

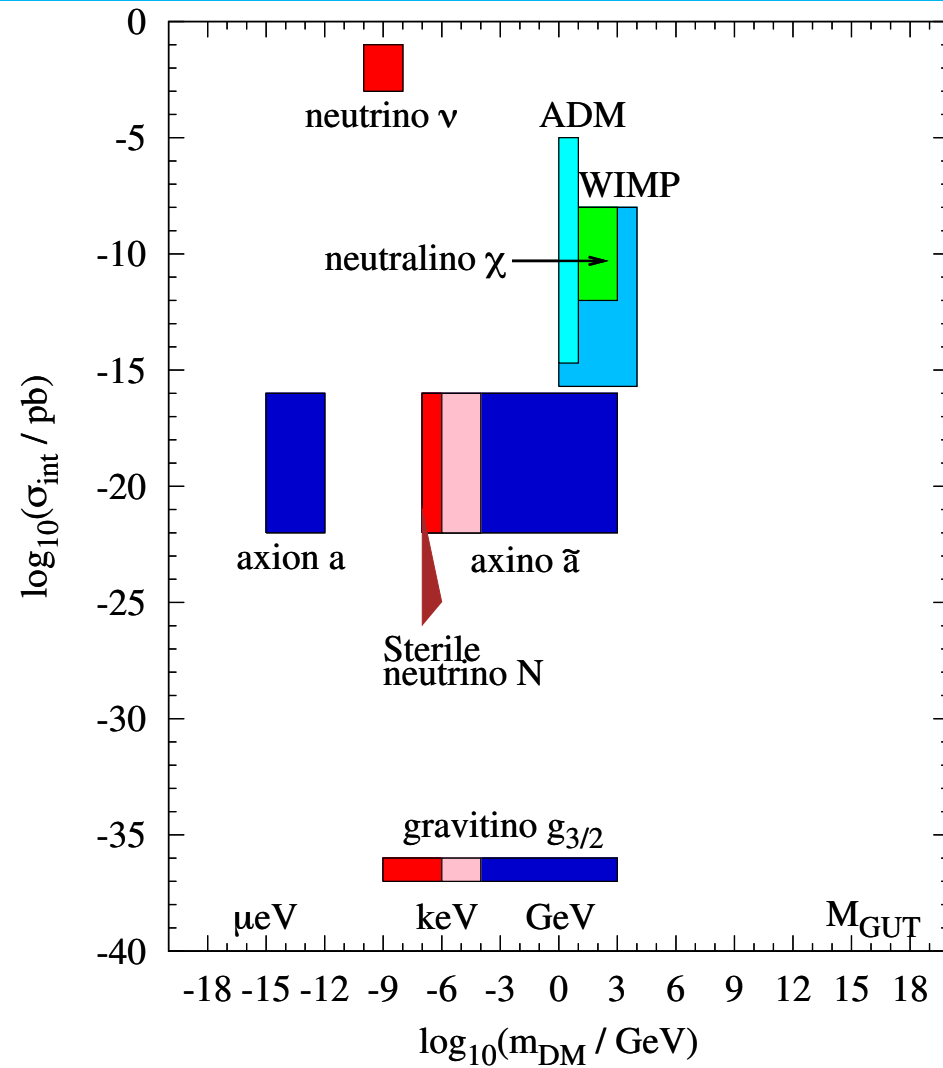
Axion Cold Dark Matter.

Andreas Ringwald (DESY)

Axion Dark Matter
Stockholm, Sweden
5-9 December 2016

Introduction

- Many cold dark matter candidates

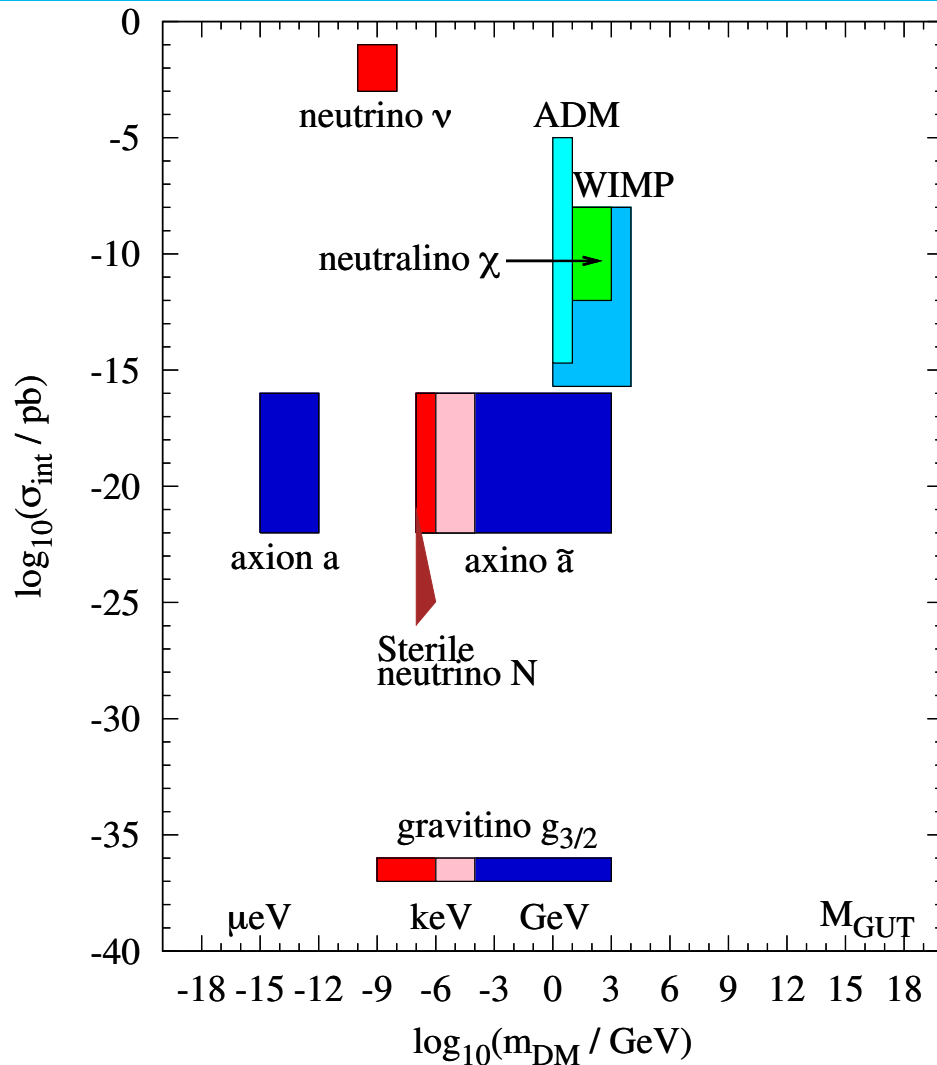


[Baer,Choi,Kim,Roszkowski 10]



Introduction

- Many cold dark matter candidates
- Best motivated are those that occur in UV completions of the Standard Model which solve also other problems

