Selectron production at an e^-e^- linear collider with transversely polarized beams

A. Bartl[1], H. Fraas[2], K. Hohenwarter-Sodek[1], T. Kernreiter[1],
G. Moortgat-Pick[3], A. Wagner[2]

[1] *Institut für Theoretische Physik, Universität Wien, A-1090 Vienna, Austria*
[2] *Institut für Theoretische Physik und Astrophysik, Universität Würzburg,*
   D-97074 Würzburg, Germany
[3] *IPPP, University of Durham, Durham DH1 3LE, UK*

Abstract

We study selectron production at an e^-e^- linear collider. With the help of transverse beam polarizations, we define CP sensitive observables in the production process $e^-e^- \to \tilde{e}_L^- \tilde{e}_R^-$. This process proceeds via $t-$channel and $u-$channel exchange of neutralinos, and is sensitive to CP violation in the neutralino sector. We present numerical results and estimate the significances to which the CP sensitive observables can be measured.

1 Introduction

Supersymmetry (SUSY) is one of the most attractive extensions of the Standard Model (SM). If SUSY particles are found at Tevatron or LHC, then one of the most important goals of the future international linear collider (ILC) will be the precise determination of the quantum numbers, masses and couplings of supersymmetric particles [1]. In addition to the e^+e^- mode of the ILC also the e^-e^- mode offers the possibility to study properties of selectrons and neutralinos [2].

In this paper we investigate the potential of e^-e^- collisions with transverse beam polarizations for the determination of SUSY CP phases. Our framework is the minimal supersymmetric standard model (MSSM) with complex parameters. The usefulness of transverse beam polarizations at the ILC has been discussed before for various observables [3–7].

We study the production process

$$e^-e^- \to \tilde{e}_L^- \tilde{e}_R^- \tag{1}$$

which proceeds via neutralino exchange in the $t-$channel and $u-$channel [1]. Therefore, it is sensitive to the complex parameters in the neutralino sector. These are (after reparametrization of the fields) the higgsino mass parameter μ and the $U(1)$ gaugino mass parameter M_1. The current experimental bounds on the electric dipole moments of electron, neutron and the atoms ^{199}Hg and ^{205}Tl suggest that the phase of μ, ϕ_μ, may be more restricted than the phase of M_1, ϕ_{M_1} (see for instance [8]). These constraints, however, are rather model dependent [9]. Therefore, it is necessary to determine the phases of the complex SUSY parameters by measurements of suitable CP sensitive observables.

We propose T-odd observables in the production process (1) by means of the azimuthal angular distribution of the selectrons. These observables require both electron beams to be transversely polarized. Without transverse beam polarization no T-odd terms involving a triple product correlation appear in the matrix element squared of the production process (1) due to the lack of three linearly independent momentum and/or polarization vectors. This remains true if the subsequent decays of the selectrons are taken into account, because the selectrons are scalar particles. We stress that in the reaction $e^+e^- \to \tilde{e}_i^+ \tilde{e}_j^-$ even for transversely polarized e^+ and e^- beams no useful CP sensitive observable can be found since in this case the CP sensitive terms are proportinal to the tiny left-right selectron mixing.

The complex parameters M_1 and/or μ (with ϕ_{M_1} and/or $\phi_\mu \neq 0, \pi$) give rise to the T-odd observables to be considered. A measurement of these T-odd observables therefore allows us to obtain information on the MSSM parameters, in addition to

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[1]The amplitude squared of selectron pair production $\tilde{e}_L^- \tilde{e}_L^-$, $\tilde{e}_R^- \tilde{e}_R^-$ does not depend on the transverse beam polarizations (see section 2).
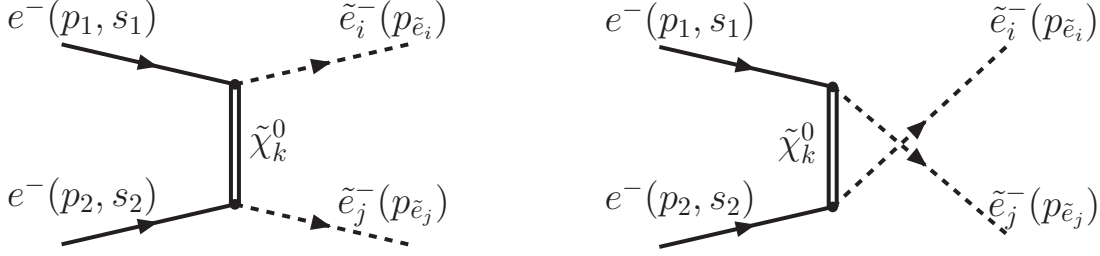
Figure 1: Feyman graphs for selectron production in e^-e^--collisions.

those which can be obtained from a measurement of suitable T-odd observables in the production process $e^+e^- \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$, $i,j = 1,\ldots,4$ [6–8,10–12]. Due to CPT invariance, at tree-level, these T-odd observables are actually CP sensitive observables.

In Section 2 we outline the calculation of the production cross section for $e^-e^- \to \tilde{e}_L^- \tilde{e}_R^-$ with arbitrary beam polarizations. In Section 3 we define our CP sensitive observables. We present numerical results in Section 4, where we also estimate the measurability of the CP sensitive observables. We summarize in Section 5.

