Polarimetry at the ILC

Robert Karl [1,2] Jenny List [2]

[1] Deutsches Elektronen-Synchrotron (DESY)

[2] University of Hamburg

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Outline

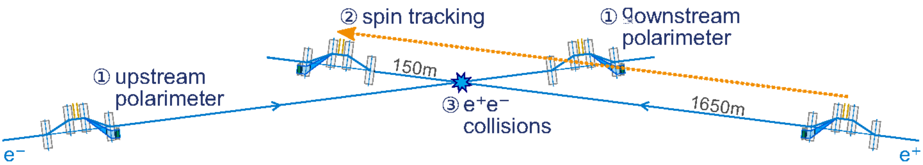
Advantages of Polarized Beams

- International Linear Collider (ILC)
  - $e^- e^+$ collider with polarized beams of $|80\%|$ and $|30\%$ - $60\%|$, respectively
  - Selectable polarization sign $\longrightarrow$ choice of spin configuration

- Advantages:
  - Sensitive to new observables (e.g. left-right-asymmetry)
  - Reduction of background processes and simultaneously increase of signal processes
  - Deep insights into the chiral structure of the weak-interaction for known and unknown particle

All event rates depend linearly on the polarization!
$\Rightarrow$ **Has to be known as precisely as the luminosity!**
$\Rightarrow$ **Requirement for permille-level precision**

ILC Polarimetry Concept



1. Measurement of the time-resolved beam polarization before and after the $e^- e^+$ IP
   - Via laser-Compton polarimeter

Ref.: Jenny List, Annika Vauth, and Benedikt Vormwald:
A Quartz Cherenkov Detector for Compton-Polarimetry at Future $e^+ e^-$ Colliders (https://bib-pubdb1.desy.de/record/221054)
A Calibration System for Compton Polarimetry at $e^+ e^-$ Colliders(https://bib-pubdb1.desy.de/record/289025)

2. Extrapolating the beam polarization to the $e^- e^+$ IP
   - Via Spin Tracking

Ref.: Moritz Beckmann, Jenny List, Annika Vauth, and Benedikt Vormwald:
Spin transport and polarimetry in the beam delivery system of the international linear collider
(http://iopscience.iop.org/article/10.1088/1748-0221/9/07/P07003/pdf)

3. Determination of the luminosity-weighed averaged polarization from collision data
   - Calculating the polarization from known standard model processes

⇒ Discussed in the following

Determination of the Polarization from Collision Data

Goal:

General strategy for the polarization determination which yields the best precision per measurement time

- Previous Work:
  - Using the information from W-pair production
    Ref.: Theses Ivan Marchesini
    (`http://pubdb.xfel.eu/record/94888`)
  - Using the information from single W, γ, Z events
    Ref.: Talk Graham W. Wilson
    (`https://agenda.linearcollider.org/event/5468/contributions/24027/`)

- Current Work:
  - Combining all relevant processes, including all uncertainties and their correlations
  - Compensating for a non-perfect helicity reversal
  - Including constraints from the polarimeter measurement

The New Unified Approach

χ^2-Method:

- Combining all suitable processes
- Considering all statistical and systematical uncertainties
- Using a full covariance Matrix (Ξ)

$$\chi^2 = \sum_{\text{process}} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)^T \Xi^{-1} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)$$

$$\vec{\sigma} := \begin{pmatrix} \sigma_{-+} & \sigma_{+-} & \sigma_{--} & \sigma_{++} \end{pmatrix}^T$$

$$\Xi := \Xi_N + \Xi_B + \Xi_\varepsilon + \Xi_\mathcal{L};$$

$$\left(\Xi_\varepsilon\right)_{ij} = \text{corr}\left(\vec{\sigma}_i^\varepsilon,\ \vec{\sigma}_j^\varepsilon\right) \frac{\partial \vec{\sigma}_i}{\partial \varepsilon_i} \frac{\partial \vec{\sigma}_j}{\partial \varepsilon_j} \Delta\varepsilon_i \Delta\varepsilon_j$$

Compensation non-perfect helicity reversal: Using 4 independent parameters

$$\underbrace{P_{e^-}^- = -80\%,}_{\text{"left"-handed } e^-\text{-beam}} \quad \underbrace{P_{e^-}^+ = 80\%,}_{\text{"right"-handed } e^-\text{-beam}} \quad \underbrace{P_{e^+}^- = -30\%,}_{\text{"left"-handed } e^+\text{-beam}} \quad \underbrace{P_{e^+}^+ = 30\%,}_{\text{"right"-handed } e^+\text{-beam}}$$

Alternative parametrization: Average Polarization and deviation

$$P_{e^\pm}^- = -\left|P_{e^\pm}\right| + \tfrac{1}{2}\delta_{e^\pm}$$
$$P_{e^\pm}^+ = \quad \left|P_{e^\pm}\right| + \tfrac{1}{2}\delta_{e^\pm}$$

Theoretical Limit of the Statistical Precision

Currently implemented processes:

Process	Channel
single $W^{\pm}$	$e\nu l\nu,\ e\nu q\bar{q}$
WW	$q\bar{q}q\bar{q},\ q\bar{q}l\nu,\ l\nu l\nu$
ZZ	$q\bar{q}q\bar{q},\ q\bar{q}ll,\ llll$
$ZZWW$Mix	$q\bar{q}q\bar{q},\ l\nu l\nu$
Z	$q\bar{q},\ ll$

- Same processes as for physics analysis (DBD)
  - → Classification
  - → Cross section
- Any combination of processes can be used
- Further Process can easily added