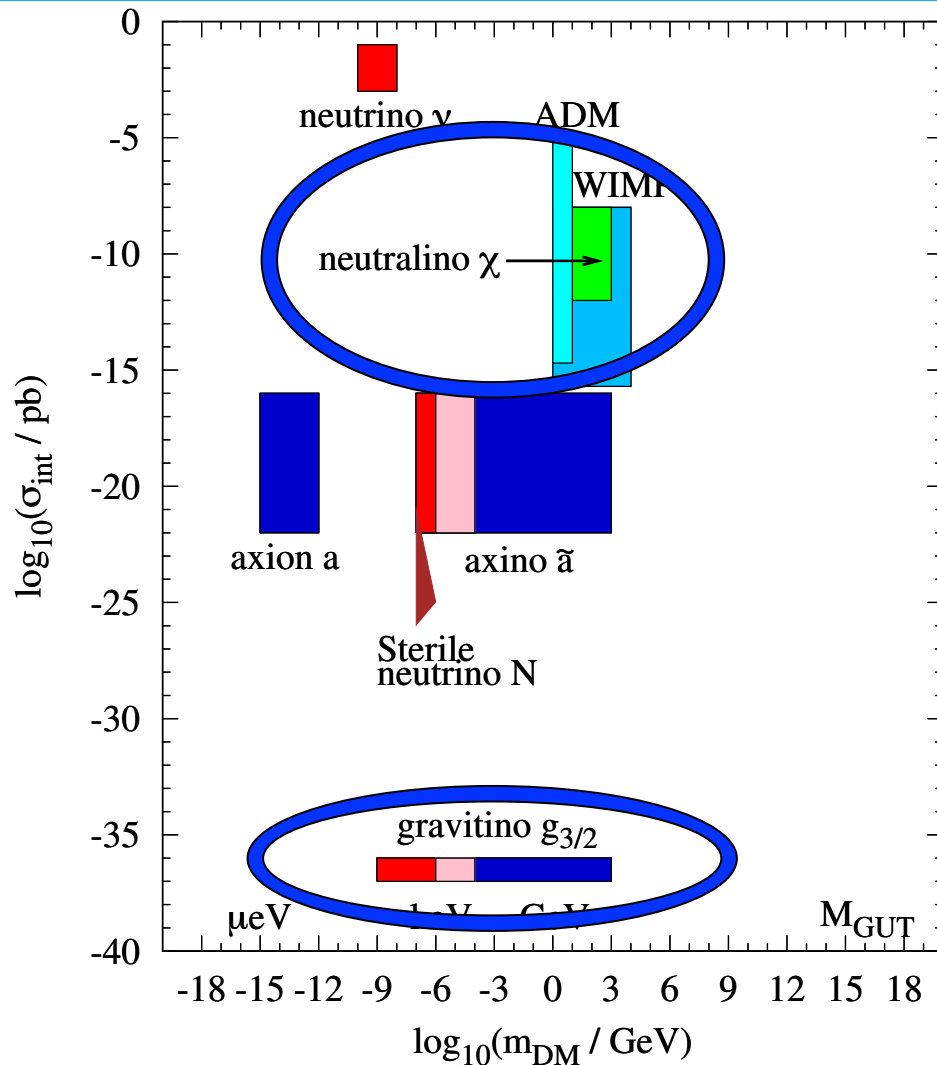


[Baer,Choi,Kim,Roszkowski 10]



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 - Hierarchy Problem
- > MSSM: Neutralino or Gravitino

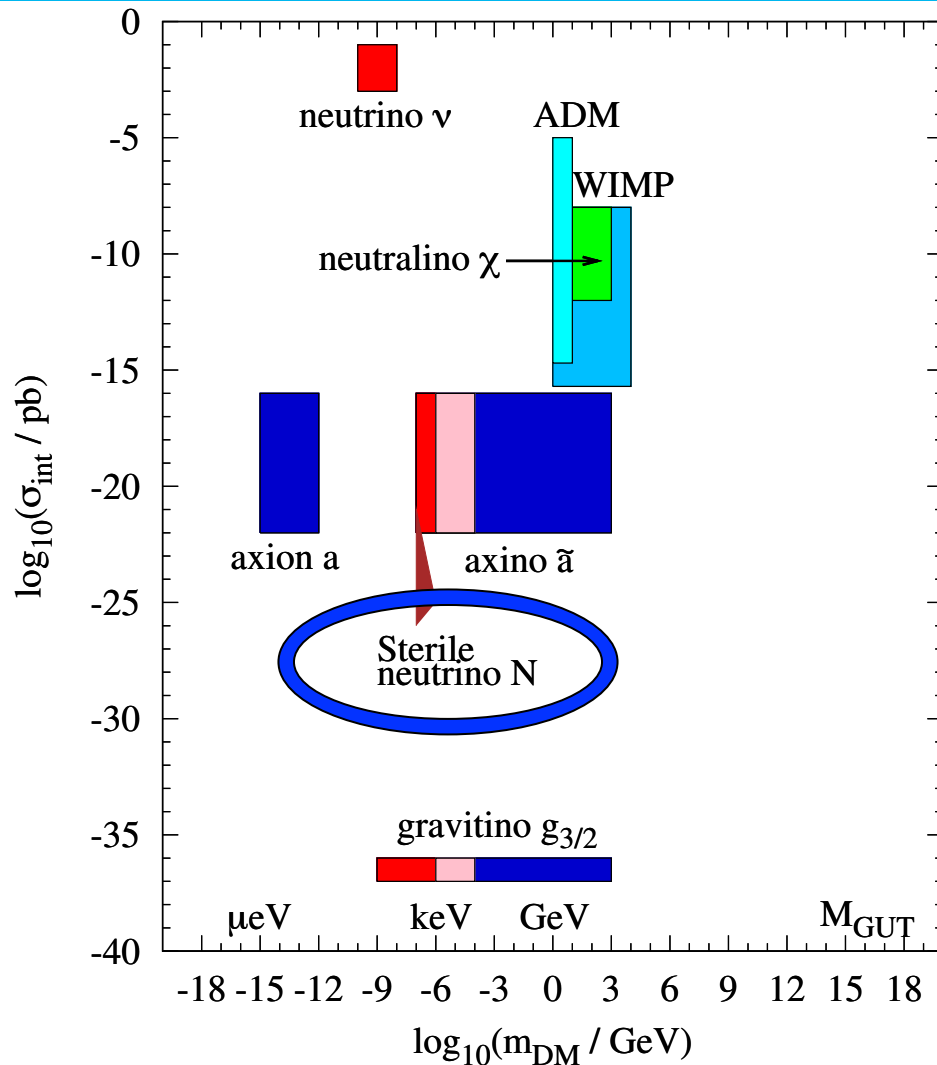


[Baer,Choi,Kim,Roszkowski 10]



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 - Baryon Asymmetry
- > MSSM: Neutralino or Gravitino
- > nuMSM: Lightest sterile neutrino

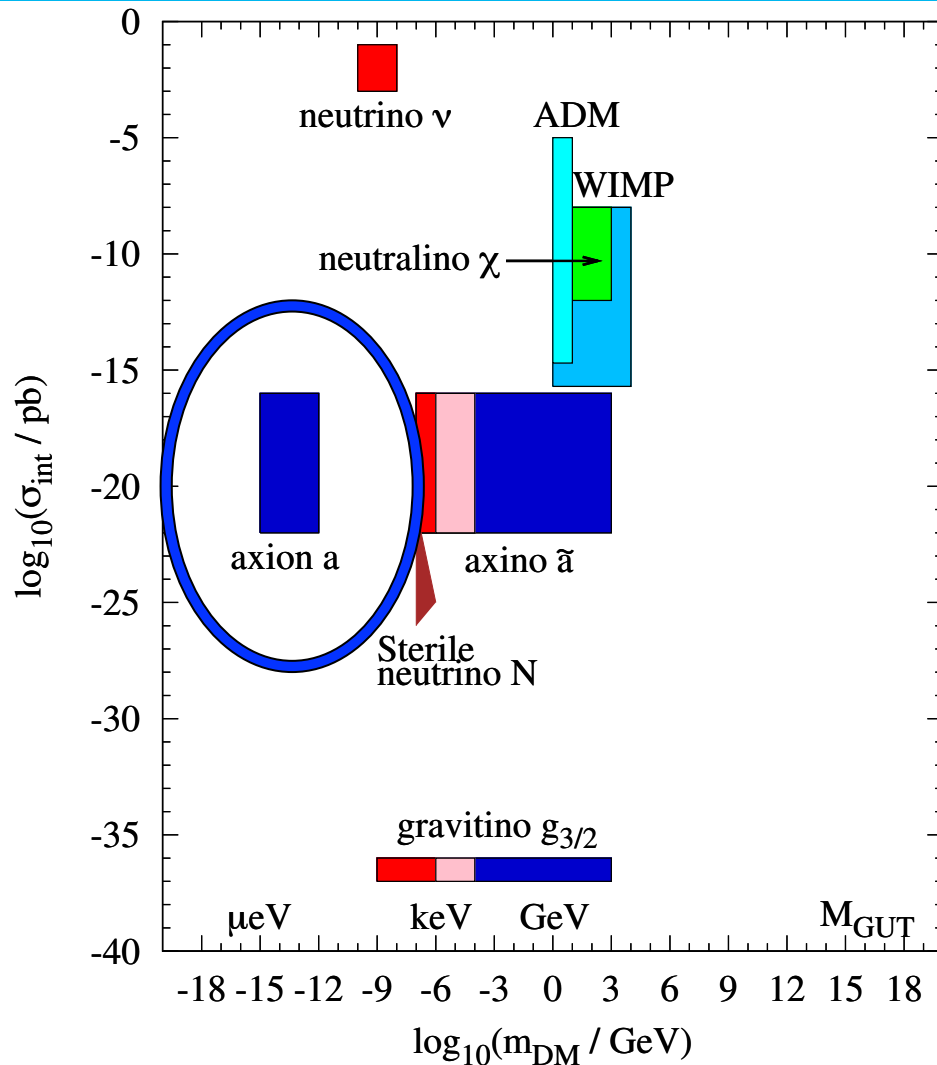


[Baer,Choi,Kim,Roszkowski 10]