2 Cross section

In the following we outline the calculation of the production cross section for $e^-e^- \to \tilde{e}_i^- \tilde{e}_j^-$, $i,j = 1,2$, for arbitrary beam polarizations, neglecting the mass of the electron. $\tilde{e}_i^-$, $i = 1,2$, is the selectron mass eigenstate, $m_{\tilde{e}_1} < m_{\tilde{e}_2}$. The selectron mixing angle is $\theta_{\tilde{e}} = 0$ $(\pi/2)$ for $\tilde{e}_1 = \tilde{e}_L$ $(\tilde{e}_R)$. We first calculate the amplitude squared for process (1). The relevant part of the Lagrangian is given by

$$\mathcal{L}_{\tilde{\chi}\tilde{e}e} = g\,\bar{e}\,(a_{kl}P_R + b_{kl}P_L)\,\tilde{\chi}_k^0\,\tilde{e}_l^- + \text{h.c.}\,, \tag{2}$$

with $P_{L,R} = 1/2(1 \mp \gamma_5)$ and g being the $SU(2)$ weak coupling constant. The couplings a_{kl} and b_{kl} in Eq. (2) contain the neutralino mixing elements N_{kl} and are given in the basis $(\tilde{B}, \tilde{W}^3, \tilde{H}_1^0, \tilde{H}_2^0)$ [13] as

$$a_{k1} = \cos\theta_{\tilde{e}} f_k^L\,, \quad a_{k2} = -\sin\theta_{\tilde{e}} f_k^L\,, \quad b_{k1} = \sin\theta_{\tilde{e}} f_k^R\,, \quad b_{k2} = \cos\theta_{\tilde{e}} f_k^R\,, \tag{3}$$

with

$$f_k^L = \frac{1}{\sqrt{2}}(N_{k2} + \tan\theta_W N_{k1})\,, \qquad f_k^R = \sqrt{2}\tan\theta_W N_{k1}^*\,, \tag{4}$$

where θ_W denotes the weak mixing angle. The amplitudes for $e^-e^- \to \tilde{e}_i^- \tilde{e}_j^-$ are

$$M_{ij} = M_{ij}^t + M_{ij}^u\,, \tag{5}$$

see Fig. 1, where M_{ij}^t is the contribution from neutralino exchange in the $t-$channel,

$$M_{ij}^t = g^2 \sum_{k=1}^4 \Delta_k^t \bar{v}(p_2,s_2)(a_{ki}^* P_L + b_{ki}^* P_R)(\slashed{p}_1 - \slashed{p}_{\tilde{e}_j} + m_k)(a_{kj}^* P_L + b_{kj}^* P_R)u(p_1,s_1)\,, \tag{6}$$

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and M_{ij}^u is the $u-$channel neutralino exchange contribution,

$$M_{ij}^u = g^2 \sum_{k=1}^{4} \Delta_k^u \bar{v}(p_2, s_2)(a_{kj}^* P_L + b_{kj}^* P_R)(\not{p}_1 - \not{p}_{\tilde{e}_i} + m_k)(a_{ki}^* P_L + b_{ki}^* P_R)u(p_1, s_1) , \quad (7)$$

where $\Delta_k^t = i/((p_1 - p_{\tilde{e}_j})^2 - m_k^2)$, $\Delta_k^u = i/((p_1 - p_{\tilde{e}_i})^2 - m_k^2)$, m_k denotes the neutralino masses, p_1 and p_2 are the 4-momenta of the incoming electrons and $p_{\tilde{e}_i}$ is the 4-momentum of the corresponding selectron.

In the treatment of beam polarizations we use the covariant projection operators [5,14] (for a different treatment see for instance [6]). In the limit of vanishing electron masses they read

$$\sum_{s_1} \bar{u}(p_1, s_1)u(p_1, s_1) = \frac{1}{2}(\mathbf{1} + P_L^1 \gamma_5 + \gamma_5 P_T^1 \not{t}_1) \not{p}_1 \quad (8)$$

and

$$\sum_{s_2} \bar{v}(p_2, s_2)v(p_2, s_2) = \frac{1}{2}(\mathbf{1} - P_L^2 \gamma_5 + \gamma_5 P_T^2 \not{t}_2) \not{p}_2 , \quad (9)$$

where $t_{1,2}$ are the transverse beam polarization 4-vectors of the e^- beams. In Eqs. (8) and (9) $P_L^{1,2}$ $[-1 \le P_L^{1,2} \le 1]$ denote the degree of the longitudinal polarizations of the e^- beams and $P_T^{1,2}$ $[0 \le P_T^{1,2} \le 1]$ denote the degree of transverse polarizations, statisfying $(P_L^{1,2})^2 + (P_T^{1,2})^2 \le 1$.

The amplitude squared for the production process $e^- e^- \to \tilde{e}_i^- \tilde{e}_j^-$ can be written as

$$|M_{ij}|^2 = |M_{ij}^t|^2 + |M_{ij}^u|^2 + 2\Re e\{M_{ij}^t M_{ij}^{u\dagger}\} , \quad (10)$$

where in the following we only give the result for the production of different mass eigenstates [2] , i.e. $i \ne j$, because otherwise only the absolute values of the couplings enter and no CP-odd term appears in the amplitude squared. As was shown in [16] the cross section for $\tilde{e}_L^- \tilde{e}_L^-$ production, although it is a CP-even observable, is quite sensitive to CP violation in the neutralino sector, representing a complementary observable.