Consider best case scenario using σ_{tot}:

- Assumption of a perfect 4π detector
- No background
- No systematic uncertainties
- Using all implemented processes

Statistical precision H-20: $\Delta P/P\,[\%]$

E	500	350	250	500	250
$\mathcal{L}$	500	200	500	3500	1500
$P_{e^-}^-$	0.2	0.3	0.1	0.08	0.09
$P_{e^-}^+$	0.05	0.06	0.03	0.02	0.02
$P_{e^+}^-$	0.1	0.1	0.06	0.04	0.04
$P_{e^+}^+$	0.2	0.3	0.1	0.08	0.08

Comparison to Previous Analyses (Statistical Uncertainty Only)

E	**500**	**250**
$\mathcal{L}$	**3500**	**1500**
$P^-_{e^-}$	0.08	0.09
$P^+_{e^-}$	0.02	0.02
$P^-_{e^+}$	0.04	0.04
$P^+_{e^+}$	0.08	0.08

E	**500**	**350**	**250**
$\mathcal{L}$	**500**	**200**	**500**
$P^-_{e^-}$	0.2	0.3	0.1
$P^+_{e^-}$	0.05	0.06	0.03
$P^-_{e^+}$	0.1	0.1	0.06
$P^+_{e^+}$	0.2	0.3	0.1

Single boson:

- $\mathcal{L} = 2000\,\text{fb}^{-1},\ E = 500\ GeV$
- No background estimation
- Fiducial cross section cuts
- Limitation on δ: $\Delta\delta < 10^{-3}$

$$P_{e^-}: 0.085\% \qquad \delta_{e^-}: 0.12\%$$
$$P_{e^+}: 0.22\% \qquad \delta_{e^+}: 0.32\%$$

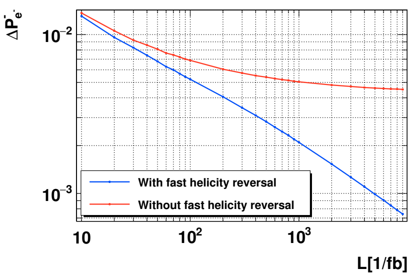
W-pairs:

- $\mathcal{L} = 500\,\text{fb}^{-1},\ E = 500\ GeV$
- Full background estimation
- Differential cross section (angular fit)
- Only using 2 free parameters

$$P_{e^-}: 0.08\% \qquad P_{e^+}: 0.34\%$$

Systematic Uncertainties and their Correlations

Systematic quantity		related to:
Integrated luminosity	$\mathcal{L}$	accelerator
Selection efficiency	ε	detector
Background estimate	B	theory



Remark:
A non-perfect helicity reversal has close to no influence on the precision due to compensation of the unified approach

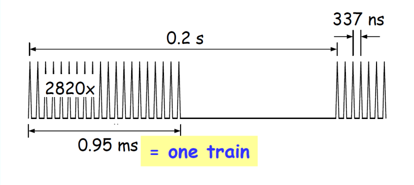
- **Uncertainties influenced by**
  - Detector calibration and alignment
  - Machine performance
  - B assumed constant and small
  - $\Rightarrow$ $\Delta\mathcal{L}$, $\Delta\varepsilon$ are time dependent
- **Correlations:**
  - Data sets taken concurrently
  - Generate correlations
  - $\Rightarrow$ Lead to cancellation of systematic uncertainties

$\Rightarrow$ **Fast helicity reversal**

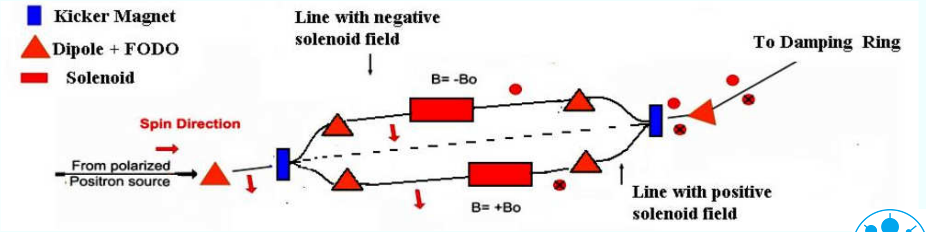
- Fast switch between $\sigma_{\pm\pm}$ measurements e.g. train-by-train
- $\Rightarrow$ Faster than changes in calibrations, alignments, etc.

Fast Helicity Reversal

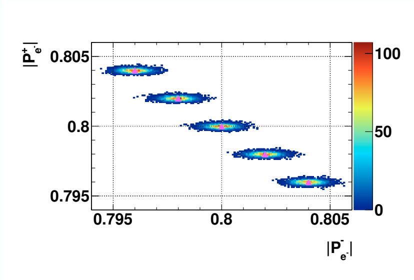
ILC Bunch Structure

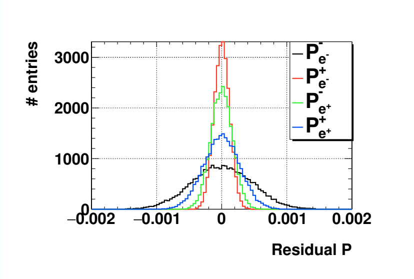


- Frequency:
  - bunch-by-bunch: switch every few bunches
  - train-by-train: switch between two trains
- Accuracy:
  - Non-perfect is realistic
  - Particularly for positron beam $\rightarrow$ two beam lines