Introduction

- > Many cold dark matter candidates
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 - Hierarchy Problem
 - Neutrino Masses and Mixing
 - Baryon Asymmetry
 - Strong CP Problem
- > MSSM: Neutralino or Gravitino
- > nuMSM: Lightest sterile neutrino
- > PQSM: Axion



[Baer,Choi,Kim,Roszkowski 10]



PQSM: Peccei-Quinn Extensions of the Standard Model

- > A singlet complex scalar field σ featuring a global $U(1)_{\text{PQ}}$ symmetry is added

- > Symmetry is broken by vev $\langle \sigma \rangle = v_{\text{PQ}}/\sqrt{2}$

$$\sigma(x) = \frac{1}{2} (v_{\text{PQ}} + \rho(x)) e^{iA(x)/v_{\text{PQ}}}$$

- Excitation of modulus: $m_\rho \sim v_{\text{PQ}}$
- Excitation of angle: NGB $m_A \ll v_{\text{PQ}}$

- > PQ charges of quarks (SM or extra) are such that $U(1)_{\text{PQ}} \times SU(3)_C \times SU(3)_C$ has chiral anomaly: NGB is called **axion**

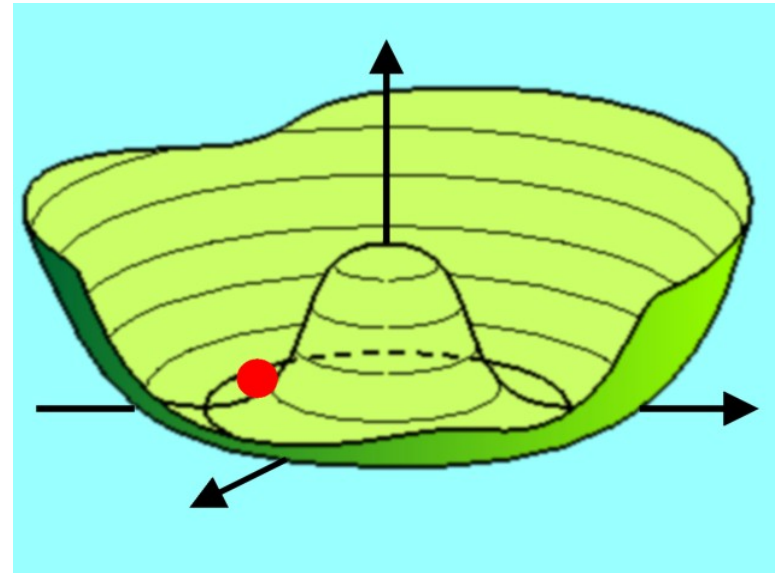
[Peccei,Quinn 77; Weinberg 78; Wilczek 78]

- > No strong CP problem, since axion field acts as x-dependent theta parameter

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi} \frac{A(x)}{f_A} G_{\mu\nu}^a \tilde{G}^{a\mu\nu}; \quad f_A = v_{\text{PQ}}/N_{\text{DW}}; \quad N_{\text{DW}} = \# \text{ quarks with PQ charge}$$

QCD dynamics: $\langle A(x) \rangle = 0$

[Kim 79; Shifman, Vainshtein, Zakharov 80; Zhitnitsky 80; Dine, Fischler, Srednicki 81; ...]



[Raffelt]

Axion Potential at High Temperatures

- Cosmological evolution of axion angular field $\theta = A/f_A$ determined by

$$\ddot{\theta} + 3H(T(t)) \dot{\theta} - \frac{\nabla^2}{a^2(t)} \theta = -\frac{1}{f_A^2} \frac{\partial}{\partial \theta} V(\theta, T(t))$$

- In absence of explicit Peccei-Quinn symmetry breaking: only source of non-trivial axion potential, $\partial V(\theta, T)/\partial \theta \neq 0$, is topological charge density in QCD,

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi} \theta(x) G_{\mu\nu}^a(x) \tilde{G}^{a,\mu\nu}(x) = -\theta(x) q(x)$$

- Axion potential equals free energy density in QCD as function of θ ,

$$V(\theta, T) = -\frac{1}{\mathcal{V}_4} \ln \left[\frac{Z(\theta, T)}{Z(0, T)} \right]$$



Axion Potential at High Temperatures

- QCD partition function can be written in terms of Fourier series of partition functions of sectors with fixed topological charge:

$$Z(\theta, T) = \sum_{Q=-\infty}^{+\infty} e^{i\theta Q} Z_Q(T); \quad Q = \int d^4x q(x)$$

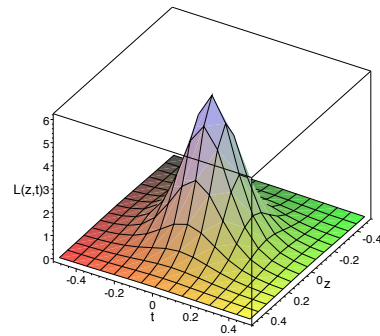
$$Z_Q(T) = \int_Q [dA][d\psi_q][d\bar{\psi}_q] \exp \left\{ -S([A, \psi_q, \bar{\psi}_q]; T) \right\}$$

- Axion mass, $m_A^2 f_A^2 \equiv \frac{\partial^2 V}{\partial \theta^2} \big|_{\theta=0}$, determined by topological susceptibility:

$$m_A^2 f_A^2 = -\frac{1}{\mathcal{V}_4} \frac{1}{Z} \frac{\partial^2 Z}{\partial \theta^2} \bigg|_{\theta=0} = \frac{1}{\mathcal{V}_4} \frac{1}{Z} \sum_{Q=-\infty}^{+\infty} Q^2 Z_Q = \frac{\langle Q^2 \rangle|_{\theta=0}}{\mathcal{V}_4} = \int d^4x \langle q(x) q(0) \rangle|_{\theta=0} \equiv \chi$$

- Need non-perturbative methods to calculate it:

- Semiclassical expansion of Euclidean path integral about minima of Euclidean action with fixed topological charge (instantons)
- Monte Carlo evaluations of lattice-regularized Euclidean path integral



Axion Potential at High Temperatures

- > At very large temperatures, far above the quark hadron phase transition, semiclassical expansion expected to be reliable
- > In this case, partition function can be described by a dilute gas of instantons and anti-instantons (minima of Euclidean action with $Q = \pm 1$)

$$Z(\theta, T) \simeq \sum_{n, \bar{n}=0}^{\infty} \frac{1}{n! \bar{n}!} (Z_1(T) e^{i\theta})^n (Z_1(T) e^{-i\theta})^{\bar{n}} = \exp \{2 Z_1(T) \cos \theta\}$$

- > Potential has cosine shape:

[Callan, Dashen, Gross '78]

$$V(\theta, T) = -\frac{1}{\mathcal{V}_4} \ln \left[\frac{Z(\theta, T)}{Z(0, T)} \right] \simeq \frac{2 Z_1(T)}{\mathcal{V}_4} (1 - \cos \theta)$$

- > All temperature dependence resides in topological susceptibility in terms of contribution of single instanton to path integral:

$$\chi(T) \simeq \frac{2 Z_1(T)}{\mathcal{V}_4} = 2 \int_0^{\infty} d\rho D(\rho) G(\pi \rho T)$$



Axion Potential at High Temperatures

- > Zero temperature instanton size distribution:

[t Hooft '76]

$$D(\rho) = \frac{d}{\rho^5} \left(\frac{2\pi}{\alpha_s(\mu_r)} \right)^{2N_c} \exp \left(-\frac{2\pi}{\alpha_s(\mu_r)} \right) (\rho \mu_r)^{\beta_0} \prod_{i=1}^{n_f} (\rho m_i(\mu_r))$$

- > At large temperature, color-electric Debye screening prohibits existence of large-scale coherent fields in plasma, leading to