We introduce a coordinate system by choosing the z-axis along the $\vec{p}_1$ direction in the c.m. system, and x and y corresponding to a right-handed coordinate system. In this coordinate system the transverse beam polarization 4-vectors in Eqs. (8) and (9) are

$$t^{1,2} = (0, \cos \phi_{1,2}, \sin \phi_{1,2}, 0) . \quad (11)$$

We first consider the case $e^- e^- \to \tilde{e}_1^- \tilde{e}_2^-$, where $\tilde{e}_1 = \tilde{e}_R$ and $\tilde{e}_2 = \tilde{e}_L$. We obtain

$$|M_{12}^t|^2 = \frac{g^4}{4} s q^2 \sin^2 \theta c_{+-} \sum_{k,l=1}^{4} f_k^{L*} f_l^L f_k^{R*} f_l^R \Delta_k^t \Delta_l^{t*} , \quad (12)$$

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[2] When neglecting selectron mixing, the result for $i = j$ is the same as given in [15] for longitudinal beam polarizations, i.e. the amplitude squared does not depend on the transverse beam polarizations in this case.

$$|M_{12}^u|^2 = \frac{g^4}{4} \, s \, q^2 \, \sin^2\theta \, c_{-+} \sum_{k,l=1}^{4} f_k^{L*} f_l^L f_k^{R*} f_l^R \Delta_k^u \Delta_l^{u*} \, , \qquad (13)$$

$$2\Re e\{M_{12}^t M_{12}^{u\dagger}\} = \frac{g^4}{2} \, P_T^1 \, P_T^2 \, s \, q^2 \sin^2\theta \sum_{k,l=1}^{4} \Delta_k^t \Delta_l^{u*}$$
$$\times \Re e\{f_k^{L*} f_l^L f_k^{R*} f_l^R [\cos(\eta - 2\phi) - i\sin(\eta - 2\phi)]\} \, , \quad (14)$$

where $c_{\pm\mp} = (1 \pm P_L^1)(1 \mp P_L^2)$, $q = \lambda^{1/2}(s, m_{\tilde{e}_1}^2, m_{\tilde{e}_2}^2)/(2\sqrt{s})$, $E_{\tilde{e}_{1,2}} = (s + m_{\tilde{e}_{1,2}}^2 - m_{\tilde{e}_{2,1}}^2)/(2\sqrt{s})$, $m_{\tilde{e}_i}$ are the selectron masses, $\eta = \phi_1 + \phi_2$, θ and ϕ being the polar angle and azimuthal angle of $\tilde{e}_2^-$. For the case $\tilde{e}_1 = \tilde{e}_L$, $\tilde{e}_2 = \tilde{e}_R$, the amplitude squared is obtained by the replacements $c_{+-} \to c_{-+}$ in Eq. (12) and $c_{-+} \to c_{+-}$ in Eq. (13), and by changing the overall sign in Eq. (14).

The differential cross section for $e^- e^- \to \tilde{e}_1^- \tilde{e}_2^-$ is given by

$$\frac{d\sigma}{d\Omega} = \frac{1}{8(2\pi)^2} \frac{q}{s^{3/2}} |M_{12}|^2 \, , \qquad (15)$$

with $d\Omega = \sin\theta d\theta d\phi$ and $|M_{12}|^2$ as given in Eq. (10). Note that the production cross section σ is independent of the transverse beam polarizations, because the appropriate contributions in the amplitude squared depend on $\cos(\eta - 2\phi)$ or on $\sin(\eta - 2\phi)$, see Eq. (14), and vanish if integrated over the whole range of the azimuthal angle ϕ.

3   CP sensitive observables

In this section we define our CP sensitive observables for the production process $e^- e^- \to \tilde{e}_L^- \tilde{e}_R^-$ with transverse e^- beam polarizations. By inspecting Eq. (14), we observe that the CP sensitive term which involves the imginary part of the couplings $f_k^{L*} f_l^L f_k^{R*} f_l^R$ is proportional to

$$\sum_{k<l}^{4} (\Delta_k^t \Delta_l^u - \Delta_l^t \Delta_k^u) \, \Im m\{f_k^{L*} f_l^L f_k^{R*} f_l^R\} \, , \qquad (16)$$

and would be zero when integrated over the whole range of the polar angle θ because of the symmetry of the propagator term. We therefore have to divide the integration over θ into two regions [6]. This amounts to a sign change of $\cos\theta$, which we can take into account by multiplying (16) by a weight function

$$\mathcal{H}_1 = \text{sign}[\sin(\eta - 2\phi)\cos\theta] \, . \qquad (17)$$

An other choice of the weight function can be given by matching the angular dependence of the term of interest in the amplitude squared [7,17]. This can be achieved with the weight function

$$\mathcal{H}_2 = \sin(\eta - 2\phi)\cos\theta\sin^2\theta . \tag{18}$$

As our CP sensitive observables, we define the expectation values of $\mathcal{H}_i, i = 1, 2$, given as

$$\langle\mathcal{H}_i\rangle = \frac{1}{\sigma}\int d\Omega\frac{d\sigma}{d\Omega}\,\mathcal{H}_i . \tag{19}$$

Due to the requirement that the statistical error of the observable should not exceed its size, we have

$$\frac{|\langle\mathcal{H}_i\rangle|}{\Delta\langle\mathcal{H}_i\rangle} > 1 , \tag{20}$$

where $\Delta\langle\mathcal{H}_i\rangle = \mathcal{N}_\sigma/\sqrt{N}\sqrt{\langle\mathcal{H}_i^2\rangle - \langle\mathcal{H}_i\rangle^2} \simeq \mathcal{N}_\sigma/\sqrt{N}\sqrt{\langle\mathcal{H}_i^2\rangle}$, with $\mathcal{N}_\sigma$ being the number of standard deviations and $N = \sigma\mathcal{L}$ the number of events, where $\mathcal{L}$ denotes the integrated luminosity. Using Eq. (20), we define an effective CP observable given as

$$\hat{O}[\mathcal{H}_i] = \sqrt{\sigma}\,\frac{\langle\mathcal{H}_i\rangle}{\sqrt{\langle\mathcal{H}_i^2\rangle}} . \tag{21}$$

$|\hat{O}[\mathcal{H}_i]| \cdot \sqrt{\mathcal{L}}$ is then the number of standard deviations to which the corresponding observable, Eq. (19), can be determined to be non-zero.

