with those listed in Eqs. (7)–(10) of Ref. [18] and will not be presented here. The corresponding formulas in the 't Hooft–Feynman gauge, with $\xi_W = \xi_Z = \xi_A = 1$, may be found in Appendix B of Ref. [19]. Generic expressions for the fermion self-energies, which were originally derived in Ref. [11], are specified in Eq. (A2) of the Appendix. The diagonal fermion wave-function renormalization constants $\delta Z_{ii}^{f,L}$ suffer from IR divergences. Therefore, we retained the infinitesimal photon mass λ in those parts of Eq. (A2), from which IR divergences may arise. In the limit $\lambda = 0$, Eq. (A2) agrees with Eqs. (B1)–(B3) of Ref. [20].² The corresponding formulas in the 't Hooft–Feynman gauge may be found in Appendix A of Ref. [21]. We do not display our analytical results for the form factors δF_a^σ because they are somewhat lengthy. They can be compared with the literature in the limiting cases $\xi_W = \xi_Z = \xi_A = 1$ [4,14] or $m_{f,i} = m_{f',j} = 0$ [18]. In the first

¹The last term in Eq. (9.6) of Ref. [14] should be multiplied by V_{ij} .

²The difference may be traced to the terms in Eq. (A2) that contain $B_0(p^2, \sqrt{\xi_A} \lambda, m_{f,i})$. If we put $\xi_A = 1$ in the argument of this function, then Eq. (A2) coincides with Eqs. (B1)–(B3) of Ref. [20]. This difference becomes relevant when IR-divergent quantities such as $\delta Z_{ii}^{f,L}$ are to be calculated, but it is immaterial for the purposes of Ref. [20].

case, we find agreement with Eqs. (27)–(29) of Ref. [4].³ In the second case, only the form factor δF_1^- survives, as may be seen from Eq. (2.3), and we find agreement with Eq. (33) of Ref. [18]. We verified the expressions for δ_b^{ew} and δ^{QCD} listed in Eqs. (35) and (37) of Ref. [4], respectively.

At this point, we should comment on a very recent paper [22] in which a new renormalization prescription for the CKM matrix is proposed. The quantity T_1 , defined in Eq. (4) of Ref. [22], corresponds to our quantity $\mathcal{M}_1^{wfif'}$, defined in Eq. (2.6) above. The authors of Ref. [22] claim that the finite part of $\mathcal{M}_1^{wfif'}$ becomes gauge dependent if δV_{ij} is omitted. In order to substantiate this claim, they introduce, in Eq. (23), the auxiliary quantity δX_{ud} , which is to represent the difference between $\mathcal{M}_1^{wfif'}$ and its counterpart for $V_{ij} = \delta_{ij}$. In our notation, this quantity reads

$$\delta X_{ij} = \frac{1}{2} V_{ij} (\delta Z_{ii}^{f,L\dagger} - \delta Z_{ii[1]}^{f,L\dagger} + \delta Z_{jj}^{f',L} - \delta Z_{jj[1]}^{f',L}) + \frac{1}{2} \left(\sum_{k \neq i} \delta Z_{ik}^{f,L\dagger} V_{kj} + \sum_{k \neq j} V_{ik} \delta Z_{kj}^{f',L} \right), \quad (2.16)$$

where the subscript “[1]” indicates that the identification $V_{ij} = \delta_{ij}$ is to be made. They find that this quantity is gauge dependent, and propose to define $\delta V_{ij} = -\delta X_{ij}$. From our above discussion, it is clear that δV_{ij} must be gauge independent, in order for the renormalized parameters V_{ij} to be gauge independent. Otherwise, the latter would not qualify as physical observables. Furthermore, we verified, by inspecting the analytic expressions, that $\mathcal{M}_1^{wfif'} + e \mathcal{M}_1^- \delta V_{ij} / (\sqrt{2} s_w)$ is gauge independent, in accordance with Refs. [11,12]. This implies that the quantity δX_{ij} , defined in Eq. (2.16), is also gauge independent, as we explicitly checked. Finally, we remark that the expression for T_1 given in Eq. (24) of Ref. [22] differs from that given in Eq. (4) *ibidem* by finite terms because $\delta g/g$ and δZ_W do depend on V_{ij} .