[Pisarski, Yaffe '80]

$$G(\pi\rho T) = \exp \left\{ -\frac{6+n_f}{3} (\pi\rho T)^2 - 18 \left[1 - \frac{n_f}{9} \right] A(\pi\rho T) \right\}$$
$$A(\pi\rho T) = -\frac{1}{12} \ln [1 + (\pi\rho T)^2/3] + \alpha [1 + \gamma(\pi\rho T)^{-2/3}]^{-8}$$

- > Cuts off integration at $\rho \sim 1/(\pi T)$ and ensures validity of dilute instanton gas approximation (DIGA) at large temperatures, at which $\alpha_s(\pi T) \ll 1$
- > Axion mass predicted to decrease power-like with temperature in DIGA:

$$m_A^2(T) f_A^2 = \chi(T) \simeq 2 \int_0^\infty d\rho D(\rho) G(\pi\rho T) \propto T^{-(7+\frac{1}{3}n_f)}$$



Axion Potential at High Temperatures

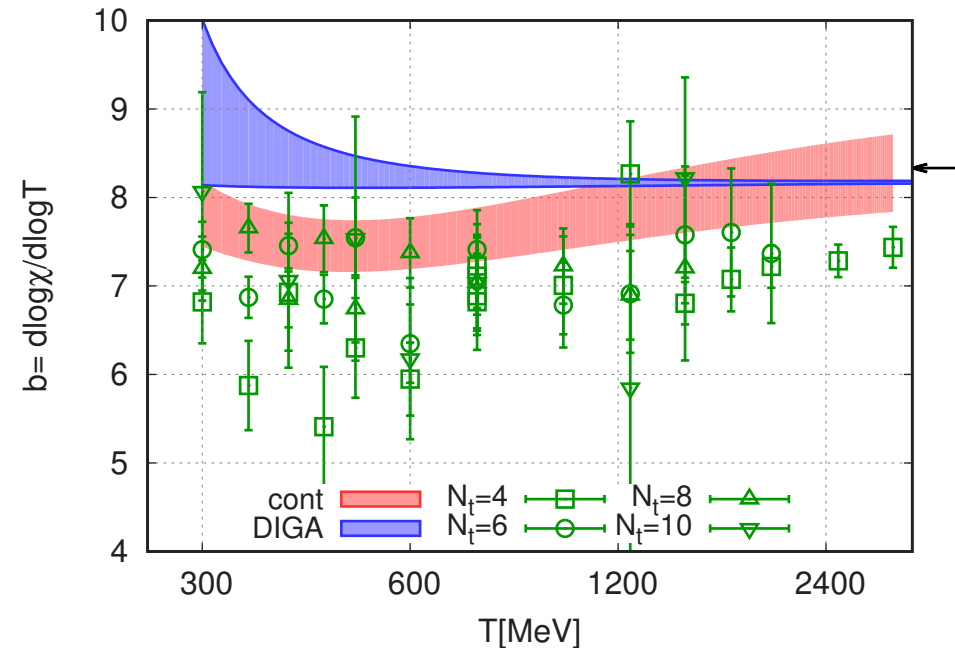
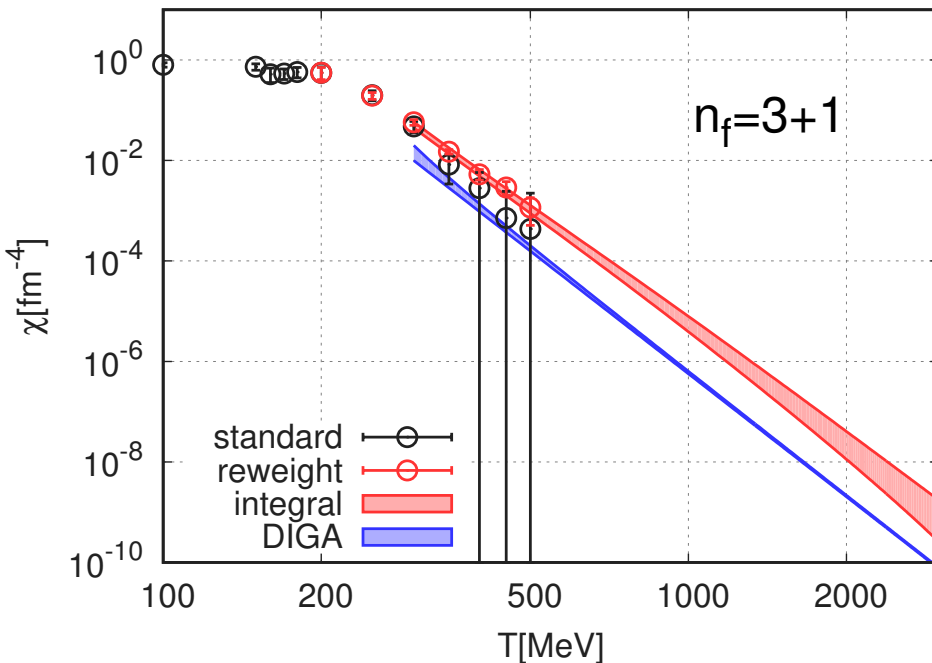
- > How good is DIGA quantitatively? Check with lattice QCD!
- > Topological susceptibility notoriously difficult to calculate on lattice
 1. Large cutoff effects when exploiting action with non-chiral quarks to calculate topological observables
 2. Tiny topological susceptibility needs extremely long simulation threads to observe enough changes of topological sectors
- > Solutions of these problems: [Borsanyi et al. '16]
 1. Eigenvalue reweighting technique: Substitute topology related eigenvalues of non-chiral quark Dirac operator with its corresponding eigenvalues in continuum
 2. Fixed sector integral technique: Measure logarithmic differential of topological susceptibility which is related to quantities to be measured in fixed topological sectors. Then integrate.



Axion Potential at High Temperatures

> Results:

[Borsanyi et al. '16]

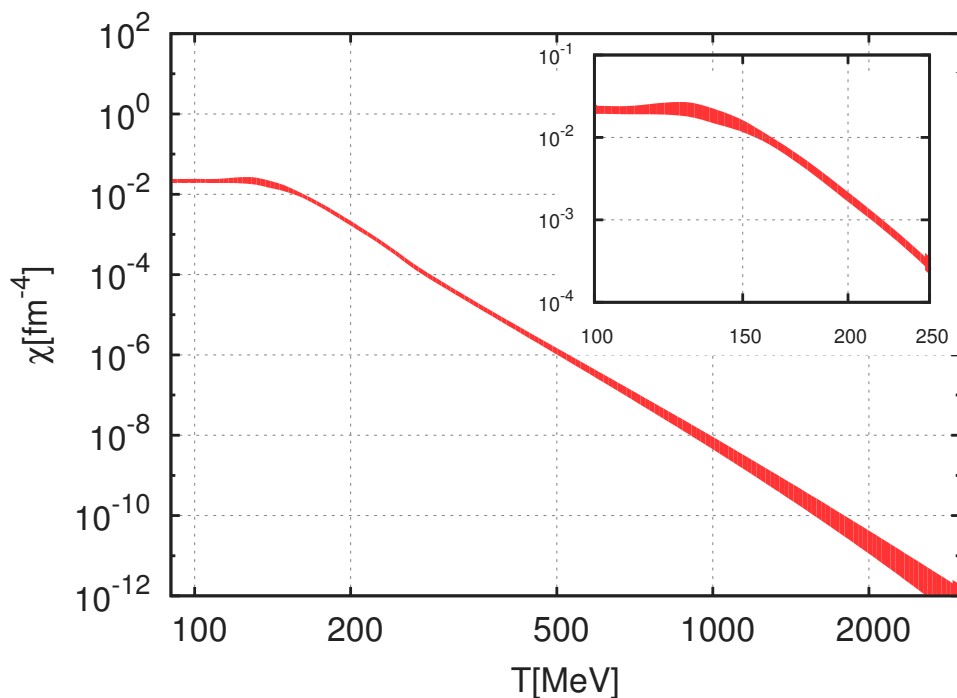


- > Temperature slope remarkably close to DIGA prediction
- > DIGA underestimates topological susceptibility by overall normalization „K factor“ of order ten (should be improved in two-loop DIGA)