Note that a measurement of the CP sensitive observables disussed above requires the reconstruction of the production plane. If all masses involved are known, this can be accomplished either in a unique way or with a two-fold ambiguity, depending on the decay pattern of the produced selectrons [6,7].

4   Numerical results

Now we analyze numerically the effective CP observables defined in Eq. (21) for the reaction $e^- e^- \to \tilde{e}_L^- \tilde{e}_R^-$ at a linear collider with $\sqrt{s} = 500$ GeV and transverse beam polarizations. We assume that a degree of transverse polarization of 90% is feasible for each of the two electron beams. Furthermore, in order to estimate the significance of a measurement of the CP sensitive observables we assume that one third of the integrated luminosity $\mathcal{L}$ of the $e^+ e^-$ mode can be achieved [2]. For our numerical analysis we choose three scenarios, A, B and C, defined in Table 1. In Table 2 we give the masses and the compositions of the neutralinos $\tilde{\chi}_i^0$ in these scenarios.
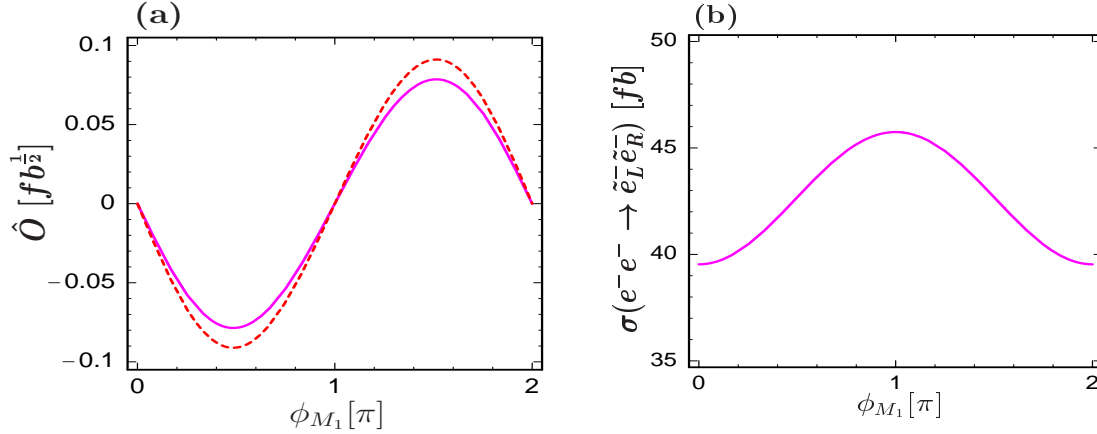
Figure 2: (a) Effective CP observable $\hat{O}[\mathcal{H}_i]$, Eq. (21), as a function of ϕ_{M_1} for scenario A of Table 1, with $\mathcal{H}_1 = \text{sign}[\cos\theta\sin(\eta - 2\phi)]$ (solid line) and $\mathcal{H}_2 = \sin^2\theta\cos\theta \ \sin(\eta - 2\phi)$ (dashed line) and (b) the corresponding cross section $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$.

Scenario	$|M_1|$	ϕ_{M_1}	M_2	$|\mu|$	ϕ_μ	$\tan\beta$	$m_{\tilde{e}_L}$	$m_{\tilde{e}_R}$
A	250.5	0.5π	500	115	0	5	200	170
B	430	0.5π	400	120	0	3	160	130
C	300	0.5π	200	160	0	3	170	120

Table 1: Input parameters $|M_1|$, ϕ_{M_1}, M_2, $|\mu|$, ϕ_μ, $m_{\tilde{e}_L}$ and $m_{\tilde{e}_R}$. All mass parameters are given in GeV.