Equation (2.13) is formulated in the pure on-shell renormalization scheme, which uses α and the physical particle masses as basic parameters. In this scheme, large electroweak corrections arise from fermion loop contributions to the renormalizations of α and s_w . As in any charged-current process, these corrections can be greatly reduced by parametrizing the lowest-order result with Fermi’s coupling constant G_F and M_W instead of α and s_w . This can be achieved with the aid of the relationship [23]

$$G_F = \frac{\pi \alpha}{\sqrt{2} s_w^2 M_W^2} \frac{1}{1 - \Delta r}, \quad (2.17)$$

where Δr contains those radiative corrections to the muon decay width which the SM introduces on top of the purely photonic corrections from within Fermi’s model. At one loop, we have [23]

$$\Delta r = \frac{\Pi^{WW}(0) - \widetilde{\text{Re}} \Pi^{WW}(M_W^2)}{M_W^2} + \frac{c_w^2}{s_w^2} \widetilde{\text{Re}} \left(\frac{\Pi^{WW}(M_W^2)}{M_W^2} - \frac{\Pi^{ZZ}(M_Z^2)}{M_Z^2} \right) + 2 \frac{c_w}{s_w} \frac{\Pi^{ZA}(0)}{M_Z^2} + \frac{\partial \Pi^{AA}(k^2)}{\partial k^2} \Big|_{k^2=0} + \frac{\alpha}{4 \pi s_w^2} \left[\left(\frac{7}{2 s_w^2} - 2 \right) \ln c_w^2 + 6 \right]. \quad (2.18)$$

The last term herein represents the vertex and box corrections to the muon decay width in the ’t Hooft–Feynman gauge. Thus, the $\Pi^{BB'}$ functions in Eq. (2.18) have to be evaluated in this gauge, too. We recall that Eq. (2.18) is gauge independent [24] and finite. The quantity $\partial \Pi^{AA}(k^2) / \partial k^2|_{k^2=0}$ receives important contributions from the light-quark flavors, which cannot be reliably predicted in perturbative QCD. This problem is usually circumvented by relating the finite and gauge-independent quantity

$$\Delta \alpha_{\text{had}}^{(5)} = \left[\frac{\partial \Pi^{AA}(k^2)}{\partial k^2} \Big|_{k^2=0} - \frac{\Pi^{AA}(M_Z^2)}{M_Z^2} \right]_{\text{udscb}} \quad (2.19)$$

via a subtracted dispersion relation to experimental data on the total cross section of inclusive hadron production in e^+e^- annihilation. In our numerical analysis, we substitute $\alpha = \sqrt{2} G_F s_w^2 M_W^2 / \pi$ in Eqs. (2.5) and (2.14) and, in turn, include the term $-\Delta r$, evaluated from Eq. (2.18) with the same formula for Δ , within the parentheses on the right-hand side of Eq. (2.13). Then, the quantity $\partial \Pi^{AA}(k^2) / \partial k^2|_{k^2=0}$ exactly cancels, so that the theoretical uncertainty in $\Delta \alpha_{\text{had}}^{(5)}$ does not affect our results.

