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Associated production of electroweak bosons and heavy mesons at LHCb and the prospects to observe double parton interactions

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The production of weak gauge bosons in association with heavy flavored mesons at the LHCb conditions is considered, and a detailed study of the different contributing processes is presented including single and double parton scattering (DPS) mechanisms. We find that the usual DPS factorization formula needs to be corrected for the limited partonic phase space, and that including the relevant corrections reduces discrepancies in the associated ZD production. We conclude finally that double parton scattering dominates the production of same-sign $W^{\pm}D^{\pm}$ states, as well as the production of W^{-} bosons associated with B mesons. The latter processes can thus be regarded as new useful DPS indicators.

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I. INTRODUCTION

In our recent publication [1] we considered the associated production of charged gauge bosons $W^{\pm}$ and charged charmed mesons $D^{(*)\pm}$ at the LHC and came to the conclusion that same-sign $W^{\pm}D^{(*)\pm}$ events could serve as an indicator of double parton interactions [2–4]. Our consideration was only restricted to the central region, i.e. to CMS [5] and ATLAS [6] kinematic conditions, since these were the only collaborations who provided the data (though not on same-sign WD configurations). To the best of our knowledge, the LHCb Collaboration is going to measure the production cross sections for all of the four WD charge combinations. Now, we feel it very tempting to foreshadow the experimental measurement with a theoretical prediction.

The planned work needs to be done with care, since the momentum conservation requirement in the large-x region may spoil the factorization hypothesis commonly used in double parton scattering (DPS) calculations. This motivates us to introduce certain corrections to the theory. We also wish to extend our analysis to the production of WB states. The latter is closely similar to the WD case in its DPS part, but the background coming from single parton scattering (SPS) is rather different and awaits a special survey.

Our previous calculation [1] was done in the k_t-factorization technique, but in the present paper we adopt "combined" approach. That is, the production of heavy systems like W or Z bosons as well as their SPS production in association with heavy quarks is done in the traditional collinear scheme, while the k_t factorization is used for solely produced $c\bar{c}$ or $b\bar{b}$ pairs (the latter constitute one branch of a double parton interaction). Then we benefit from easily including higher-order radiative corrections which are taken into account in the form of k_t-dependent parton densities. Thus, we rely on a combination of two techniques, with each of them being used at the kinematics where it is most suitable (W and Z at large x, $c\bar{c}$ and $b\bar{b}$ at small x).

The outline of the paper is as follows. In Sec. II, we reconsider the DPS formalism in the forward (LHCb) kinematics and introduce corrections matching the momentum conservation requirement. Then we test our theory by applying it to the associated ZD production, where the existing data [7] form grounds for a comparison. We further use the corrected formalism to make predictions on the charm-associated $W^{\pm}$ production in Sec. III and on the beauty-associated $W^{\pm}$ production in Sec. IV. Our findings are summarized in Sec. V.

II. DOUBLE PARTON SCATTERING IN THE LARGE-x REGION

As far as the SPS contributions are concerned, the calculations are straightforward and need no special explanation. Throughout this paper, all calculations are

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based on the following parameter setting. We employ the k_t-factorization approach [8,9] for relatively light states ($c\bar{c}$ or $b\bar{b}$) and collinear factorization for states containing W or Z bosons. We used Kimber-Martin-Ryskin [10] parametrization for unintegrated quark and gluon distributions with Martin-Stirling-Thorne-Watt (MSTW) [11] collinear densities taken as input (or pure MSTW densities for collinear calculations); we used running strong and electroweak coupling constants normalized to $\alpha_s(m_Z^2) = 0.118$; $\alpha(m_Z^2) = 1/128$; $\sin^2\Theta_W = 0.2312$; the factorization and renormalization scales were chosen as $\mu_R^2 = \mu_F^2 = m_T^2(W) \equiv m_W^2 + p_T^2(W)$ or $m_T^2(Z)$ for all processes involving W and Z bosons, and $\mu_R^2 = \mu_F^2 = m_Q^2$ for the production of sole $Q\bar{Q}$ pairs ($Q = c, b$); the quark masses were set to $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, $m_t = 175$ GeV, c and b quarks were converted into D^+ and B mesons using the Peterson fragmentation function [12] with $\epsilon_c = 0.06$ and $\epsilon_b = 0.006$, respectively, and normalized to $f(c \to D^+) = 0.268$ [13], $f(b \to B^-) = 0.40$ and $f(b \to \bar{B}^0) = 0.40$.