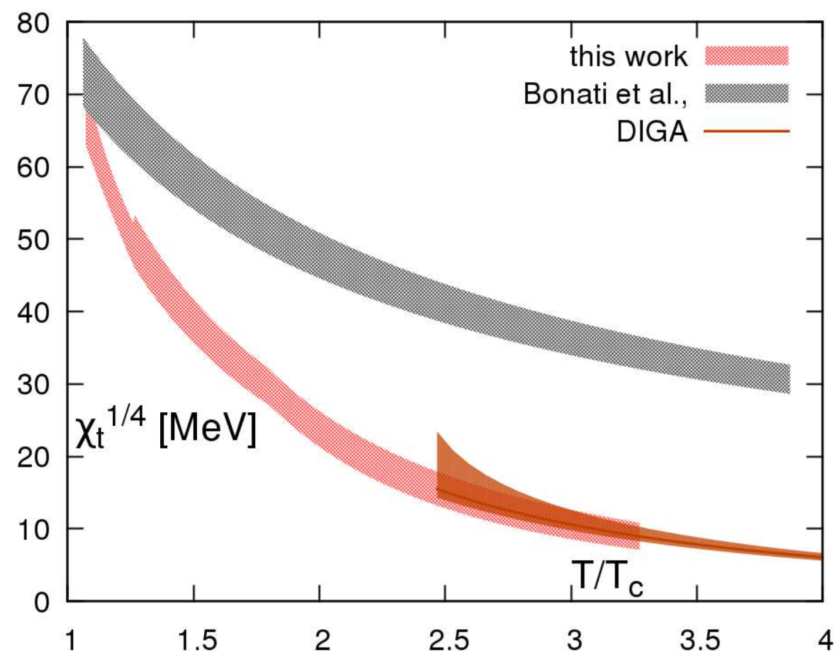


Axion Potential at High Temperatures

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[Borsanyi et al. '16]



[Petreczky et al. '16]

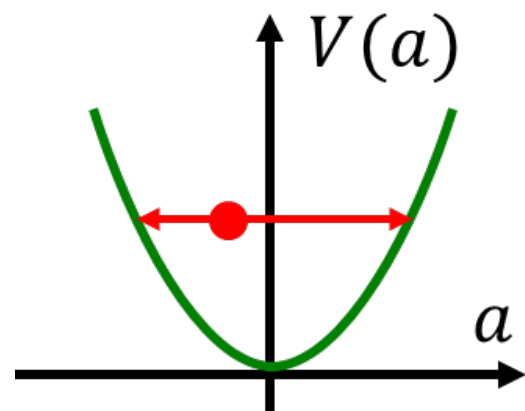
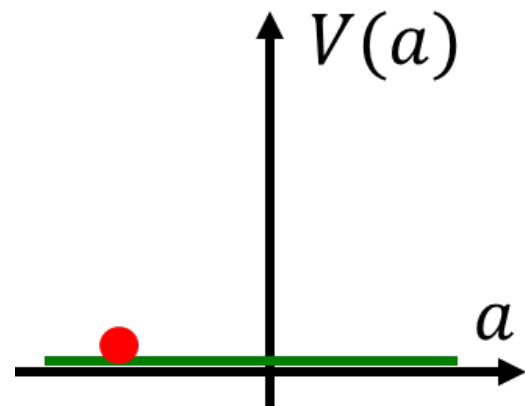


Axion Cold Dark Matter

> DM from vacuum realignment:

[Preskill,Wise,Wilczek 83; Abbott,Sikivie 83; Dine,Fischler 83,...]

- In early universe, axion frozen at random initial value
- Later, field feels pull of mass towards zero and oscillates around it



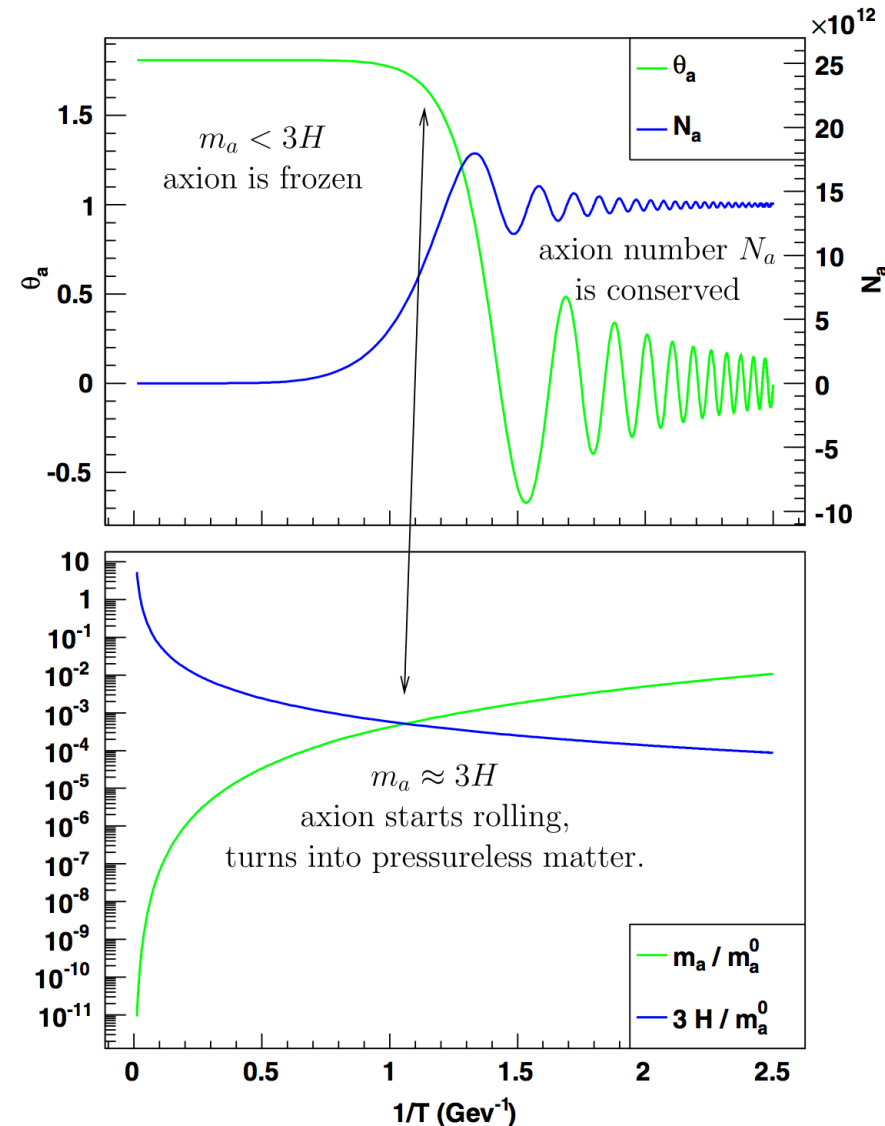
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- Spatially uniform oscillating classical field = coherent state of many, extremely non-relativistic particles = CDM



[Wantz,Shellard '09]



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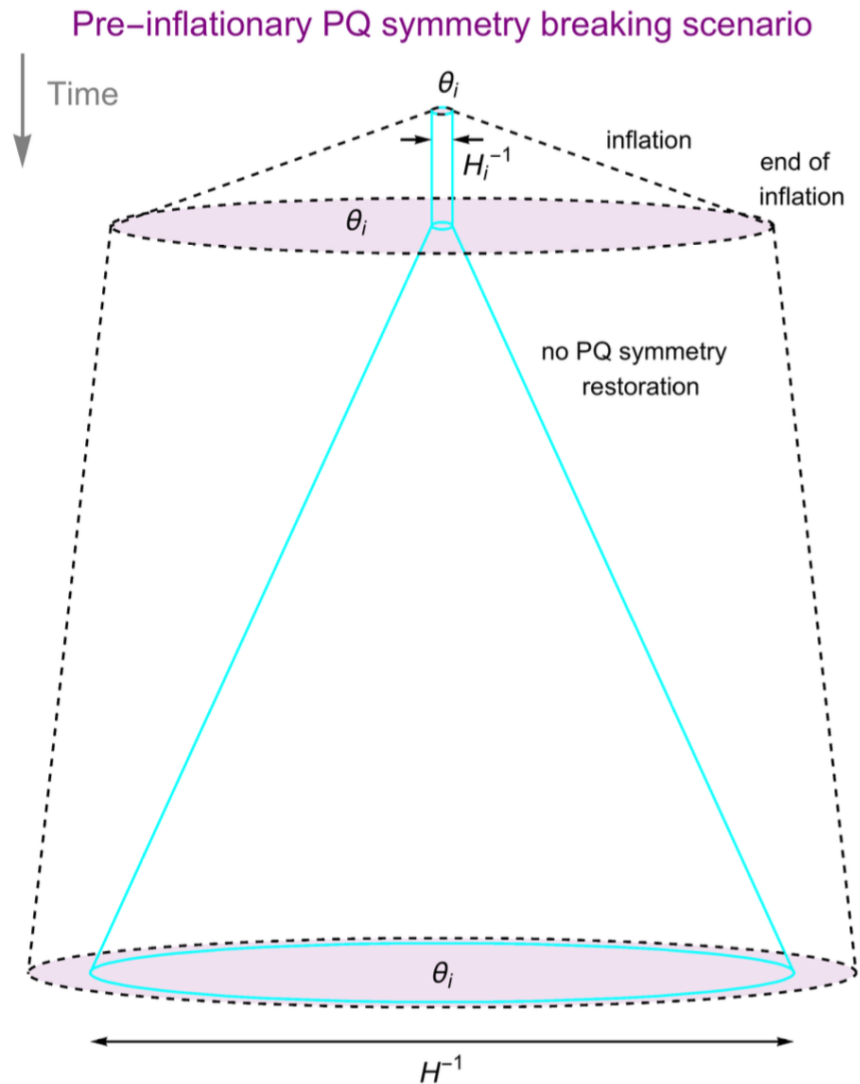
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- Axion CDM density depends on single initial angle during inflation and f_A