III. $\overline{\text{MS}}$ DEFINITION OF THE CKM MATRIX

The relationship between the on-shell and $\overline{\text{MS}}$ definitions of the CKM matrix may be conveniently revealed by considering the identity

$$V_{ij}^0 = V_{ij} + \delta V_{ij} = \bar{V}_{ij} + \delta \bar{V}_{ij}, \quad (3.1)$$

where the superscript “0” labels bare quantities, and $\overline{\text{MS}}$ quantities are marked by an overbar. By definition, $\delta \bar{V}_{ij}$ is the UV-divergent part of δV_{ij} , proportional to $1/\epsilon + \ln(4\pi) - \gamma_E$, where γ_E is Euler’s constant. We thus obtain the relationship

$$\bar{V}_{ij}(\mu) = V_{ij} + \Delta V_{ij}(\mu), \quad (3.2)$$

where

$$\Delta V_{ij}(\mu) = \delta V_{ij} - \delta \bar{V}_{ij} \quad (3.3)$$

³The function in the seventh line of Eq. (29) should carry the superscript “ σ ” instead of “ $-$.”

is a finite shift, which depends on the 't Hooft mass scale μ of DR. Inserting Eq. (A2) in Eq. (2.9) and performing the $\overline{\text{MS}}$ subtraction, we find

$$\begin{aligned} \Delta V_{ij}(\mu) = & \sum_{k \neq i} \sum_l V_{il} V_{lk}^\dagger V_{kj} \frac{m_{f,k}^2 + m_{f,i}^2}{m_{f,k}^2 - m_{f,i}^2} f(m_{f',l}) \\ & + \sum_{k \neq j} \sum_l V_{ik} V_{kl}^\dagger V_{lj} \frac{m_{f',k}^2 + m_{f',j}^2}{m_{f',k}^2 - m_{f',j}^2} f(m_{f,l}), \end{aligned} \quad (3.4)$$

where

$$\begin{aligned} f(m) = & \frac{\alpha}{16\pi s_w^2} \frac{x}{2} \left[3 \ln \frac{\mu^2}{M_W^2} - \frac{5x-11}{2(x-1)} \right. \\ & \left. + \frac{3x(x-2)}{(x-1)^2} \ln x \right]_{x=m^2/M_W^2}. \end{aligned} \quad (3.5)$$

The μ dependence of $\bar{V}_{ij}(\mu)$ is described by the β function

$$\beta_{V_{ij}} = \frac{d}{d \ln \mu} \bar{V}_{ij}(\mu). \quad (3.6)$$

Inserting Eq. (3.2) into Eq. (3.6), we obtain the one-loop expression

$$\begin{aligned} \beta_{V_{ij}} = & \frac{3\alpha}{16\pi s_w^2 M_W^2} \left[\sum_{k \neq i} \sum_l V_{il} V_{lk}^\dagger V_{kj} m_{f',l}^2 \frac{m_{f,k}^2 + m_{f,i}^2}{m_{f,k}^2 - m_{f,i}^2} \right. \\ & \left. + \sum_{k \neq j} \sum_l V_{ik} V_{kl}^\dagger V_{lj} m_{f,l}^2 \frac{m_{f',k}^2 + m_{f',j}^2}{m_{f',k}^2 - m_{f',j}^2} \right], \end{aligned} \quad (3.7)$$

which agrees with the one given in Ref. [13].

The standard parametrization of V_{ij} utilizes three angles, θ_{12} , θ_{23} , and θ_{13} , and one phase, δ_{13} [25]. A popular approximation that emphasizes the hierarchy in the size of the angles, $\sin \theta_{12} \gg \sin \theta_{23} \gg \sin \theta_{13}$, is due to Wolfenstein [26], where one sets $\lambda = \sin \theta_{12}$, the sine of the Cabibbo angle, and then writes the other CKM matrix elements in terms of powers of λ . Through $O(\lambda^3)$, one has [25]

$$V = \begin{pmatrix} 1 - \lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1 - \lambda^2/2 & A\lambda^2 \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix}, \quad (3.8)$$

where A , ρ , and η are real numbers, which were intended to be of order unity. The relationships between the parameters λ , A , ρ , and η in the on-shell scheme and their counterparts in the $\overline{\text{MS}}$ scheme may be obtained with the aid of Eq. (3.2) and read