a) Case with M_1/M_2 GUT-relation

In Fig. 2 we show the effective CP observables, Eq. (21), that are based on the weight functions $\mathcal{H}_1$ and $\mathcal{H}_2$ and the associated production cross section as a function of ϕ_{M_1} for scenario A, given in Table 1. In this scenario we assume the GUT-inspired relation $|M_1| = 5/3\tan^2\Theta_W M_2$. Table 2 shows that in scenario A, $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ are mainly higgsinos, $\tilde{\chi}_3^0$ is mainly a bino and $\tilde{\chi}_4^0$ is mainly a wino. For the parameters chosen, the leading contribution to $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$ stems from $\tilde{\chi}_3^0$ exchange in the t−channel and u−channel, since $\tilde{e}_L$ couples to the bino and wino components of the neutralinos and $\tilde{e}_R$ to their bino component, see Eq. (4). On the other hand, because $\tilde{\chi}_1^0$ has an appreciable bino component (3.6%), the leading CP violating contribution to the CP sensitive observables, Eq. (19), is due to the interference of the $\tilde{\chi}_1^0$ and $\tilde{\chi}_3^0$ exchange amplitudes, Eq. (14). In Figs. 2a and b we can clearly see the antisymmetric dependence of the CP sensitive observables on the phase ϕ_{M_1} while the production cross section is symmetric in ϕ_{M_1}. Thus, both kinds of observables are needed for an unambiguous determination of ϕ_{M_1}. However, in order to probe the CP sensitive observables $\langle\mathcal{H}_1\rangle$ and $\langle\mathcal{H}_2\rangle$ at 3σ, integrated luminosities of $\mathcal{L} = 1500$ fb^{-1} and $\mathcal{L} = 1042$ fb^{-1} would be required for scenario A.
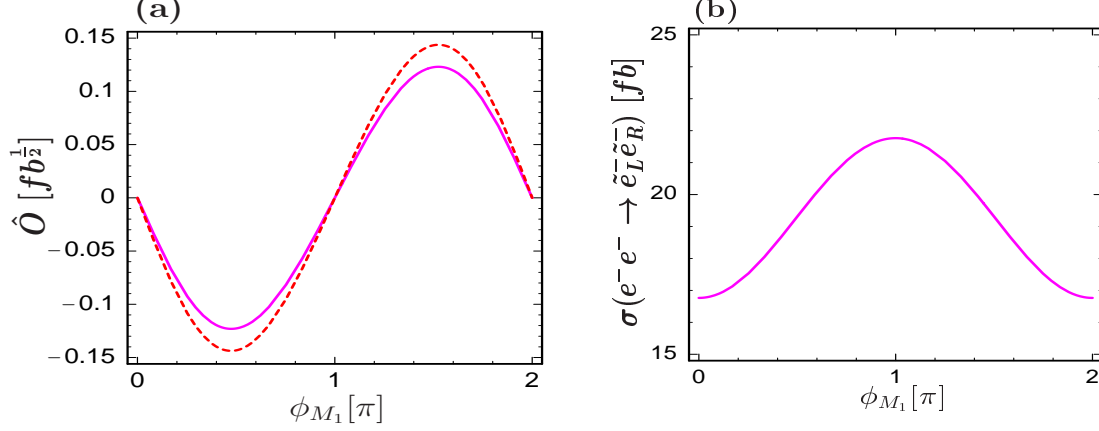
Figure 3: (a) Effective CP observable $\hat{O}[\mathcal{H}_i]$, Eq. (21), as a function of ϕ_{M_1} for scenario B of Table 1, with $\mathcal{H}_1 = \mathrm{sign}[\cos\theta\sin(\eta - 2\phi)]$ (solid line) and $\mathcal{H}_2 = \sin^2\theta\cos\theta\ \sin(\eta - 2\phi)$ (dashed line) and (b) the corresponding cross section $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$.

A	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$	B	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$
$\tilde{B}$	0.036	0.010	0.953	0.000	$\tilde{B}$	0.009	0.002	0.140	0.849
$\tilde{W}^3$	0.023	0.007	0.003	0.967	$\tilde{W}^3$	0.052	0.006	0.823	0.120
$\tilde{H}_1$	0.515	0.473	0.007	0.005	$\tilde{H}_1$	0.508	0.476	0.009	0.006
$\tilde{H}_2$	0.426	0.510	0.036	0.028	$\tilde{H}_2$	0.431	0.516	0.028	0.025
Mass	100.8	118.0	259.2	514.4	Mass	102.2	122.4	416.6	437.5

C	$\tilde{\chi}_1^0$	$\tilde{\chi}_2^0$	$\tilde{\chi}_3^0$	$\tilde{\chi}_4^0$
$\tilde{B}$	0.015	0.011	0.088	0.887
$\tilde{W}^3$	0.336	0.013	0.619	0.033
$\tilde{H}_1$	0.390	0.472	0.114	0.024
$\tilde{H}_2$	0.259	0.505	0.180	0.056
Mass	106.2	162.7	251.4	311.2

Table 2: Neutralino compositions and mass spectra [GeV] for the scenarios A, B and C.

(a) $\hat{O}[\mathcal{H}_2]$ $[\text{fb}^{\frac{1}{2}}]$

(b) $\sigma(e^-e^- \to \tilde{e}_L^- \tilde{e}_R^-)$ $[\text{fb}]$
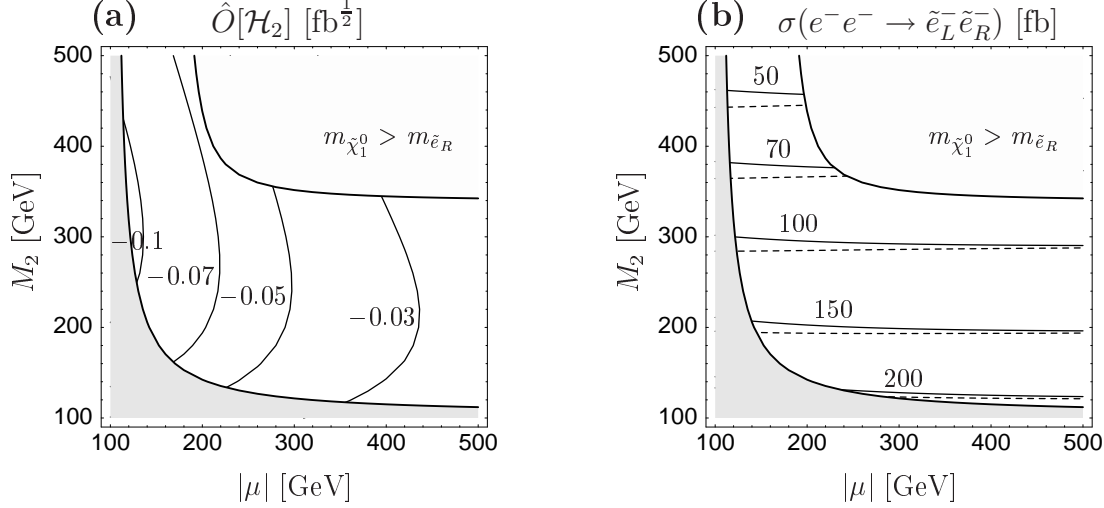
Figure 4: Contour lines (a) of the effective CP observable $\hat{O}[\mathcal{H}_2]$ for $\phi_{M_1} = 0.5\pi$ and (b) of the corresponding cross section $\sigma(e^-e^- \to \tilde{e}_L^- \tilde{e}_R^-)$ for $\phi_{M_1} = 0$ (dashed line) and $\phi_{M_1} = 0.5\pi$ (solid line), in the $|\mu|$–M_2 plane . The parameters which are not varied are as given in scenario A of Table 1. In the light-gray region $m_{\tilde{\chi}_1^\pm} < 104$ GeV and the region in the top right corner is excluded because there $m_{\tilde{\chi}_1^0} > m_{\tilde{e}_R}$.