Our choice of renormalization scale is slightly different from the conventional one by using m_Q rather than $m_T(Q)$, but we then can fit the experimental data [see below, Eqs. (4), (15) and (22); otherwise, with $\mu_R^2 = m_T^2(Q) \equiv m_Q^2 + p_T^2(Q)$, the calculations would lie slightly below the data points]. We do not mind developing here a rigorous theory of heavy quark production, but are rather interested in understanding the relative importance of the different contributions. Our simple prescription would suffice for that purpose.

To calculate the DPS contributions, one commonly makes use of a simple factorization formula (for details see the reviews [2–4] and the references therein),

$$\sigma_{\rm DPS}^{WD} = \sigma_{\rm SPS}^{W}\sigma_{\rm SPS}^{D}/\sigma_{\rm eff}, \tag{1}$$

where $\sigma_{\rm eff}$ is a normalization constant that encodes all "DPS unknowns" into a single phenomenological parameter. Deriving this formula relies on two simplifying approximations, that (i) the double parton distribution functions can be decomposed into longitudinal and transverse components, and (ii) the longitudinal component $D_p^{ij}(x_1, x_2; Q_1^2, Q_2^2)$ reduces to the diagonal product of two independent single parton distribution functions:

$$D_p^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_p^i(x_1; Q_1^2) D_p^j(x_2; Q_2^2) \tag{2}$$

(here x_1 and x_2 are the longitudinal momentum fractions of the partons i and j entering the hard subprocesses at the probing scales Q_1 and Q_2). The latter approximation is acceptable for such collider experiments where only small-x values are probed; however, this cannot be said of the LHCb conditions, especially with respect to heavy systems as electroweak bosons. At the LHCb conditions, the probed x values are not far from the phase space boundary where the evident restriction on the total parton momentum $x_1 + x_2 \le 1$ violates the DPS factorization ansatz.

Setting the boundary condition in the form of theta function $\Theta(1 - x_1 - x_2)$ would result in a steplike discontinuity at the edge of the phase space. This does not seem physically consistent for the parton densities. In a more accurate approach [14–21],

$$D_p^{ij}(x_1, x_2; Q_1^2, Q_2^2) = D_p^i(x_1; Q_1^2) D_p^j(x_2; Q_2^2) \times (1 - x_1 - x_2)^n, \tag{3}$$

the kinematical constraints are smoothly put into play with the correction factor $(1 - x_1 - x_2)^n$, where $n > 0$ is a parameter to be fixed phenomenologically. The integrand and its derivative remain continuous at the phase space border. One often chooses $n = 2$. This choice of the phase space factor can be partly justified [14,16] in the framework of perturbative QCD and gives double parton distribution functions which satisfy the momentum sum rules [15] reasonably well. To feel the size of the possible effect we also tried $n = 3$. The case of unconstrained phase space is presented in Table I as $n = 0$. A numerical value of $\sigma_{\rm eff} \simeq 15$ mb has earlier been obtained empirically from fits to $p\bar{p}$ and pp data. This will be taken as the default value throughout the paper. As we will see, variations within some reasonable range $\sigma_{\rm eff} \simeq 15 \pm 5$ mb would affect our DPS predictions (with the respective errors presented in the tables), though without changing our basic conclusions.

Now we are ready to compare predictions with the data. For the LHCb fiducial phase space [7] we obtain

$$\sigma_{\rm incl}(D^+) + \sigma_{\rm incl}(D^0) = 670\ \mu{\rm b}, \tag{4}$$

$$Br^{Z\to ll}\sigma_{\rm incl}(Z^0) = 75\ {\rm pb}, \tag{5}$$

in excellent agreement with Ref. [22], reporting $Br^{Z\to ll}\sigma_{\rm incl}(Z^0) = 76$ pb.

TABLE I. Comparison of the measured and predicted cross sections (in pb) for Z bosons produced in association with open charm mesons in the fiducial region $p_T(\mu^\pm) > 20$ GeV, $2 < \eta(\mu^\pm) < 4.5$, $2 < p_T(D) < 12$ GeV, $2 < y(D) < 4$. The SPS and DPS contributions are shown separately, with n indicating the power of the correction factor in Eq. (3).