Testing for a Non-Perfect Helicity Reversal





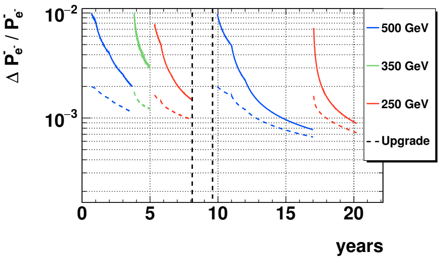
- **Variation in the absolute polarization**
  - Toy Measurement for 5 different polarization discrepancies for both beams
  - Nominal initial polarizations: $|P_{e-}| = 80\%$, $|P_{e+}| = 30\%$
  - Statistical uncertainties only
- **χ^2-Fit:**
  - Correct determination of the 4 polarization values
  - No noticeable changes in the uncertainties

$\Rightarrow$ **Non-Perfect Helicity Reversal:**

$\Rightarrow$ No noticeable impact on polarization precision using total cross sections

$\Rightarrow$ In addition:
consider polarimeter information

Consider Constraints from the Polarimeter Measurement



E	500	350	250	500	250
$\mathcal{L}$	500	200	500	3500	1500
Without Constraint					
$P_{e^-}^-$	0.2	0.3	0.1	0.08	0.09
With Constraint					
$P_{e^-}^-$	0.1	0.1	0.1	0.07	0.07

Simplified approach: (as a first step)

- Neglect spin transport
- Using $\Delta P/P = 0.25\%$:
- Gaussian distribution
  - Mean: $|P_{e^-}| = 80\%, |P_{e^+}| = 30\%$
  - Width: ΔP

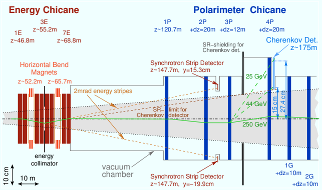
Implementation:

$$\chi^2 += \sum_P \left[\frac{\left(P^{\pm}_{e^{\pm}} - \mathcal{P}^{\pm}_{e^{\pm}}\right)^2}{\Delta \mathcal{P}^2} \right]$$

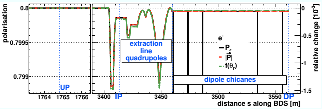
- $P^{\pm}_{e^{\pm}}$: 4 fitted parameters
- $\mathcal{P}^{\pm}_{e^{\pm}}$: Polarimeter measurement
- $\Delta\mathcal{P}$: Polarimeter uncertainty

Prospective Implementations of Polarimeter Constraints

- **Consideration of the upstream polarimeter:**
  - Using the information of both polarimeters
  - → Compensation of misalignments in the BDS (Spin tracking)
  - → Compensation for beam collision effects

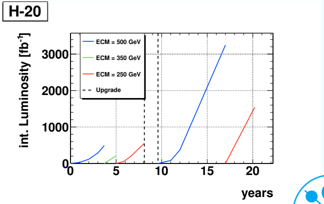


- **Time-dependence of the beam polarization:**
  - Currently, there is always the assumption of a time-independent beam polarization
  - → e.g. 7 years run ⟶ not realistic scenario
  - → Correction for potential time-dependencies



⇒ **Implementation in the unified approach:**

- ⇒ Straight forward



Outlook

Considering systematic quantities for each process

- Determination of the selection efficiency ε and background estimation B
- Determination of the systematic uncertainties: ΔB, $\Delta\varepsilon$, $\Delta\mathcal{L}$
- Determination of realistic correlation factors

Implementing differential cross sections within the χ^2-method

- Using the angular information of a process to further improve the precision
- Consider a well separation from possible BSM effects

Conclusion

- **Polarization provides a deep insight in the chiral structure of the standard model and beyond**

  $\Rightarrow$ A permille-level precision of the luminosity-weighted average polarization at the IP is required

- **New unified approach combing all suitable cross sections and the polarimeter measurement**

$$\chi^2 = \sum_{\text{process}} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)^T \Xi^{-1} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right) + \sum_{P} \left[\frac{\left(P^{\pm}_{e^{\pm}} - \mathcal{P}^{\pm}_{e^{\pm}}\right)^2}{\Delta \mathcal{P}^2} \right]$$

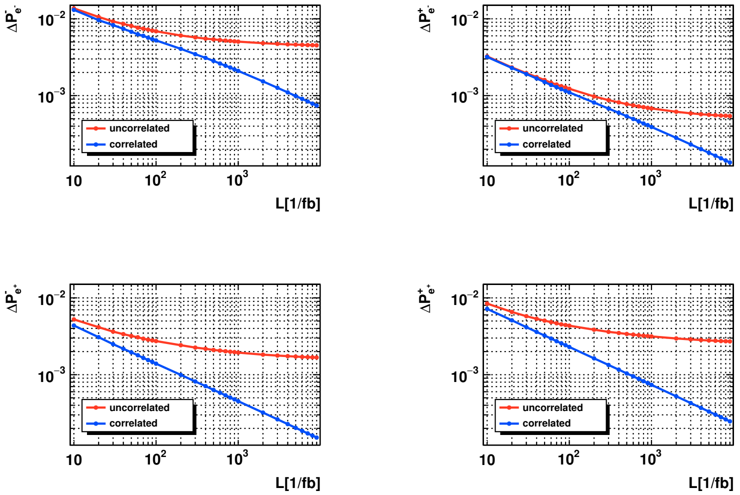
  $\Rightarrow$ Statistical precision of a permille-level is achievable

  $\Rightarrow$ Impact of time-dependent systematic uncertainties can be reduced due to a fast helicity reversal

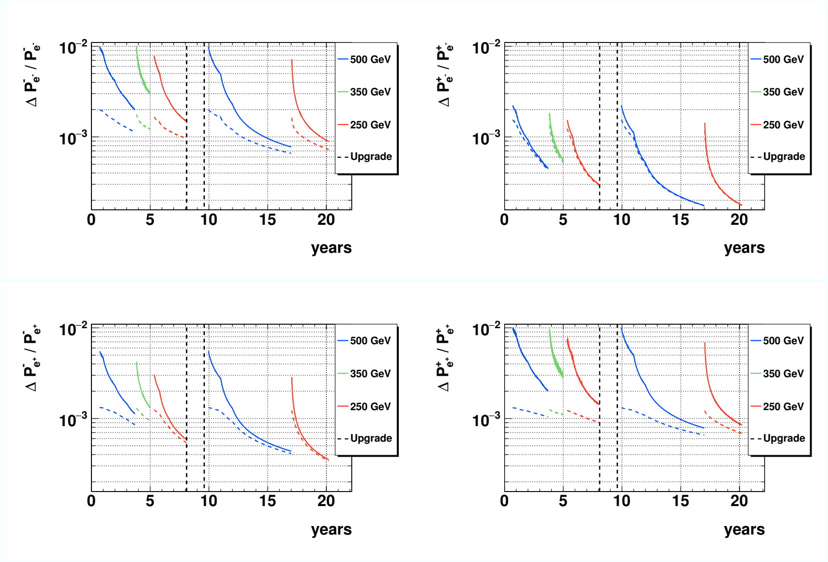
  $\Rightarrow$ Non-perfect helicity reversal has no impact on the statistical precision

- **Further improvement due to polarimeter constraints**
  $\rightarrow$ particular for low integrated luminosity

Backup Slides



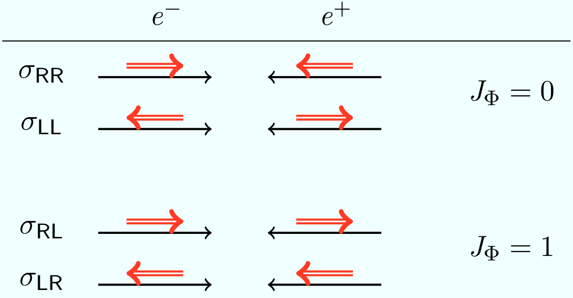
$\Delta P^-_{e^-}$
$\Delta P^+_{e^-}$
$\Delta P^-_{e^+}$
$\Delta P^+_{e^+}$
uncorrelated
correlated
L[1/fb]


$\Delta P_{e^-}^- / P_{e^-}^-$
$\Delta P_{e^-}^+ / P_{e^-}^+$
$\Delta P_{e^+}^- / P_{e^+}^-$
$\Delta P_{e^+}^+ / P_{e^+}^+$
10^{-2}
10^{-3}
0
5
10
15
20
years
500 GeV
350 GeV
250 GeV
Upgrade

E	500	350	250	500	250
$\mathcal{L}$	500	200	500	3500	1500
Without Constraint					
$P^{-}_{e^-}$	0.2	0.3	0.1	0.08	0.09
$P^{+}_{e^-}$	0.05	0.06	0.03	0.02	0.02
$P^{-}_{e^+}$	0.1	0.1	0.06	0.04	0.04
$P^{+}_{e^+}$	0.2	0.3	0.1	0.08	0.08
With Constraint					
$P^{-}_{e^-}$	0.1	0.1	0.1	0.07	0.07
$P^{+}_{e^-}$	0.04	0.05	0.03	0.02	0.02
$P^{-}_{e^+}$	0.09	0.09	0.05	0.04	0.03
$P^{+}_{e^+}$	0.1	0.1	0.09	0.07	0.07

Polarization at a $e^- e^+$ Collider

- Helicity is the projection of the spin vector on the direction of motion
- In case of massless particles, helicity is equal to chirality
- If $E_{\text{kin}} \gg E_0 \quad \longrightarrow \quad m_e \approx 0$

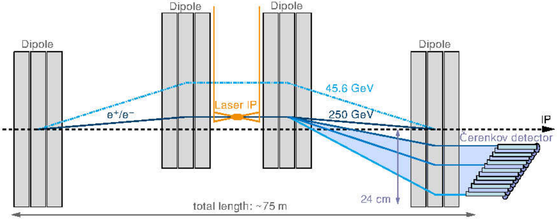


- For a bunch of particles the polarization is defined as:

$$P := \frac{N_R - N_L}{N_R + N_L}$$

Laser-Compton Polarimeters

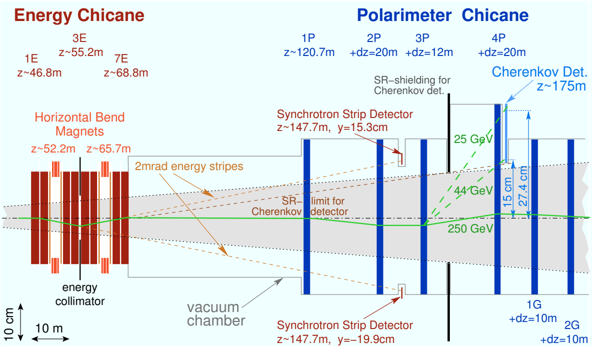
Magnetic chicane of the upstream polarimeters



- Compton scattering of the beam with a polarized Laser
- $\mathcal{O}(10^3)$ particles per bunch $(2 \cdot 10^{10})$ are scattered
- Magnetic chicane: energy spectrum $\Rightarrow$ spatial distribution

- Energy spectrum measurement:
  $\Rightarrow$ Counting the scattered particles at different positions
- Design of the magnetic Chicane:
  - Laser-bunch interaction point moves with beam energy
    $\longrightarrow$ position of the Compton edge stays the same
  - Orbit of the non-scattered particles is unaffected by the magnetic chicane

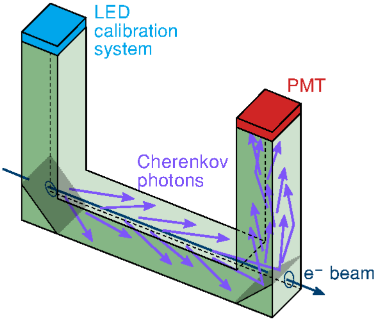
Downstream Polarimeter

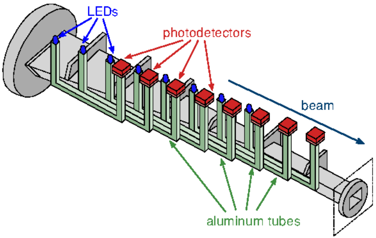


Difference to Upstream Polarimeter due to a large disturbed beam

- Stronger banding of the beam after γ-IP
- 2 additional magnets to restore the beam orbit
- Measuring one bunch per train

Cherenkov Detectors: Basic Concept

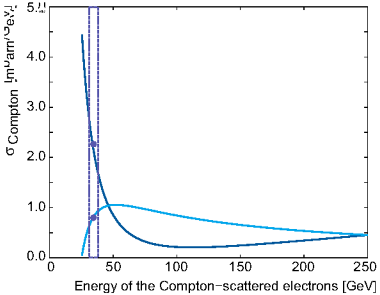




- U-shape channels filled with gas:
  e.g. perfluorobutane
- Concept
  - Scattered particles propagates through the bottom
  - Produced Cherenkov light is reflected to one end of the channel
  - Light measurement with photomultiplier tube (PMT)
- At the other end: LED for PMT calibration
- Sampling of the energy distribution
  → Number of Cherenkov detector
- Energy resolution
  → Thickness of a Cherenkov detector
- Quartz Cherenkov detector concept:
  Ref.: Theses Annika Vauth

http://bib-pubdb1.desy.de/record/171400

Differential Compton Cross Section



Energy dependence:

$$\frac{\mathrm{d}\sigma_C}{\mathrm{d}y_C} = \frac{2\pi r_e^2}{x_C}\left(a_C + \lambda\mathcal{P}\cdot b_C\right); \quad y_C := 1 - \frac{E'}{E}$$

e^- Polarization: $\mathcal{P}$; Laser Polarization: λ

DarkBlue: $\lambda\mathcal{P} = +1$

Cyan: $\lambda\mathcal{P} = -1$

Calculating $\mathcal{P}_i$ of the i-th channel with asymmetry A_i, analysing power Π_i

$$A_i := \frac{N_i^- - N_i^+}{N_i^- + N_i^+}; \qquad \Pi_i = \frac{\mathcal{I}_i^- - \mathcal{I}_i^+}{\mathcal{I}_i^- + \mathcal{I}_i^+}; \qquad \mathcal{I}_i^{\pm} := \int_{E_i-\Delta/2}^{E_i+\Delta/2} \left.\frac{\mathrm{d}\sigma_C}{\mathrm{d}y_C}\right|_{\lambda\mathcal{P}=\pm 1} \mathrm{d}y_C$$

$N^{\pm} := \#e_{\text{Compton}}$ for $\lambda\mathcal{P} = \pm 1$; E_i : energy of i-th channel; Δ : energy width

$$\Rightarrow \quad \lambda\mathcal{P}_i = \frac{A_i}{\Pi_i} \quad \Rightarrow \quad \mathcal{P} = \langle \mathcal{P}_i \rangle$$

Compton Scattering Cross Section: Formulary

$$\frac{d\sigma}{dy_C} = \frac{2\pi r_e^2}{x_C}\left(a_C + \lambda\mathcal{P}\cdot b_C\right)$$

$$y_C := 1 - \frac{E'_\gamma}{E}; \quad x_C := \frac{4EE_\gamma}{m_e^2}\cos^2\left(\frac{\vartheta_0}{2}\right)$$

$$r_C := \frac{y_C}{x_C\left(1-y_C\right)}$$

$$a_C := \left(1-y_C\right)^{-1} + 1 - y_C - 4r_C\left(1-r_C\right)$$

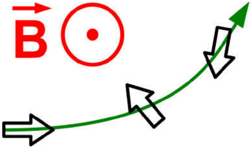
$$b_C := r_C x_C\left(1-2r_C\right)\left(2-y_C\right)$$

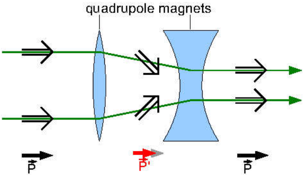
$E,\ E_\gamma$:	e^-,γ energy before Compton scattering
$E',\ E'_\gamma$:	e^-,γ energy after Compton scattering
$m_e,\ r_e$:	mass, classical radius of e^-
ϑ_0 :	crossing angle between e^-,γ
$\mathcal{P}$:	longitudinal polarization of e^-
λ :	circular polarization of γ_{Laser}

Characteristic Point:

$$E'_{\text{crossover}} = \frac{E}{1+x_C/2}, \qquad E'_{\text{ComptonEdge}} = E'_{\text{min}} = \frac{E}{1+x_C}$$

Spin Precession





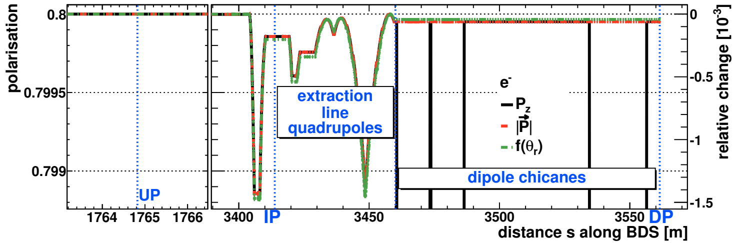
- Polarimeters are 1.65 km and 150 m away from IP
  - → Particles propagate through magnets
  - → Magnets influence the spin, as well
  - → Described by Thomas precession
- if $\vec{B}_{\parallel} = \vec{E} = 0$:

$$\frac{\mathrm{d}}{\mathrm{d}t}\vec{S} = -\frac{q}{m\gamma}\left((1 + a\gamma)\,\vec{B}_{\perp}\right) \times \vec{S}$$

- Effects from focusing and defocusing can cancel
- For a series of quadrupole magnets $\mathcal{P}$ described by the angular divergence θ_r

$$f(\theta_r) = |\vec{\mathcal{P}}|_{\mathsf{max}} \cdot \cos\left((1 + a\gamma) \cdot \theta_r\right)$$

Spin Tracking



Further causes of longitudinal beam polarization change:

- *Bremsstrahlung:*
  Deceleration by passing through matter $\longrightarrow$ negligible for colliders
- *Beamstrahlung:*
  Deflection by the em-field of the oncoming bunch during collision
- *Synchrotron radiation:*
  Deflection by the em-field of accelerator magnets

Systematic Polarization Uncertainty

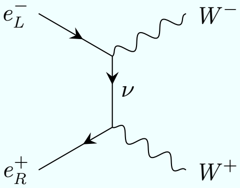
contribution	uncertainty $[10^{-3}]$
Beam and polarization alignment at polarimeters and IP ($\Delta\vartheta_{\text{bunch}} = 50\,\mu$rad, $\Delta\vartheta_{\text{pol}} = 25\,$mrad)	0.72
Variation in beam parameters (10 % in the emittances)	0.03
Bunch rotation to compensate the beam crossing angle	< 0.01
Longitudinal precession in detector magnets	0.01
Emission of synchrotron radiation	0.005
Misalignments (10 μ) without collision effects	0.43
Total (quadratic sum)	0.85
Collision effects in absence of misalignments	< 2.2

[Ref.:] Thesis Moritz Beckmann (http://bib-pubdb1.desy.de/record/155874)

Concept

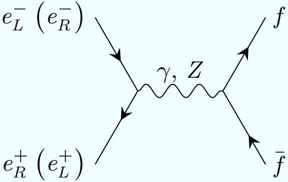
Example Processes:

W-pair production:



$\sigma_{\text{LL}} = \sigma_{\text{RR}} = \sigma_{\text{RL}} = 0$

s-channel spin-1 particle:



$\sigma_{\text{LL}} = \sigma_{\text{RR}} = 0$

- Calculation of the P from polarized σ measurement of well known SM-process
  - $\rightarrow$ Using the information of their chiral structure
- Requirement to consider a process:
  - Theoretical very well known
    - $\rightarrow$ Reduction of theoretical uncertainties
  - High absolute cross section (high rate)
    - $\rightarrow$ Minimizing the statistical error
  - Large left-right-asymmetry
    - $\rightarrow$ Minimizing the influence of systematic uncertainties
  - Well separable from possible BSM-effects
- Feature of the ILC:
  Using 4 different polarization configuration
  ($\rightarrow$ signs of the polarizations)

$\Rightarrow$ Task: Providing the absolute scale calibration

Special Case: The Modified Blondel Scheme (MBS)

- Constraints for the Modified Blondel Scheme:
  - Process must fulfill: $\sigma_{\text{LL}} \equiv \sigma_{\text{RR}} \equiv 0$
  - Perfect helicity reversal: $+|P| \longleftrightarrow -|P| \quad \Rightarrow \quad |P| \equiv$ const.
- Unique solution:

  4 possible cross section measurements: $\sigma_{-+},\ \sigma_{+-},\ \sigma_{--},\ \sigma_{++}$

  Maximal 4 unknown quantities: $\sigma_{\text{LR}},\ \sigma_{\text{RL}},\ |P_{e^-}|,\ |P_{e^+}|$
- Solve for $|P_{e^\mp}|$:

$$\sigma_{\pm\pm} = \frac{\left(1\pm|P_{e^-}|\right)}{2}\frac{\left(1\mp|P_{e^+}|\right)}{2}\cdot\sigma_{RL} + \frac{\left(1\mp|P_{e^-}|\right)}{2}\frac{\left(1\pm|P_{e^+}|\right)}{2}\cdot\sigma_{LR}$$

- Modified Blondel-Scheme:

$$|P_{e^\mp}| = \sqrt{\frac{\left(\sigma_{-+}+\sigma_{+-}-\sigma_{--}-\sigma_{++}\right)\left(\pm\sigma_{-+}\mp\sigma_{+-}+\sigma_{--}-\sigma_{++}\right)}{\left(\sigma_{-+}+\sigma_{+-}+\sigma_{--}+\sigma_{++}\right)\left(\pm\sigma_{-+}\mp\sigma_{+-}-\sigma_{--}+\sigma_{++}\right)}}$$

- Uncertainties are calculated via analytic error propagation

The Unified Approach: χ^2-Method

- Desire for a more general approach:
  - Consider any process with a polarization dependence + using several processes at once
  - Compensate non-perfect helicity reversal: $+\left|P^R\right| \longleftrightarrow -\left|P^L\right|$
- Consider a χ^2-Method: Using all 4 chiral cross sections

$$\chi^2 = \sum_{\text{process}} \left\{ \sum_{\pm\pm} \left[\frac{\left(\sigma^{\text{data}} - \sigma^{\text{theory}}\right)^2}{\Delta\sigma^2} \right] \right\}$$

- Compensate non-perfect helicity reversal: 4 free parameters

$$\underbrace{P_L^- = -80\%,}_{\text{left-handed } e^-\text{-beam}} \quad \underbrace{P_R^- = 80\%,}_{\text{right-handed } e^-\text{-beam}} \quad \underbrace{P_L^+ = -30\%,}_{\text{left-handed } e^+\text{-beam}} \quad \underbrace{P_R^+ = 30\%,}_{\text{right-handed } e^+\text{-beam}}$$

- Error determination via toy experiments

Polarized Cross Section

- Theoretical polarized cross section:

$$\sigma\left(P_{e^-}, P_{e^+}\right) = \frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\mathrm{LL}} + \frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\mathrm{RR}}$$
$$+\frac{\left(1-P_{e^-}\right)}{2}\frac{\left(1+P_{e^+}\right)}{2}\cdot\sigma_{\mathrm{LR}} + \frac{\left(1+P_{e^-}\right)}{2}\frac{\left(1-P_{e^+}\right)}{2}\cdot\sigma_{\mathrm{RL}}$$

- Measured polarized cross section:

$$\sigma\left(P_{e^-}, P_{e^+}\right) = \frac{N}{\varepsilon\cdot\mathcal{L}} = \frac{D-\langle B\rangle}{\varepsilon\cdot\mathcal{L}};$$

Statistic quantity: selected data D, number of events N

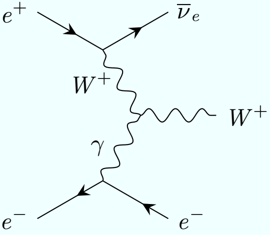
Systematic quantity: background B, selection efficiency ε, integrated luminosity $\mathcal{L}$

- Cross section of the 4 polarization configurations

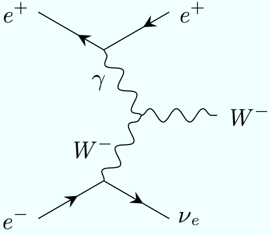
$$\sigma_{--} := \sigma\left(-|P_{e^-}|, -|P_{e^+}|\right) \qquad \sigma_{++} := \sigma\left(+|P_{e^-}|, +|P_{e^+}|\right)$$
$$\sigma_{-+} := \sigma\left(-|P_{e^-}|, +|P_{e^+}|\right) \qquad \sigma_{+-} := \sigma\left(+|P_{e^-}|, -|P_{e^+}|\right)$$

Previous Single $W^{\pm}$, Z, γ Study: Leading Diagrams

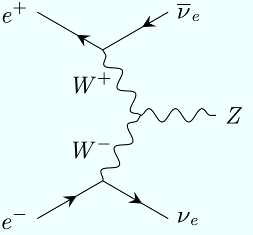
Single W^+



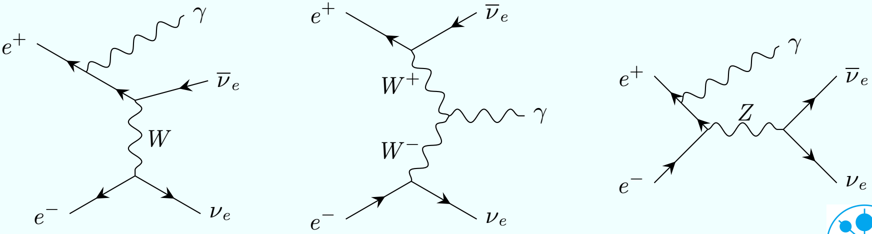
Single W^-



Single Z



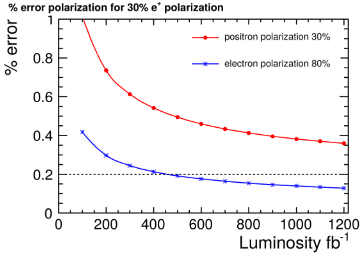
Single γ



Comparison to the Previous W-Pair Study

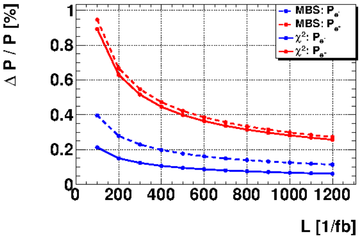
Study by Ivan Marchesini:

- Using $e^- e^+ \to W^+ W^- \to q\bar{q}l\nu$
- Statistical uncertainties only
- Consider equal absolute polarizations (MBS)
- Including full background study



Adjustment of the current study:

- Limited to $e^- e^+ \to W^+ W^- \to q\bar{q}l\nu$
- Forced equal absolute polarizations $\left(\left|P^L\right| \equiv \left|P^R\right|\right)$
- Including same background estimation and selection efficiency



Comparison:

$\Rightarrow$ χ^2-method yields better precision under same conditions than the MBS

Comparison to Previous Single $W^{\pm}$, γ, Z Study

Study by Graham W. Wilson

- Using 4 Processes simultaneously:

$$e^- e^+ \to \nu\bar{\nu}\gamma; \qquad e^- e^+ \to \nu\bar{\nu}Z$$
$$e^- e^+ \to e^+ \nu W^- \to e^+ \nu \mu^- \bar{\nu}$$
$$e^- e^+ \to e^- \bar{\nu} W^+ \to e^- \bar{\nu} \mu^+ \nu$$

- Consider equal absolute polarizations
  2 Parameters: P_{e^-}, P_{e^+}
- Consider deviations: 4 Parameters

$$P^L_{e^\pm} = -|P_{e^\pm}| + \frac{1}{2}\delta_\pm$$
$$P^R_{e^\pm} = \quad |P_{e^\pm}| + \frac{1}{2}\delta_\pm$$

parameters		$\Delta P/P,\ \mathcal{L} = 2\text{ab}^{-1}$	
#	P	**Previous**	**Current**
2	P_{e^-}	0.07%	0.051%
	P_{e^+}	0.22%	0.21%
4	P_{e^-}	0.085%	0.088%
	δ_{e^-}	0.12%	0.19%
	P_{e^+}	0.22%	0.23%
	δ_{e^+}	0.32%	0.56%

$\mathcal{L}$ equally distributed between $\sigma_{\pm\pm}$

Statistical precision only

Comparison to Current analysis

- Differences:

Previous: Constraint on δ: $\Delta\delta < 10^{-3}$

Current: direct fit of $P^{L,R}_{e^\pm}$

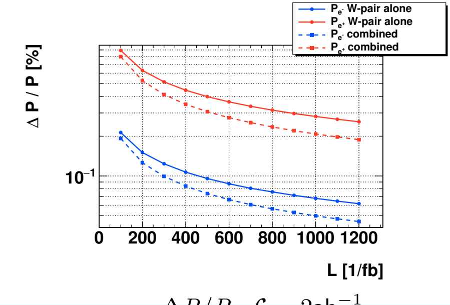
- Very similar precision even without additional constraint on δ

Combining W-Pair + Single W, Z, γ

Combined vs. W-Pairs alone

- W-Pair yields only enough information for 2 parameter fit P_{e-}, P_{e+}
- Large improvement
  $\rightarrow$ due to additional processes
- Combined: fit of 4 parameters is possible P^L_{e-}, P^R_{e-}, P^L_{e+}, P^R_{e+}

$\Rightarrow$ Compensation for a non-perfect helicity reversal



Combined vs. Single Boson

- The 4 processes Single $W^{\pm}$, Single Z, Single γ yields a large analysis power
- Combined precision dominated by single boson processes

$\Delta P/P,\ \mathcal{L} = 2\text{ab}^{-1}$

	single W, Z, γ	Combined
P_{e-}	0.088%	0.079%
δ_{e-}	0.19%	0.18%
P_{e+}	0.23%	0.16%
δ_{e+}	0.56%	0.51%

Consider Correlated Uncertainty

Implementing correlated uncertainty:

$$\chi^2 = \sum_{\text{process}} \sum_{i\in\pm\pm} \frac{\left(\sigma_i^{\text{data}} - \sigma_i^{\text{theory}}\right)^2}{\Delta\sigma_i^2} \quad\longrightarrow\quad \sum_{\text{process}} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)^T \Xi^{-1} \left(\vec{\sigma}_{\text{data}} - \vec{\sigma}_{\text{theory}}\right)$$

$$\vec{\sigma} := \begin{pmatrix}\sigma_{-+} & \sigma_{+-} & \sigma_{--} & \sigma_{++}\end{pmatrix}^T$$

$$\Xi := \Xi_N + \Xi_B + \Xi_\varepsilon + \Xi_{\mathcal{L}}; \qquad \text{e.g. } \left(\Xi_\varepsilon\right)_{ij} = \text{corr}\left(\vec{\sigma}_i^\varepsilon,\ \vec{\sigma}_j^\varepsilon\right) \frac{\partial\vec{\sigma}_i}{\partial\varepsilon_i}\frac{\partial\vec{\sigma}_j}{\partial\varepsilon_j}\Delta\varepsilon_i\Delta\varepsilon_j$$

Occurrence of correlated uncertainties:

- Fast switch between $\sigma_{\pm\pm}$
- Faster than change in e.g $\delta\mathcal{L}$

→ $\Delta\sigma_{\pm\pm}$ $(\Delta\mathcal{L})$ becomes correlated

⇒ corr $\left(\vec{\sigma}_i^{\mathcal{L}},\ \vec{\sigma}_j^{\mathcal{L}}\right) \neq 0 \quad \forall i \neq j$

Consider disadvantageous situation:

- $\varepsilon = 0.6$
- $\Delta\varepsilon/\varepsilon = 0.01$
- $\Delta\mathcal{L}/\mathcal{L} = 0.001$

→ Studying the impact of correlations

Outlook

- **Open issues**
  - Implementing fiducial cuts for all processes $\rightarrow$ correct description of all systematics
  - Including a complete background analyses
- **Further Improvement**
  - Consider also differential cross sections
  - Study the possibility to use fiducial and differential cross sections simultaneously