[Saikawa]



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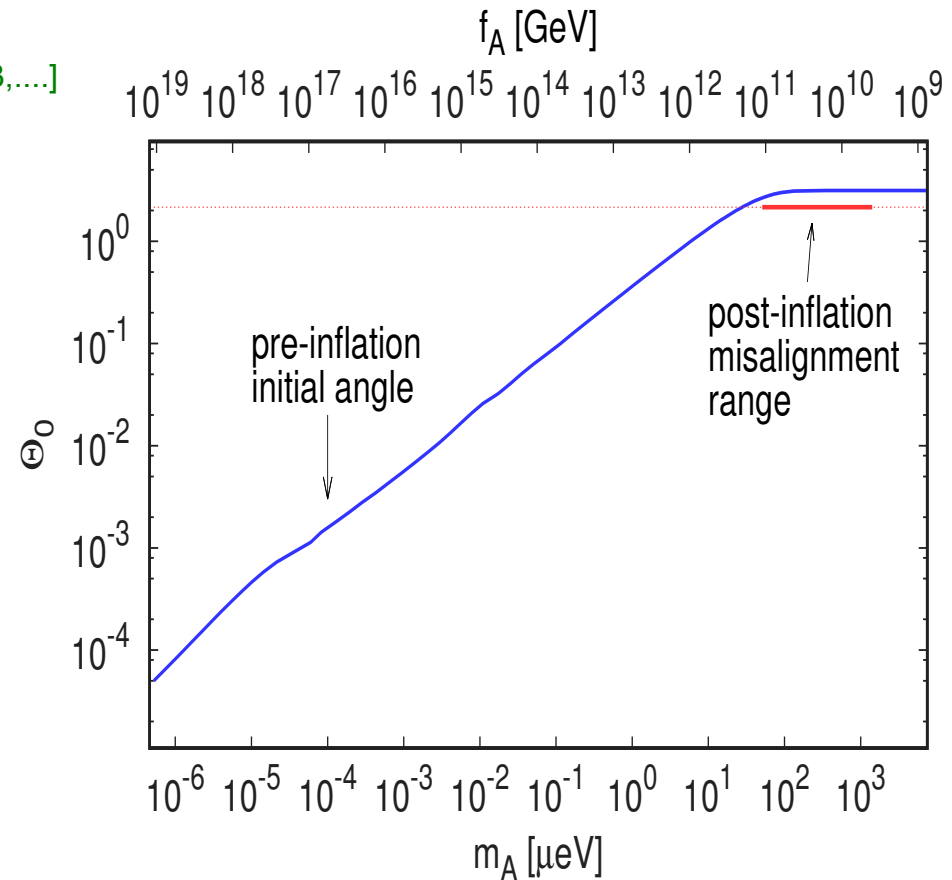
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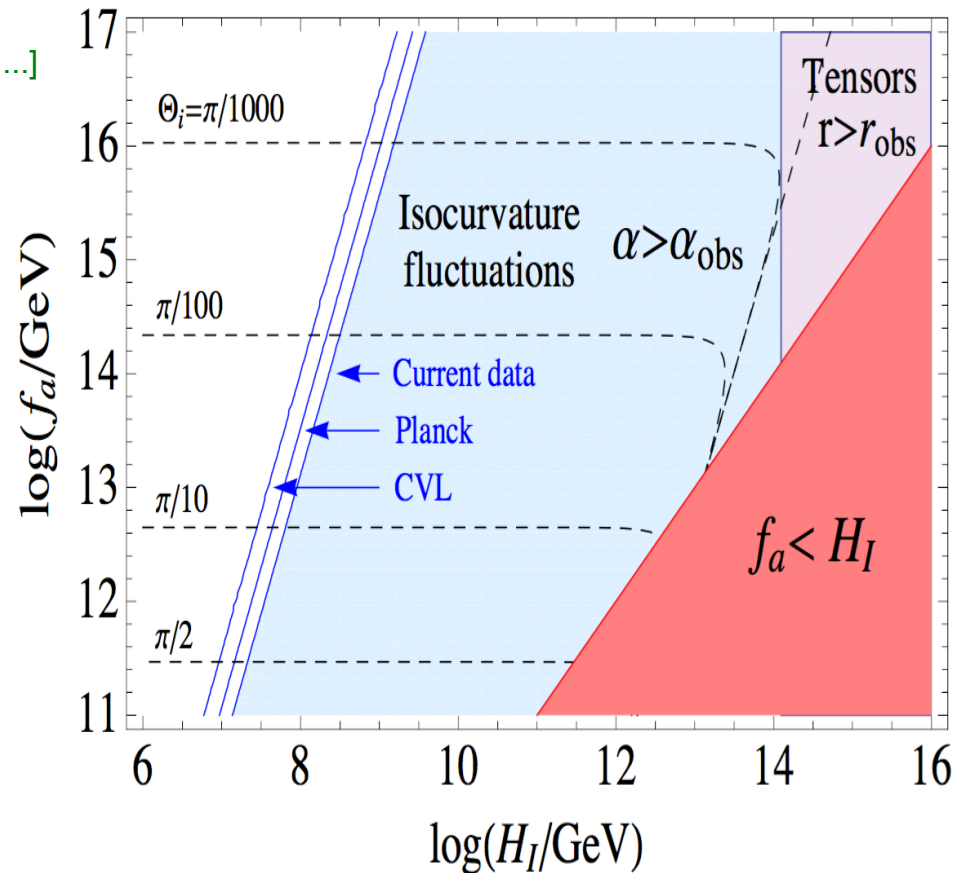
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- Axion CDM density depends on single initial angle during inflation and f_A
- Axion is present during inflation and creates isocurvature fluctuations which are not erased after inflation



[Hamann et al. '09]

[Wilczek,Turner '91; Beltran et al. 06; Hertzberg,Tegmark,Wilczek 08; Visinelli,Gondolo 09; Hamann et al. 09; Wantz,Shellard 09]



Axion Cold Dark Matter

> If Peccei-Quinn symmetry restored after inflation (post-inflationary PQ breaking scenario)

- Vacuum realignment contribution depends on spatially averaged initial misalignment angle and f_A

$$\Omega_{A,\text{real}} h^2 = (3.8 \pm 0.6) \times 10^{-3} \times \left(\frac{f_A}{10^{10} \text{ GeV}} \right)^{1.165}$$