Channel	Data	SPS	DPS($n = 0$)	DPS($n = 2$)	DPS($n = 3$)
Z^0D^0	2.50	0.6	2.4 ± 0.6	1.15 ± 0.38	0.95 ± 0.32
Z^0D^+	0.44	0.25	0.95 ± 0.32	0.50 ± 0.17	0.40 ± 0.13
Sum	2.94	0.85	3.35 ± 0.92	1.65 ± 0.55	1.35 ± 0.45

As the experimental statistics is very limited (7 ZD^0 events and 4 ZD^+ events) it is more reasonable not to consider the ZD^0 and ZD^+ cross sections separately, but rather to rely on the sum of them. Taken separately, the ZD^0 and ZD^+ data are at variance with other measurements. There exist independent publications [13,23,24] (including the one by the LHCb Collaboration) which all agree with each other showing the ratio $\sigma(D^0)/\sigma(D^+) \sim 2.5$, in contrast with $\sigma(ZD^0)/\sigma(ZD^+) \sim 5.5$ seen in [7]. In fact, the authors of [7] seem to greatly underestimate their statistical errors.

So, we calculate the $Zc\bar{c}$ production cross section at the quark level and then convert c quarks into D^0 and D^+ mesons with the overall probability normalized to 85% (with the remaining 15% left for D_s and Λ_c). We estimate the yields from the different subprocesses as

$$\sigma(u\bar{u} \to Zc\bar{c}) = 5 \text{ pb}, \tag{6}$$

$$\sigma(d\bar{d} \to Zc\bar{c}) = 2.6 \text{ pb}, \tag{7}$$

$$\sigma(gu \to Zuc\bar{c}) = 11.4 \text{ pb}, \tag{8}$$

$$\sigma(gd \to Zdc\bar{c}) = 5.2 \text{ pb}, \tag{9}$$

$$\sigma(gg \to Zc\bar{c}) = 2.5 \text{ pb}. \tag{10}$$

Summing up and multiplying by the quark fragmentation probability and by the $Z \to \mu^+\mu^-$ branching fraction we arrive at $\sigma^{\mathrm{SPS}}(ZD^0, ZD^+) = 0.85$ pb. This result is consistent with the theoretical calculation presented in [7] under the name of “MCFM massive.” Adding the DPS contribution in the form (1) gives $\sigma^{\mathrm{SPS+DPS}}(ZD^0, ZD^+) = 4.2$ pb, which significantly exceeds the data. After applying the correction factor (3) the agreement becomes rather satisfactory (see Table I).

III. CHARM-ASSOCIATED $W^{\pm}$ PRODUCTION

The production of opposite-sign $W^{\pm}D^{\mp}$ states is dominated by the quark-gluon scattering at $\mathcal{O}(\alpha_s\alpha)$,

$$g + q \to W^- + c \quad \text{or} \quad g + \bar{q} \to W^+ + \bar{c}, \tag{11}$$

where the main role belongs to strange quarks, $q = s$. Among the variety of processes contributing to both opposite-sign and same-sign WD states, the most important ones are the quark-antiquark annihilation at $\mathcal{O}(\alpha_s^2\alpha)$,

$$\begin{aligned} &u + \bar{d} \to W^+ + c + \bar{c} \quad \text{or} \\ &d + \bar{u} \to W^- + c + \bar{c}, \end{aligned} \tag{12}$$

and quark-gluon scattering at $\mathcal{O}(\alpha_s^3\alpha)$,

$$\begin{aligned} &g + u \to W^+ + d + c + \bar{c} \quad \text{or} \\ &g + d \to W^- + u + c + \bar{c}. \end{aligned} \tag{13}$$

In addition to that, there present indirect contributions from the production of top-quark pairs

$$g + g \to t + \bar{t} \quad \text{and} \quad q + \bar{q} \to t + \bar{t} \tag{14}$$

followed by a long chain of decays: $t \to W^+b$, $W^+ \to c\bar{s}$, $b \to cX$ or $b \to c\bar{c}s$ (and the charge conjugated modes). All other possible processes are suppressed by extra powers of coupling constants or by Kobayashi-Maskawa mixing matrix. Subprocesses $q\bar{q} \to W^- c\bar{s}$ and $q\bar{q} \to W^+ s\bar{c}$, though formally of the same order as (12), are heavily suppressed by the gluon propagator having virtuality of order m_W^2 rather than m_{cc}^2.