b) Case without M_1/M_2 GUT-relation

In Fig. 3 we plot the effective CP observables, Eq. (21), that are based on the weight functions $\mathcal{H}_1$ and $\mathcal{H}_2$ and the associated production cross section as a function of ϕ_{M_1} for scenario B, given in Table 1. In scenario B, $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ are again mainly higgsinos (see Table 2), however, we do not assume the GUT-relation between the gaugino mass parameters $|M_1|$ and M_2. For scenario B the maximum (minimum) values of the effective CP observables are reached at $\phi_{M_1} \approx 0.5\pi$ (1.5π). The integrated luminosity required for a measurement of the associated CP sensitive observables $\langle \mathcal{H}_1 \rangle$ and $\langle \mathcal{H}_2 \rangle$ at 3σ is $\mathcal{L} = 667$ fb^{-1} and $\mathcal{L} = 416$ fb^{-1}, respectively.

For scenario B we now compare our results for $e^-e^- \to \tilde{e}_L^- \tilde{e}_R^-$ with the T-odd asymmetry A_T studied in [10] for neutralino production $e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$ followed by the three-body decay $\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+\ell^-$, $\ell = e, \mu$, at the ILC operating at $\sqrt{s} = 500$ GeV. For the optimal choice of $(P_{e^-}, P_{e^+}) = (-0.9, +0.6)$ for the longitudinal beam polarizations we obtain $A_T = 0.0013$ and for the cross section of the combined process $\sigma(e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0) \cdot \sum_\ell B(\tilde{\chi}_2^0 \to \tilde{\chi}_1^0 \ell^+\ell^-) = 11.9$ fb. The integrated luminosity necessary to measure this asymmetry in $e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$ at 3σ would be $\mathcal{L} = 4.5 \times 10^5$ fb^{-1}. This example illustrates the potential of the e^-e^- mode for an identification of CP violation in the neutralino sector. In this context we remark that it may be the case that the reaction $e^+e^- \to \tilde{\chi}_1^0 \tilde{\chi}_2^0$ is kinematically not allowed because the threshold is too high, however, $e^-e^- \to \tilde{e}_L^- \tilde{e}_R^-$ is accessible. In such a case the reaction $e^-e^- \to \tilde{e}_L^- \tilde{e}_R^-$ and the CP sensitive observables defined in Eq. (19) may be a suitable way to determine the phases ϕ_{M_1}.

(a) $\hat{O}[\mathcal{H}_2]$ $[\mathrm{fb}^{\frac{1}{2}}]$ **(b)** $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$ $[\mathrm{fb}]$
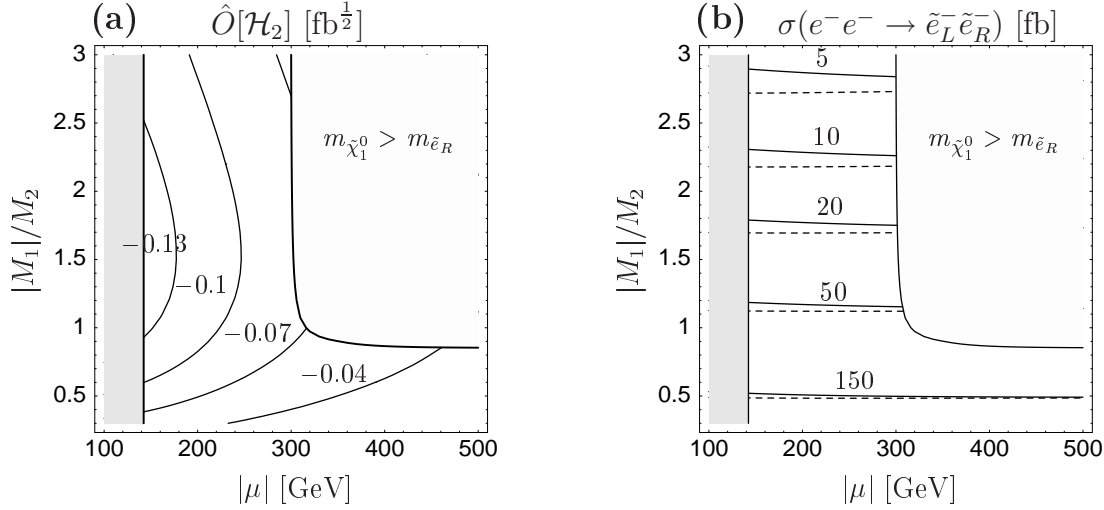
Figure 5: Contour lines (a) of the effective CP observable $\hat{O}[\mathcal{H}_2]$ for $\phi_{M_1} = 0.5\pi$ and (b) of the corresponding cross section $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$ for $\phi_{M_1} = 0$ (dashed line) and $\phi_{M_1} = 0.5\pi$ (solid line) in the $|\mu|$–$|M_1|/M_2$ plane with $M_2 = 200$ GeV. The parameters which are not varied are as given in scenario A of Table 1. In the light-gray region $m_{\tilde{\chi}_1^\pm} < 104$ GeV and the region in the top right corner is excluded because there $m_{\tilde{\chi}_1^0} > m_{\tilde{e}_R}$.

c) Dependence on the gaugino/higgsino mass parameters

In Fig. 4a we show contour lines of the effective CP observable, Eq. (21), for the weight function $\mathcal{H}_2$ in the $|\mu| - M_2$ plane, where the other parameters are as in scenario A with $|M_1| = 5/3 \tan^2 \Theta_W M_2$. As one can see, the effective CP observable is larger for $|\mu| \lesssim M_2$, because the terms in the amplitude squared which are not sensitive to CP violation are smaller than the CP sensitive terms. For the largest absolute value of the effective CP observable ($|\hat{O}[\mathcal{H}_2]| = 0.1$ $\mathrm{fb}^{1/2}$) the integrated luminosity necessary to measure the corresponding CP sensitive observable $\langle\mathcal{H}_2\rangle$ at 3σ is $\mathcal{L} = 888$ fb^{-1}. Fig. 4b shows the associated production cross section in the $|\mu|$–M_2 plane for scenario A and for comparison also for the CP conserving case $\phi_{M_1} = 0$. As can be seen, the production cross section is almost independent of $|\mu|$, because the leading contributions are due to the exchange of neutralinos with dominant bino and wino components. The production cross section, however, sensitively depends on $|M_1|$, and decreases for increasing $|M_1|$, because then the heavier neutralino states are dominantly binos and winos.

In Fig. 5a we show the contour lines of the effective CP observable $\hat{O}[\mathcal{H}_2]$ in the $|\mu|$–$|M_1|/M_2$ plane, fixing $M_2 = 200$ GeV. The remaining parameters are as in scenario A, see Table 1. The absolute value of the effective CP observable is increased if the ratio $|M_1|/M_2$ is increased from a value of 0.5 $\mathrm{fb}^{1/2}$ to 1.5 $\mathrm{fb}^{1/2}$. In order to probe $\langle\mathcal{H}_2\rangle$ at 3σ, an integrated luminosity of at least $\mathcal{L} = 519$ fb^{-1} ($|\hat{O}[\mathcal{H}_2]| = 0.13$ $\mathrm{fb}^{1/2}$) is required in this case. In Fig. 5b the production cross
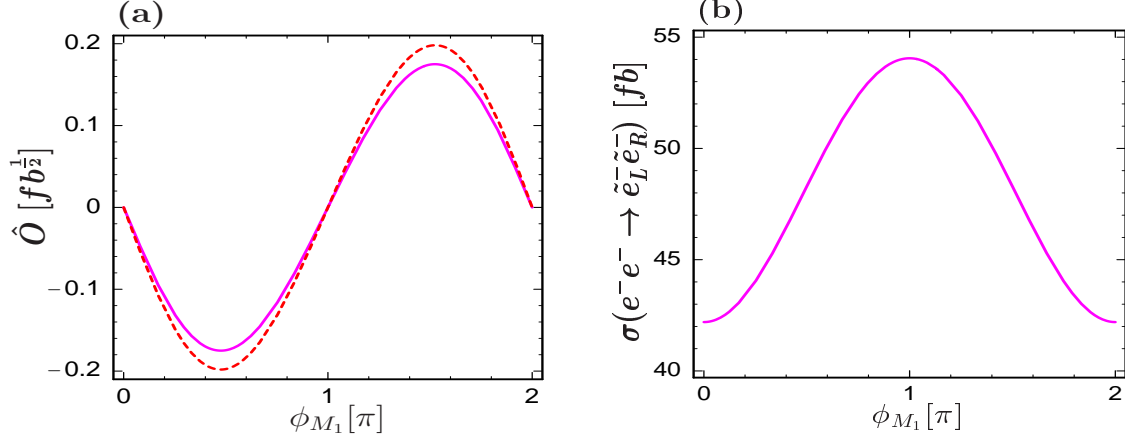
Figure 6: (a) Effective CP observable $\hat{O}[\mathcal{H}_i]$, Eq. (21), as a function of ϕ_{M_1} for scenario C of Table 1, with $\mathcal{H}_1 = \text{sign}[\cos\theta\sin(\eta - 2\phi)]$ (solid line) and $\mathcal{H}_2 = \sin^2\theta\cos\theta\,\sin(\eta - 2\phi)$ (dashed line) and (b) the corresponding cross section $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$.

section for the reaction $e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-$ in the $|\mu|$–$|M_1|/M_2$ plane is displayed for $\phi_{M_1} = \frac{\pi}{2}$ and $\phi_{M_1} = 0$. Again the cross section is almost independent of the value of $|\mu|$ and decreases when $|M_1|/M_2$ is increased, since the leading contribution is due to the exchange of $\tilde{\chi}_4^0$ which is mainly a bino.

d) Dependence on $\tan\beta$ and ϕ_μ

We have also studied the $\tan\beta$ and ϕ_μ dependences of the CP sensitive observables. For larger values of $\tan\beta$ for the scenarios A and B the effective CP observable is somewhat reduced, because in this case the degree of the higgsino admixture to $\tilde{\chi}_1^0$ is decreased. The influence of ϕ_μ on the CP sensitive observables is less strong, especially in scenario A. In order to understand this point qualitatively, one can use approximative fromulae of the neutralino mixing matrix elements (see e.g. [12]) that enter the relevant coupling $\Im m\{f_1^{L*}f_3^L f_1^{R*}f_3^R\}$, showing that the leading term is proportional to $\sin\phi_{M_1}$ and the $\sin\phi_\mu$ dependence is less pronounced. Furthermore, the measurabilities of $\langle\mathcal{H}_1\rangle$ and $\langle\mathcal{H}_2\rangle$, Eq. (19), increase for smaller selectron masses in which case the partial cancellation of t−channel and u−channel contributions is smaller, see Eq. (16).