$$\bar{\lambda} = \lambda [1 + (1 - \lambda^2)F(m_u, m_c, m_d, m_s)],$$

$$\bar{A} = A [1 - 2F(m_u, m_c, m_d, m_s) + F(m_c, m_t, m_s, m_b)],$$

$$\begin{aligned} \bar{\rho} = & \rho [1 - F(m_u, m_c, m_d, m_s) - F(m_c, m_t, m_s, m_b) \\ & + F(m_u, m_t, m_d, m_b)] \\ & + \left(\frac{m_u^2 + m_c^2}{m_u^2 - m_c^2} - \frac{m_u^2 + m_t^2}{m_u^2 - m_t^2} \right) [f(m_d) - f(m_s)] \\ & + \left(\frac{m_s^2 + m_b^2}{m_s^2 - m_b^2} - \frac{m_d^2 + m_b^2}{m_d^2 - m_b^2} \right) [f(m_c) - f(m_t)], \end{aligned}$$

$$\begin{aligned} \bar{\eta} = & \eta [1 - F(m_u, m_c, m_d, m_s) - F(m_c, m_t, m_s, m_b) \\ & + F(m_u, m_t, m_d, m_b)], \end{aligned} \quad (3.9)$$

where

$$\begin{aligned} F(m_1, m_2, m_3, m_4) = & \left\{ \frac{m_1^2 + m_2^2}{m_1^2 - m_2^2} [f(m_3) - f(m_4)] \right. \\ & \left. + \frac{m_3^2 + m_4^2}{m_3^2 - m_4^2} [f(m_1) - f(m_2)] \right\}. \end{aligned} \quad (3.10)$$

Exploiting the strong hierarchy among the up- and down-type quark masses, which satisfy $m_u \ll m_c \ll m_t$ and $m_d \ll m_s \ll m_b$, respectively, and the significant mass splittings within the second and third quark generations, $m_s \ll m_c$ and $m_b \ll m_t$, respectively, we can derive the following approximation formulas:

$$\begin{aligned} \bar{\lambda} &= \lambda [1 + (1 - \lambda^2)f(m_c)], \\ \bar{A} &= A [1 + f(m_t)], \\ \bar{\rho} &= \rho, \\ \bar{\eta} &= \eta. \end{aligned} \quad (3.11)$$

For $\mu = M_W$, these approximations agree with the exact results, evaluated from Eq. (3.9), within an error of less than 10^{-5} .

IV. NUMERICAL RESULTS

We now describe our numerical analysis of Eq. (2.13), with the modifications specified at the end of Sec. II. We evaluated the A_0 , B_0 , and C_0 functions with the aid of the FORTRAN program package FF [27], which is embedded into the MATHEMATICA environment through the program package LOOPTOOLS [28]. We performed several checks for the correctness and the stability of our numerical results. As we demonstrated in Sec. II, Eq. (2.13) is IR and UV finite and gauge independent, as a consequence of cancellations among

the various terms contained in Eqs. (2.14) and (2.15). We can check the IR finiteness and gauge independence numerically by varying the IR regulator λ and the gauge parameters ξ_W , ξ_Z , and ξ_A , respectively. In the physical limit $D \rightarrow 4$, the UV divergences appear as terms proportional to $1/\epsilon + \ln(4\pi) - \gamma_E$, which are accompanied by a term $\ln(\mu^2/M^2)$, with M being a characteristic mass scale of the considered loop integral. Although we nullified $1/\epsilon + \ln(4\pi) - \gamma_E$ in our computer program, we can check the UV finiteness numeri-

cally by varying the 't Hooft mass scale μ . A further check on the stability of our numerical analysis can be obtained by directly evaluating the two- and three-point tensor integrals with the aid of LOOPTOOLS instead of applying the Passarino-Veltman reduction algorithm [29]. Our numerical analysis passed all these checks. Finally, we managed to reproduce the numerical results of Ref. [14] after adopting the definition of δV_{ij} , the choice of gauge, and the values of the input parameters from there.