Our parameter setting was basically described in Sec. II. For the indirect contributions we also assumed 100% branching fraction for $t \to bW$ and used the inclusive branching fractions $Br(\bar{B}^0 \to D^+X) = 37\%$, $Br(B^0 \to D^+X) = 3\%$, $Br(B^- \to D^+X) = 10\%$ and $Br(B^+ \to D^+X) = 2.5\%$ listed in the Particle Data Book [25].

The evaluation of the DPS contributions is done in accordance with the explanations given in the previous section. The individual inclusive SPS cross sections $\sigma(D^{\pm})$ and $\sigma(W^{\pm})$ have been calculated as in Refs. [13] and [26], respectively. For the LHCb fiducial phase space our expectations read

$$\sigma_{\mathrm{incl}}(D^+) = \sigma_{\mathrm{incl}}(D^-) = 190\ \mu\text{b}, \tag{15}$$

$$Br^{W\to l\nu}\sigma_{\mathrm{incl}}(W^+) = 970 \text{ pb}, \tag{16}$$

$$Br^{W\to l\nu}\sigma_{\mathrm{incl}}(W^-) = 680 \text{ pb}, \tag{17}$$

in good agreement with [7] and [27], respectively. Our results for the SPS and DPS channels are displayed in Table II. All DPS contributions are presented there without phase space corrections; they have to be multiplied by a correction factor of 0.48 for $n = 2$ or 0.38 for $n = 3$.

The indirect contributions, though small already, can be further suppressed using a well-known experimental technique based on the property that the secondary b-decay vertex is displaced with respect to the primary interaction vertex. Summing up the direct contributions, we see that the predicted same-sign WD production rates with and without DPS channels differ by a significant factor. This difference is sensible enough to warrant interpretation of the forthcoming LHCb data as giving conclusive evidence for double parton interactions.

IV. BEAUTY-ASSOCIATED $W^{\pm}$ PRODUCTION

Associated WB production is not simply a repetition of the WD case with a different quark mass. Indeed, the

TABLE II. Predicted WD production cross sections times the $W \to l\nu$ branching (in pb) integrated over the fiducial region $p_T(l) > 20$ GeV, $2 < \eta(l) < 4.5$, $2 < p_T(D) < 12$ GeV, $2 < \eta(D) < 4$.

Subprocess	W^+D^+	W^+D^-	W^-D^-	W^-D^+
Double parton scattering contributions				
$gg \to c\bar{c}, u\bar{d} \to W^+$	12.3 ± 4.1	12.3 ± 4.1	$\cdots$	$\cdots$
$gg \to c\bar{c}, d\bar{u} \to W^-$	$\cdots$	$\cdots$	8.9 ± 3.0	8.9 ± 3.0
Single parton scattering contributions				
$g\bar{s}, g\bar{d} \to W\bar{c}$	$\cdots$	1.7	$\cdots$	$\cdots$
$gs, gd \to Wc$	$\cdots$	$\cdots$	$\cdots$	2.0
$u\bar{d} \to Wc\bar{c}$	0.8	0.8	$\cdots$	$\cdots$
$d\bar{u} \to Wc\bar{c}$	$\cdots$	$\cdots$	0.4	0.4
$gu \to Wdc\bar{c}$	1.9	1.9	$\cdots$	$\cdots$
$g\bar{d} \to W\bar{u}c\bar{c}$	0.16	0.16	$\cdots$	$\cdots$
$gd \to Wuc\bar{c}$	$\cdots$	$\cdots$	0.8	0.8
$g\bar{u} \to W\bar{d}c\bar{c}$	$\cdots$	$\cdots$	0.14	0.14
$gg \to t\bar{t} \to$ decays	0.01	0.01	0.01	0.01
$q\bar{q} \to t\bar{t} \to$ decays	0.015	0.02	0.015	0.02

TABLE III. Predicted WB production cross sections times the $W \to l\nu$ branching (in picobarns) integrated over the fiducial region $p_T(l) > 20$ GeV, $2 < \eta(l) < 4.5$, $2 < \eta(B) < 4.5$. Here B^+ and B^- denote the sum of B^+ and B^0 and the sum of the B^- and $\bar{B}^0$ mesons, respectively.