e) Scenario with light neutralinos $\tilde{\chi}_i^0$

In Fig. 6 we show the effective CP observables, Eq. (21), that are based on the weight functions $\mathcal{H}_1$ and $\mathcal{H}_2$ and the associated production cross section as a function of ϕ_{M_1} for scenario C, given in Table 1. In this scenario $\tilde{\chi}_1^0$ and $\tilde{\chi}_2^0$ are mainly higgsinos, $\tilde{\chi}_4^0$ is mainly a bino and $\tilde{\chi}_3^0$ is mainly a wino with a pronounced bino admixture (see Table 2). Due to the moderate values of the heavier neutralino masses the $\tilde{\chi}_3^0$–$\tilde{\chi}_4^0$ interference term in Eq. (14) gives the leading contribution to the CP sensitive observables $\langle\mathcal{H}_1\rangle$ and $\langle\mathcal{H}_2\rangle$. As can be seen in Fig. 6a, the effective CP observables reach their minimum (maximum) value at $\phi_{M_1} \approx 0.5\pi$ (1.5π). For these values the integrated luminosities necessary to probe the corresponding CP sensitive observables $\langle\mathcal{H}_1\rangle$ and $\langle\mathcal{H}_2\rangle$ at 3σ are $\mathcal{L} = 284$ fb^{-1} and $\mathcal{L} = 240$ fb^{-1}.
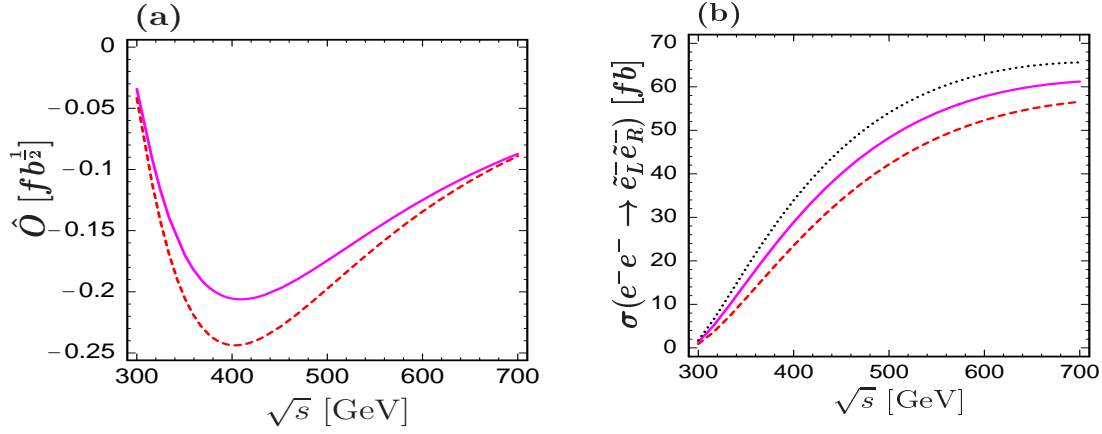
Figure 7: (a) Effective CP observable $\hat{O}[\mathcal{H}_i]$, Eq. (21), as a function of $\sqrt{s}$ for scenario C of Table 1, with $\mathcal{H}_1 = \text{sign}[\cos\theta\sin(\eta - 2\phi)]$ (solid line) and $\mathcal{H}_2 = \sin^2\theta\cos\theta \; \sin(\eta - 2\phi)$ (dashed line) and (b) the corresponding cross section $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$ for $\phi_{M_1} = 0$ (dashed line), $\phi_{M_1} = 0.5\pi$ (solid line) and $\phi_{M_1} = \pi$ (dotted line).

In Fig. 7a the effective CP observables, Eq. (21), are plotted as a function of the center of mass energy $\sqrt{s}$ for scenario C. As can be seen in Fig. 7a the minimum values of the effective CP observables are reached at $\sqrt{s} \approx 400$ GeV. At this point the integrated luminosities necessary to probe $\langle\mathcal{H}_1\rangle$ and $\langle\mathcal{H}_2\rangle$ at 3σ decrease to $\mathcal{L} = 206$ fb^{-1} and $\mathcal{L} = 156$ fb^{-1} compared to the case $\sqrt{s} = 500$ GeV. In Fig. 7b we show the $\sqrt{s}$ behavior of the production cross section $\sigma(e^-e^- \to \tilde{e}_L^-\tilde{e}_R^-)$ for scenario C including the CP conserving cases $\phi_{M_1} = 0, \pi$ for comparison.

5   Summary

We have proposed and analyzed CP sensitive observables by means of the azimuthal angular distribution of the produced selectrons at an e^-e^- linear collider with transverse beam polarizations. These observables are non-vanishing due to the CP violating phases ϕ_{M_1} and ϕ_μ in the neutralino sector. We have numerically studied the MSSM parameter dependence of these observables and of the production cross section $\sigma(e^-e^- \to \tilde{e}_L\tilde{e}_R)$. Moreover, we have also estimated the measurability of the proposed CP sensitive observables. The best significances (at the 3σ level) for their measurement are obtained in scenarios where the GUT-inspired relation $|M_1| = 5/3\tan^2\theta_W M_2$ does not hold. In such a case two exchanged neutralinos can have a significant bino component, where the interference term of the corresponding amplitudes gives the dominant contribution to the CP sensitive observables.

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