We use the following input parameters [25,30]:

$$\begin{aligned}
 \alpha &= 1/137.03599976, & G_F &= 1.16639 \times 10^{-5} \text{ GeV}^{-2}, & \alpha_s^{(5)}(M_Z) &= 0.1181, \\
 M_W &= 80.419 \text{ GeV}, & M_Z &= 91.1871 \text{ GeV}, \\
 m_e &= 0.510998902 \text{ MeV}, & m_\mu &= 105.658357 \text{ MeV}, & m_\tau &= 1777.03 \text{ MeV}, \\
 m_u &= 3 \text{ MeV}, & m_d &= 6 \text{ MeV}, & m_s &= 123 \text{ MeV}, \\
 m_c &= 1.5 \text{ GeV}, & m_b &= 4.6 \text{ GeV}, & m_t &= 174.3 \text{ GeV}, \\
 s_{12} &= 0.223, & s_{23} &= 0.040, & s_{13} &= 0.004.
 \end{aligned} \tag{4.1}$$

Here, m_u , m_d , and m_s correspond to current-quark masses, and m_c , m_b , and m_t to pole masses. Since the contributions from the light quarks $q = u, d, s, c, b$ come with the suppression factors m_q^2/M_W^2 , the considerable uncertainties in the values of m_q do not jeopardize the reliability of our theoretical predictions. We take the neutrinos to be massless. For simplicity, we assume that $\delta_{13} = 0$. Then, the values for $s_{ij} = \sin \theta_{ij}$ provided in the last row of Eq. (4.1) lead to

$$\begin{aligned}
 V_{ud} &= 0.975, & V_{us} &= 0.223, & V_{ub} &= 0.004, \\
 V_{cd} &= -0.223, & V_{cs} &= 0.974, & V_{cb} &= 0.040, \\
 V_{td} &= 0.005, & V_{ts} &= -0.040, & V_{tb} &= 0.999.
 \end{aligned} \tag{4.2}$$

These values approximately satisfy the unitarity condition $V_{ik}V_{kj}^\dagger = \delta_{ij}$.⁴ We evaluate $\alpha_s^{(n_f)}(\mu)$ appearing in δ^{QCD} at the renormalization scale $\mu = M_W$ with $n_f = 5$ active quark flavors from the one-loop relation

$$\alpha_s^{(n_f)}(M_W) = \frac{\alpha_s^{(n_f)}(M_Z)}{1 + \alpha_s^{(n_f)}(M_Z) \beta_0 \ln(M_W^2/M_Z^2)/\pi}, \tag{4.3}$$

where $\beta_0 = 11/4 - n_f/6$. For the Higgs-boson mass, we consider the values $M_H = 100, 250$, and 600 GeV.

We now present our numerical results. We first investigate the quantitative significance of the definition of δV_{ij} . Toward this end, we compare, in Table I, our results for the partial widths of the various hadronic W -boson decay channels with those obtained in the 't Hooft-Feynman gauge

with the definition of δV_{ij} proposed in Ref. [10], assuming $M_H = 250$ GeV. The relative deviations are largest for the final states involving the b quark, where they are of order $\alpha m_b^2/(\pi M_W^2) \approx 10^{-5}$. Although small against the present experimental accuracies [25,30], they are of the same order as the entire shifts due to the renormalization of the CKM matrix [10]. We stress that the numbers in the second column of Table I do depend on the choice of gauge. However, this gauge dependence turns out to be feeble. In Table II, we present our tree-level and one-loop results for the leptonic and hadronic partial decay widths of the W boson, assuming $M_H = 100, 250$, or 600 GeV. In the leptonic channels, the radiative corrections are nearly flavor independent and amount to approximately -0.3% . In the hadronic channels, the corrections range between 3.5% and 3.8% and are dominantly of QCD origin. In all cases, the M_H dependence is feeble, of relative order 10^{-5} . Finally, we determine the uncertainties in our theoretical prediction for the total W -boson decay width Γ_W due to the errors on our input parameters. Specifically, the variations of G_F , $\alpha_s^{(5)}(M_Z)$, M_W , M_Z , m_c ,

TABLE I. Partial widths (in GeV) of the hadronic W -boson decays at one loop, for $M_H = 250$ GeV. The results obtained in the 't Hooft-Feynman gauge with δV_{ij} as defined in Ref. [10] are compared with ours.

Partial width	Ref. [10]	Refs. [11,12]
$\Gamma(W \rightarrow ud)$	0.67122024	0.67122024
$\Gamma(W \rightarrow us) \times 10$	0.35125894	0.35125894
$\Gamma(W \rightarrow ub) \times 10^4$	0.11273928	0.11273992
$\Gamma(W \rightarrow cd) \times 10$	0.35113436	0.35113436
$\Gamma(W \rightarrow cs)$	0.67001209	0.67001209
$\Gamma(W \rightarrow cb) \times 10^2$	0.11279438	0.11279502
$\Gamma(W \rightarrow \text{hadrons})$	1.41261088	1.41261088

⁴Notice that the indices of V_{ij} refer to generations rather than quark flavors. For example, we have $V_{12} = V_{us}$, $V_{21} = V_{cd}$, and $V_{12}^\dagger = V_{21}^* = V_{cd}^*$.

TABLE II. Partial decay widths (in GeV) of the W boson at the tree level and at one loop, for $M_H = 100, 250$, or 600 GeV.

Partial width	Born	One loop		
		$M_H = 100$ GeV	$M_H = 250$ GeV	$M_H = 600$ GeV
$\Gamma(W \rightarrow e \nu_e)$	0.2275641	0.2268336	0.2268444	0.2268458
$\Gamma(W \rightarrow \mu \nu_\mu)$	0.2275635	0.2268331	0.2268439	0.2268452
$\Gamma(W \rightarrow \tau \nu_\tau)$	0.2273974	0.2266717	0.2266823	0.2266836
$\Gamma(W \rightarrow \text{leptons})$	0.6825249	0.6803384	0.6803706	0.6803746
$\Gamma(W \rightarrow ud)$	0.6487322	0.6711894	0.6712202	0.6712242
$\Gamma(W \rightarrow us) \times 10$	0.3394894	0.3512428	0.3512589	0.3512610
$\Gamma(W \rightarrow ub) \times 10^4$	0.1086947	0.1127355	0.1127399	0.1127402
$\Gamma(W \rightarrow cd) \times 10$	0.3392503	0.3511185	0.3511344	0.3511363
$\Gamma(W \rightarrow cs)$	0.6473169	0.6699818	0.6700121	0.6700158
$\Gamma(W \rightarrow cb) \times 10^2$	0.1086359	0.1127907	0.1127950	0.1127953
$\Gamma(W \rightarrow \text{hadrons})$	1.3650203	1.4125466	1.4126109	1.4126190
Γ_W	2.0475452	2.0928850	2.0929814	2.0929936

m_b , m_t , s_{12} , s_{23} , and s_{13} by $\pm 1 \times 10^{-10} \text{ GeV}^{-2}$, ± 0.002 , $\pm 38 \text{ MeV}$, $\pm 2.1 \text{ MeV}$, $\pm 0.1 \text{ GeV}$, $\pm 0.2 \text{ GeV}$, $\pm 5.1 \text{ GeV}$, ± 0.004 , ± 0.003 , and ± 0.002 [25,30] shift Γ_W by ± 0.02 , ± 0.9 , ± 3.0 , $\pm 7.5 \times 10^{-4}$, ± 0.02 , $\pm 1 \times 10^{-4}$, ± 0.033 , $\pm 3 \times 10^{-5}$, $\pm 3 \times 10^{-4}$, and $\pm 3 \times 10^{-5} \text{ MeV}$, respectively. The residual parametric uncertainties are marginal.

V. CONCLUSIONS

We calculated the partial decay widths of the W boson at one loop in the SM using the on-shell scheme endowed with the gauge-independent definition of the CKM matrix V_{ij} recently proposed in Refs. [11,12]. Working in the R_ξ gauge, we explicitly verified that the final expressions are independent of the gauge parameters. In particular, the renormalization constant $\delta V_{ij}/V_{ij}$, Eq. (2.9), and the one-loop amplitude $\mathcal{M}_1^{wf_i f'_j}$, Eq. (2.6), of the W -boson decay to quarks, Eq. (2.1), with this renormalization constant removed are separately gauge independent. In this respect, we disagree with the findings of Ref. [22]. The difference between our analysis and the corresponding one with the gauge-dependent definition of $\delta V_{ij}/V_{ij}$ from Ref. [10] is of the same order as the entire effect due to the renormalization of the CKM matrix, but it is small compared to the present experimental precision. Furthermore, we established the relationship be-

tween the on-shell and $\overline{\text{MS}}$ definitions of the CKM matrix, both in its generic form [25] and in the Wolfenstein parametrization [26]. As a by-product of our analysis, we recovered the beta function of the CKM matrix [13].

Note added. In the meantime, a revised version of Ref. [22] has appeared, in which the weaknesses pointed out in Sec. II have been remedied.

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APPENDIX: FERMION TWO-POINT FUNCTIONS

The unrenormalized two-point function describing the fermion transition $f_j \rightarrow f_i$, at four-momentum p , may be decomposed as

$$\Gamma_{ij}^f(p) = i[\delta_{ij}(\not{p} - m_{f,i}) + \not{p} \omega_- \Sigma_{ij}^{f,L}(p^2) + \not{p} \omega_+ \Sigma_{ij}^{f,R}(p^2) + (m_{f,i} \omega_- + m_{f,j} \omega_+) \Sigma_{ij}^{f,S}(p^2)]. \quad (\text{A1})$$

At one loop in the R_ξ gauge, we find, for the coefficient functions herein,

$$\begin{aligned}
\Sigma_{ij}^{f,L}(p^2) = & -\frac{\alpha}{4\pi} \left\{ \delta_{ij} Q_f^2 \left\{ 2B_1(p^2, m_{f,i}, \lambda) + 1 + B_0(p^2, \lambda, m_{f,i}) - \xi_A B_0(p^2, \sqrt{\xi_A} \lambda, m_{f,i}) \right. \right. \\
& + \frac{p^2 - m_{f,i}^2}{\lambda^2} [B_1(p^2, \lambda, m_{f,i}) - B_1(p^2, \sqrt{\xi_A} \lambda, m_{f,i})] \left. \right\} \\
& + \delta_{ij} (g_f^-)^2 \left\{ 2B_1(p^2, m_{f,i}, M_Z) + 1 + B_0(p^2, m_{f,i}, M_Z) - \xi_Z B_0(p^2, m_{f,i}, \sqrt{\xi_Z} M_Z) \right. \\
& + \frac{p^2 - m_{f,i}^2}{M_Z^2} [B_1(p^2, M_Z, m_{f,i}) - B_1(p^2, \sqrt{\xi_Z} M_Z, m_{f,i})] \left. \right\} + \delta_{ij} \frac{m_{f,i}^2}{4s_w^2 M_W^2} [B_1(p^2, m_{f,i}, \sqrt{\xi_Z} M_Z) + B_1(p^2, m_{f,i}, M_H)] \\
& + \frac{1}{2s_w^2} \sum_k V_{ik} V_{kj}^\dagger \left[2B_1(p^2, m_{f',k}, M_W) + \frac{m_{f',k}^2}{M_W^2} B_1(p^2, m_{f',k}, \sqrt{\xi_W} M_W) + 1 + B_0(p^2, m_{f',k}, M_W) \right. \\
& - \xi_W B_0(p^2, m_{f',k}, \sqrt{\xi_W} M_W) + \frac{p^2 - m_{f',k}^2}{M_W^2} [B_1(p^2, M_W, m_{f',k}) - B_1(p^2, \sqrt{\xi_W} M_W, m_{f',k})] \left. \right] \left. \right\}, \\
\Sigma_{ij}^{f,R}(p^2) = & -\frac{\alpha}{4\pi} \left\{ \delta_{ij} Q_f^2 \left\{ 2B_1(p^2, m_{f,i}, \lambda) + 1 + B_0(p^2, \lambda, m_{f,i}) - \xi_A B_0(p^2, \sqrt{\xi_A} \lambda, m_{f,i}) \right. \right. \\
& + \frac{p^2 - m_{f,i}^2}{\lambda^2} [B_1(p^2, \lambda, m_{f,i}) - B_1(p^2, \sqrt{\xi_A} \lambda, m_{f,i})] \left. \right\} \\
& + \delta_{ij} (g_f^+)^2 \left\{ 2B_1(p^2, m_{f,i}, M_Z) + 1 + B_0(p^2, m_{f,i}, M_Z) - \xi_Z B_0(p^2, m_{f,i}, \sqrt{\xi_Z} M_Z) \right. \\
& + \frac{p^2 - m_{f,i}^2}{M_Z^2} [B_1(p^2, M_Z, m_{f,i}) - B_1(p^2, \sqrt{\xi_Z} M_Z, m_{f,i})] \left. \right\} + \delta_{ij} \frac{m_{f,i}^2}{4s_w^2 M_W^2} [B_1(p^2, m_{f,i}, \sqrt{\xi_Z} M_Z) + B_1(p^2, m_{f,i}, M_H)] \\
& + \frac{m_{f,i} m_{f',j}}{2s_w^2 M_W^2} \sum_k V_{ik} V_{kj}^\dagger B_1(p^2, m_{f',k}, \sqrt{\xi_W} M_W) \left. \right\}, \\
\Sigma_{ij}^{f,S}(p^2) = & -\frac{\alpha}{4\pi} \left\{ \delta_{ij} Q_f^2 [4B_0(p^2, m_{f,i}, \lambda) - 2 - B_0(p^2, m_{f,i}, \lambda) + \xi_A B_0(p^2, m_{f,i}, \sqrt{\xi_A} \lambda)] \right. \\
& + \delta_{ij} g_f^+ g_f^- [4B_0(p^2, m_{f,i}, M_Z) - 2 - B_0(p^2, m_{f,i}, M_Z) + \xi_Z B_0(p^2, m_{f,i}, \sqrt{\xi_Z} M_Z)] \\
& + \delta_{ij} \frac{m_{f,i}^2}{4s_w^2 M_W^2} [B_0(p^2, m_{f,i}, \sqrt{\xi_Z} M_Z) - B_0(p^2, m_{f,i}, M_H)] + \frac{1}{2s_w^2 M_W^2} \sum_k V_{ik} V_{kj}^\dagger m_{f',k}^2 B_0(p^2, m_{f',k}, \sqrt{\xi_W} M_W) \left. \right\},
\end{aligned} \tag{A2}$$

where

$$B_0(p^2, m_0, m_1) = \frac{(2\pi\mu)^{4-D}}{i\pi^2} \int \frac{d^D q}{(q^2 - m_0^2)[(q+p)^2 - m_1^2]}, \tag{A3}$$

$$B_1(p^2, m_0, m_1) = \frac{m_1^2 - m_0^2}{2p^2} [B_0(p^2, m_0, m_1) - B_0(0, m_0, m_1)] - \frac{1}{2} B_0(p^2, m_0, m_1).$$

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