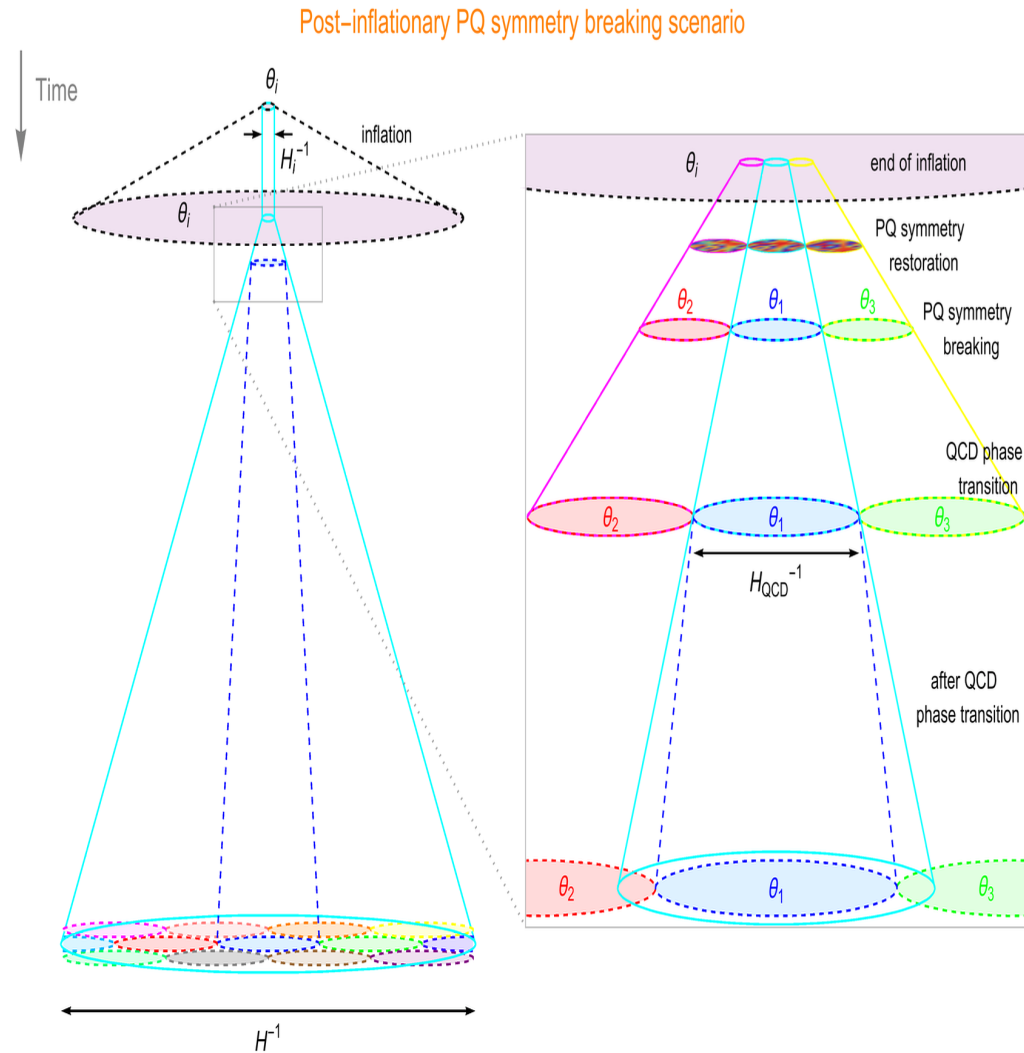
- Upper limit on f_A from requirement that realignment contribution should not exceed DM abundance gives lower limit on axion mass:

$$m_A > 28(2) \text{ } \mu\text{eV}$$

[Borsanyi et al. '16]

using lattice result on $\chi(0)$,

$$m_A = 57.0(7) \left(\frac{10^{11} \text{ GeV}}{f_A} \right) \mu\text{eV}$$



[Saikawa]

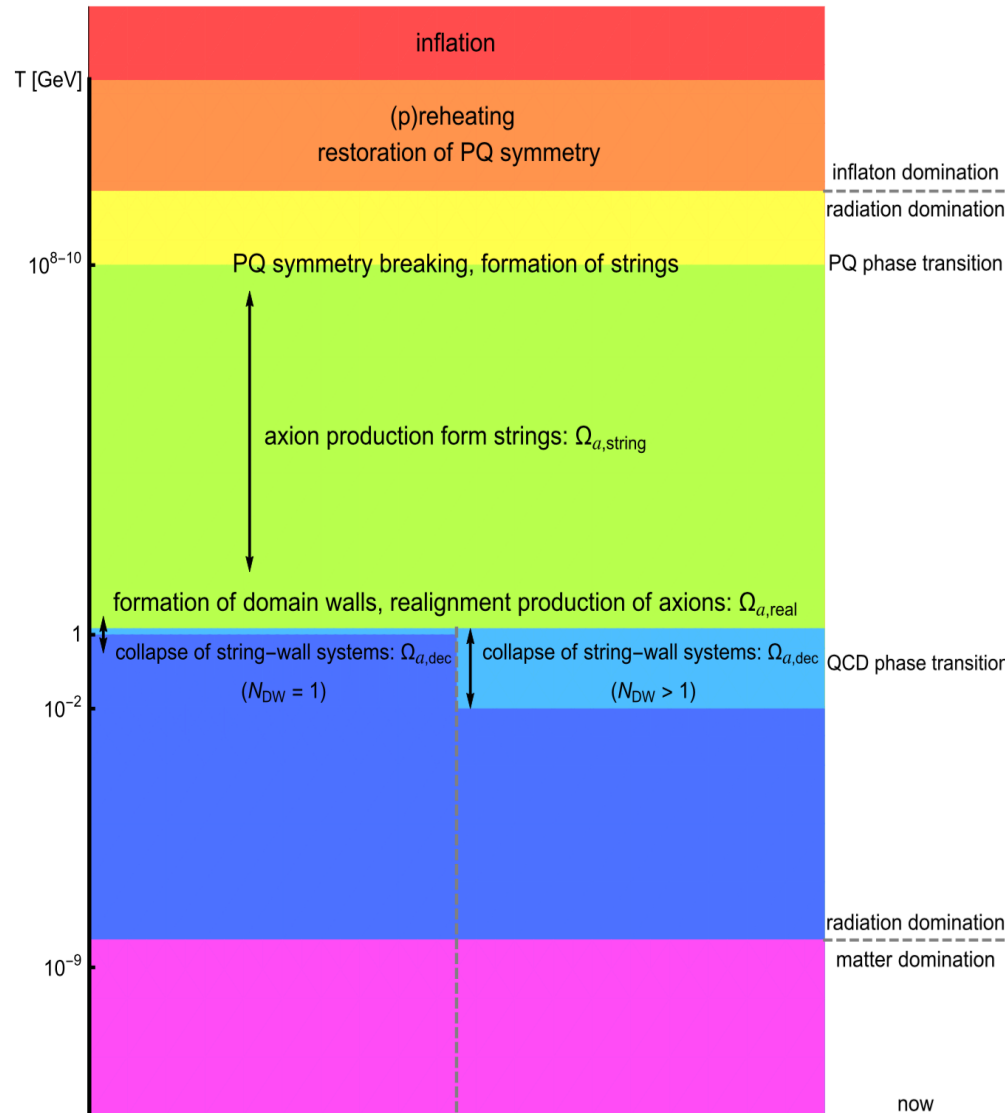


Axion Cold Dark Matter

- If Peccei-Quinn symmetry restored after inflation (post-inflationary PQ breaking scenario)

- Additional contributions arise from decay of topological defects (axion strings and axion domain walls)

$$\Omega_{A,\text{tot}} h^2 = \Omega_{A,\text{real}} h^2 + \Omega_{A,\text{string}} h^2 + \Omega_{A,\text{wall}} h^2$$



[Saikawa]



Axion Cold Dark Matter

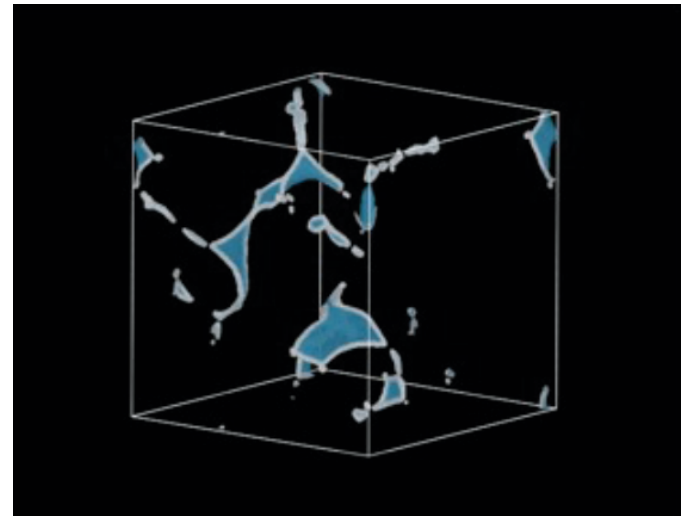
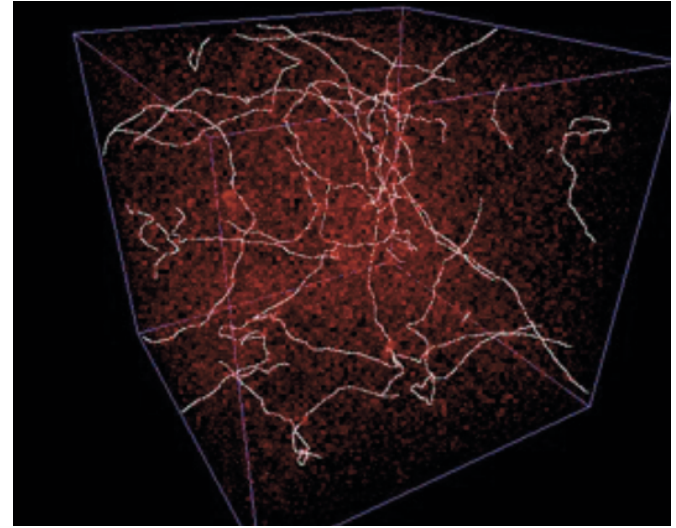
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- Latter determined by lattice simulations of cosmic string and wall networks