Subprocess	W^+B^+	W^+B^-	W^-B^-	W^-B^+
Double parton scattering contributions				
$gg \to b\bar{b}, u\bar{d} \to W^+$	5.5 ± 1.8	5.5 ± 1.8	$\cdots$	$\cdots$
$gg \to b\bar{b}, d\bar{u} \to W^-$	$\cdots$	$\cdots$	4.0 ± 1.3	4.0 ± 1.3
Single parton scattering contributions				
$u\bar{d} \to Wb\bar{b}$	1.2	1.2	$\cdots$	$\cdots$
$d\bar{u} \to Wb\bar{b}$	$\cdots$	$\cdots$	0.5	0.5
$gu \to Wdb\bar{b}$	2.7	2.7	$\cdots$	$\cdots$
$g\bar{d} \to W\bar{u}b\bar{b}$	0.22	0.22	$\cdots$	$\cdots$
$gd \to Wub\bar{b}$	$\cdots$	$\cdots$	1.1	1.1
$g\bar{u} \to W\bar{d}b\bar{b}$	$\cdots$	$\cdots$	0.2	0.2
$gg \to t\bar{t} \to WWb\bar{b}$	0.030	0.045	0.030	0.045
$q\bar{q} \to t\bar{t} \to WWb\bar{b}$	0.055	0.060	0.055	0.060
$u\bar{d} \to t\bar{b} \to Wb\bar{b}$	0.0018	0.0042	0.0018	0.0042
$d\bar{u} \to b\bar{t} \to W\bar{b}b$	0.0002	0.0005	0.0002	0.0005

contributing parton subprocesses are significantly different. First, there is no analog to process (11), as the Cabibbo-Kobayashi-Maskawa couplings of a b quark to the quarks of two lighter generations are really negligible. Second, the feed down from top-quark decays now must be regarded as a direct contribution, as it shows no secondary decay vertex (and, therefore, cannot be rejected experimentally).

The full list of processes included in the present analysis reads as follows: quark-antiquark annihilation at $\mathcal{O}(\alpha_s^2\alpha)$,

$$u + \bar{d} \to W^+ + b + \bar{b} \quad \text{or}$$
$$d + \bar{u} \to W^- + b + \bar{b}; \tag{18}$$

quark-gluon scattering at $\mathcal{O}(\alpha_s^3\alpha)$,

$$g + u \to W^+ + d + b + \bar{b} \quad \text{or}$$
$$g + d \to W^- + u + b + \bar{b}; \tag{19}$$

strong production of top-quark pairs

$$g + g \to t + \bar{t} \quad \text{and} \quad q + \bar{q} \to t + \bar{t} \tag{20}$$

followed by their decays $t \to W^+ b$, $\bar{t} \to W^- \bar{b}$; and, finally, weak production of $t\bar{b}$ and $\bar{t}b$ states

$$u + \bar{d} \to t + \bar{b} \quad \text{or} \quad d + \bar{u} \to b + \bar{t}, \tag{21}$$

also followed by t decays.

With the parameter setting described in Sec. III, we estimate the inclusive production of b quarks in the LHCb domain as

$$\sigma_{\mathrm{incl}}(b) = \sigma_{\mathrm{incl}}(\bar{b}) = 95\mu\mathrm{b}. \tag{22}$$

This number is compatible with the experimental result [28]

$$\sigma(B^+) + \sigma(B^0) + \sigma(B_s) = 39 + 38 + 10 = 87\mu b; \tag{23}$$

at least, it lies within the usual theoretical uncertainty related to the choice of the interaction scale and quark mass. Combining this result with Eqs. (16) and (17) we obtain the DPS cross section for Wb. Table III represents our predictions for unconstrained phase space of Eq. (1); they have to be corrected by a factor of 0.45 for $n = 2$ or 0.36 for $n = 3$.

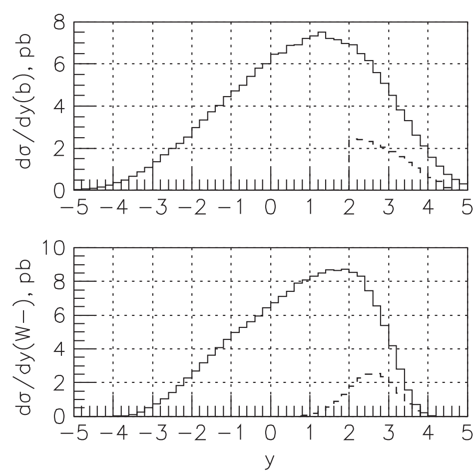

FIG. 1. Rapidity distributions of the b quarks (upper panel) and W^- bosons (lower panel) produced in association in the process $d\bar{u} \to W^- b\bar{b}$. Solid curves, original spectra; dashed curves, left after imposing the LHCb kinematic cuts.

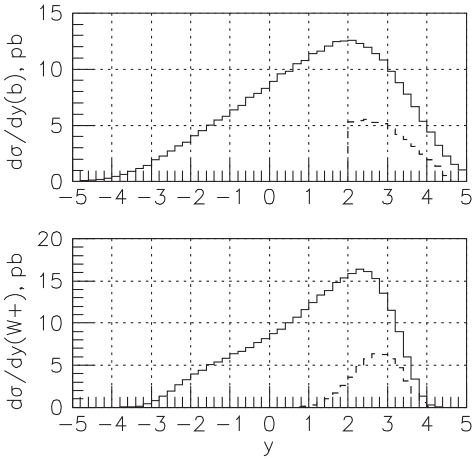

FIG. 2. Rapidity distributions of the b quarks (upper panel) and W^+ bosons (lower panel) produced in association in the process $u\bar{d} \to W^+ b\bar{b}$. Solid curves, original spectra; dashed curves, left after imposing the LHCb kinematic cuts.

We find it worth saying a few words on the specific properties of SPS and DPS kinematics at the LHCb conditions. Parton momentum configurations in the SPS channels are very asymmetric. To produce a heavy Wb system with both W and b having large positive rapidity, the positive light-cone momentum fraction of the incoming parton must be large. On the average, valence u quarks carry a larger x than valence d quarks, thus favoring the production of W^+ in comparison with W^- bosons in subprocesses (18) and (19). This property is illustrated in Figs. 1 and 2. Sea quarks are mainly concentrated in the small-x region and cannot contribute at a significant level.

In general, the desired large positive light-cone momentum is easier to get with two independent partons in DPS than with a single parton in SPS. This explains the relative suppression of the SPS channels seen in Table III. Especially pleasant are negligible contributions from top-quark decays. Their rapidity distributions are shown in Fig. 3. DPS clearly and unambiguously dominates the production of $W^\pm B$ states, making them very informative observables.

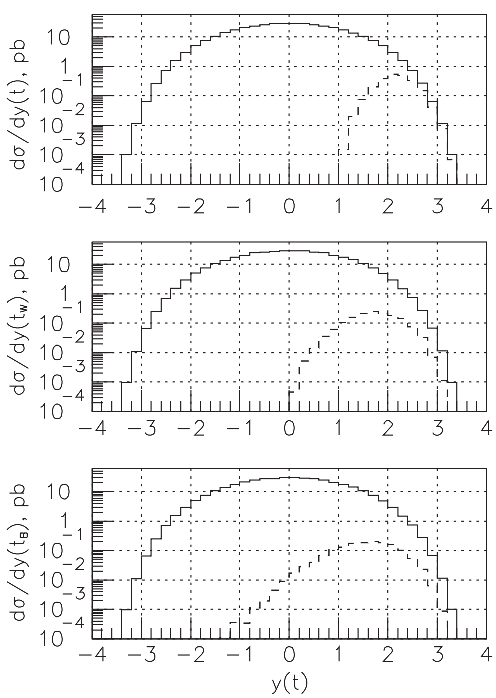

FIG. 3. Rapidity distributions of the top quarks or antiquarks. Opposite-sign $W^\pm b^\mp$ events: top quarks converting into a $W^\pm b^\mp$ pair (upper panel). Same sign $W^\pm b^\pm$ events: top quarks producing W bosons (middle panel); top quarks producing beauty quarks (lower panel). Solid curves, original spectra; dashed curves, left after imposing the LHCb kinematic cuts.

V. CONCLUSIONS

Having considered the production of $Z^0 D$, $W^\pm D$ and $W^\pm B$ states at the LHCb conditions, we deduce the following assessments.

(i) As a general rule for the production of electroweak bosons in the DPS channel, the simple DPS factorization formula needs to be corrected for the limited partonic phase space. Numerically, these corrections amount to a factor of 2 in the total rates and, when taken into account, lead to better agreement with the available data on $Z^0 D$ production than there seemed to be before.

(ii) Similarly to what has been observed earlier for the central region (ATLAS and CMS), the production of same-sign $W^\pm D^\pm$ states in the forward region is also dominated by the DPS mechanism. Once again, this process can be recommended as a DPS indicator.

(iii) Along with that, LHCb kinematics opens doors for a still new indicative process, which is the beauty-associated production of weak gauge bosons W^-. The charge of the accompanying b quark is irrelevant. Here we benefit from the asymmetric rapidity selection cuts, which correspond to large positive light-cone momentum values of the incoming partons. The essential values can easier be reached with two independent partons in DPS than with a single parton in SPS, thus giving favor to DPS production. Another useful feature of the LHCb kinematics as compared to the ATLAS and CMS conditions is in much lower p_t cuts for inclusive open flavor production. This enhances the visible inclusive cross sections $\sigma_{\rm incl}(D)$ and $\sigma_{\rm incl}(B)$ and, consequently, the DPS channel in associated production with gauge bosons.

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[1] S. P. Baranov, M. A. Malyshev, A. V. Lipatov, A. M. Snigirev, and N. P. Zotov, Phys. Lett. B **746**, 100 (2015).

[2] P. Bartalini *et al.*, arXiv:1111.0469.

[3] H. Abramowicz *et al.*, arXiv:1306.5413.

[4] S. Bansal *et al.*, arXiv:1410.6664.

[5] S. Chatrchyan *et al.* (CMS Collaboration), J. High Energy Phys. 02 (2014) 013.

[6] G. Aad *et al.* (ATLAS Collaboration), J. High Energy Phys. 05 (2014) 068.

[7] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 04 (2014) 091.

[8] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rep. **100**, 1 (1983); E. M. Levin, M. G. Ryskin, Y. M. Shabelsky, and A. G. Shuvaev, Sov. J. Nucl. Phys. **53**, 657 (1991); S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. B **242**, 97 (1990); Nucl. Phys. **B366**, 135 (1991); J. C. Collins and R. K. Ellis, Nucl. Phys. **B360**, 3 (1991).

[9] B. Andersson *et al.* (Small x Collaboration), Eur. Phys. J. C **25**, 77 (2002); , Eur. Phys. J. C **35**, 67 (2004); **48**, 53 (2006).

[10] M. A. Kimber, A. D. Martin, and M. G. Ryskin, Phys. Rev. D **63**, 114027 (2001).

[11] A. D. Martin, W. J. Stirling, R. S. Thorne, and G. Watt, Eur. Phys. J. C **63**, 189 (2009).

[12] C. Peterson, D. Schlatter, I. Schmitt, and P. M. Zerwas, Phys. Rev. D **27**, 105 (1983).

[13] H. Jung, M. Kraemer, A. V. Lipatov, and N. P. Zotov, J. High Energy Phys. 01 (2011) 085.

[14] V. L. Korotkikh and A. M. Snigirev, Phys. Lett. B **594**, 171 (2004).

[15] J. R. Gaunt and W. J. Stirling, J. High Energy Phys. 03 (2010) 005.

[16] A. M. Snigirev, Phys. Rev. D **83**, 034028 (2011).

[17] H.-M. Chang, A. V. Manohar, and W. J. Waalewijn, Phys. Rev. D **87**, 034009 (2013).

[18] M. Rinaldi, S. Scopetta, and V. Vento, Phys. Rev. D **87**, 114021 (2013).

[19] K. Golec-Biernat and E. Lewandowska, Phys. Rev. D **90**, 014032 (2014).

[20] F. A. Ceccopieri, Phys. Lett. B **734**, 79 (2014).

[21] A. M. Snigirev, N. A. Snigireva, and G. M. Zinovjev, Phys. Rev. D **90**, 014015 (2014).

[22] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 02 (2013) 106.

[23] R. Aaij *et al.* (LHCb Collaboration), Nucl. Phys. **B871**, 1 (2013).

[24] L. Gladilin, Eur. Phys. J. C **75**, 19 (2015).

[25] K. A. Olive *et al.* (Particle Data Group), Chin. Phys. C **38**, 090001 (2014).

[26] S. P. Baranov, A. V. Lipatov, and N. P. Zotov, Phys. Rev. D **78**, 014025 (2008).

[27] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 06 (2012) 058; 12 (2014) 079.

[28] R. Aaij *et al.* (LHCb Collaboration), J. High Energy Phys. 08 (2013) 117.