[Hiramatsu et al. 12; Kawasaki et al. 15]



[Hiramatsu et al. 12]

Axion Cold Dark Matter

➤ If Peccei-Quinn symmetry restored after inflation (post-inflationary PQ breaking scenario)

- For $N_{\text{DW}} = 1$, string-wall systems short lived, leading to

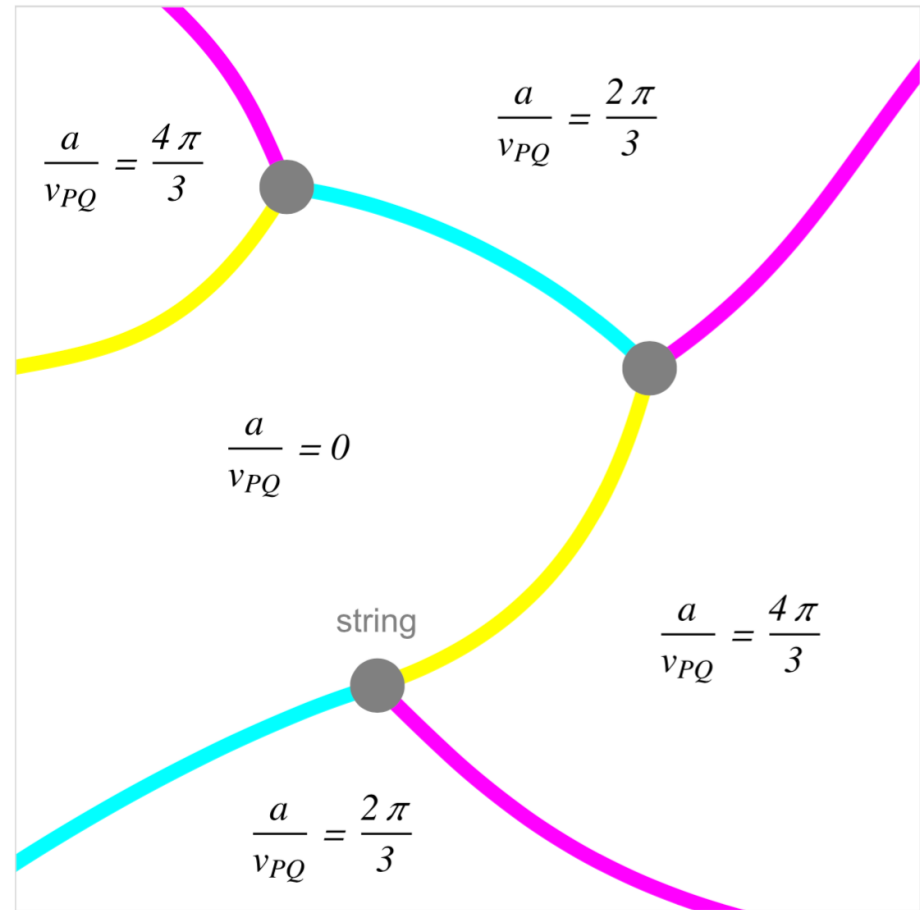
$$\Omega_{A,\text{tot}} h^2 \approx 1.6_{-0.7}^{+1.0} \times 10^{-2} \times \left(\frac{f_A}{10^{10} \text{ GeV}} \right)^{1.165}$$

CDM explained for

$$m_A \approx (50-200) \mu\text{eV}$$

- For $N_{\text{DW}} > 1$, string-wall systems absolutely stable, eventually overclosing Universe, unless PQ symmetry explicitly broken e.g. by Planck suppressed operators

$$\Delta V = g \sigma^N / M_{\text{P}}^{N-4}$$



[Saikawa]



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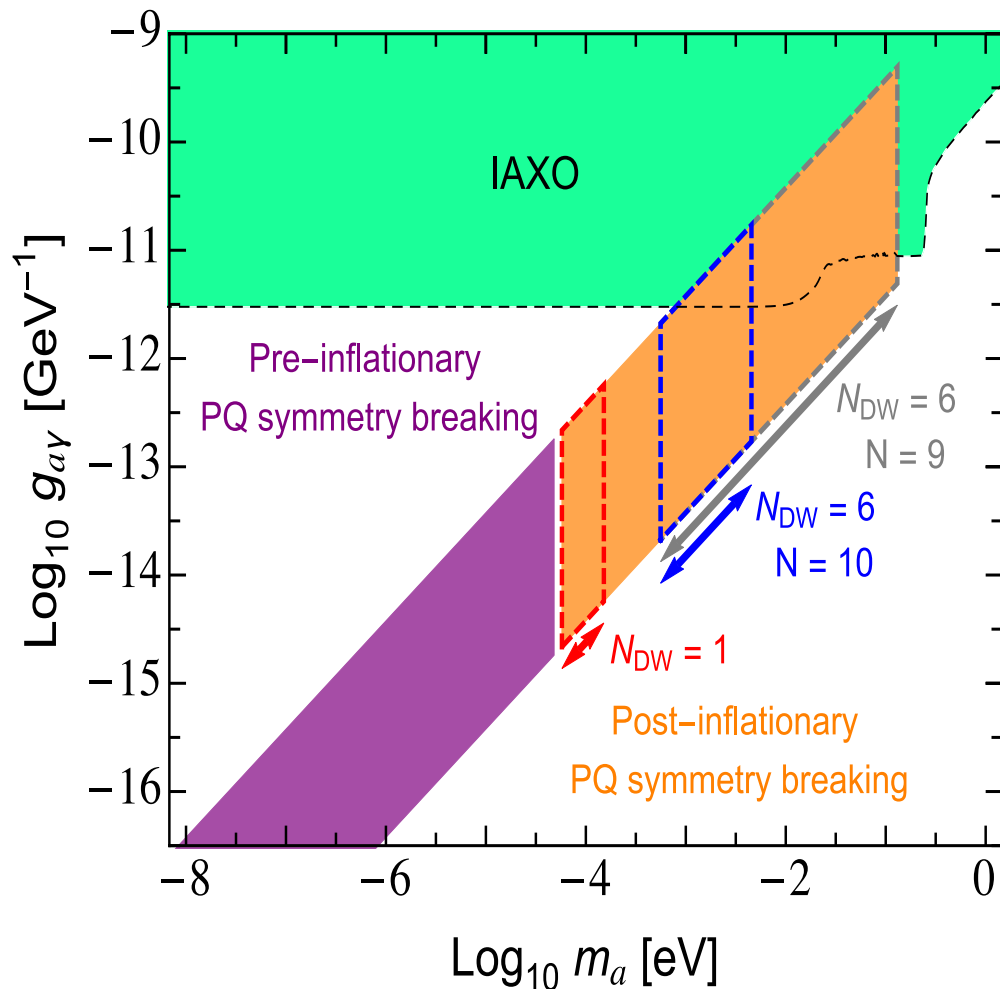
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[Saikawa]



Summary

- Lattice results on topological susceptibility: Vacuum realignment contribution to axion cold dark matter abundance can be predicted now with $O(10\%)$ accuracy
- Lower limit on axion mass in post-inflationary PQ breaking scenario:

$$m_A > 28(2) \mu\text{eV}$$

- Lot of search space for axion cold dark matter:

