Axion Cold Dark Matter.

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Axion Dark Matter
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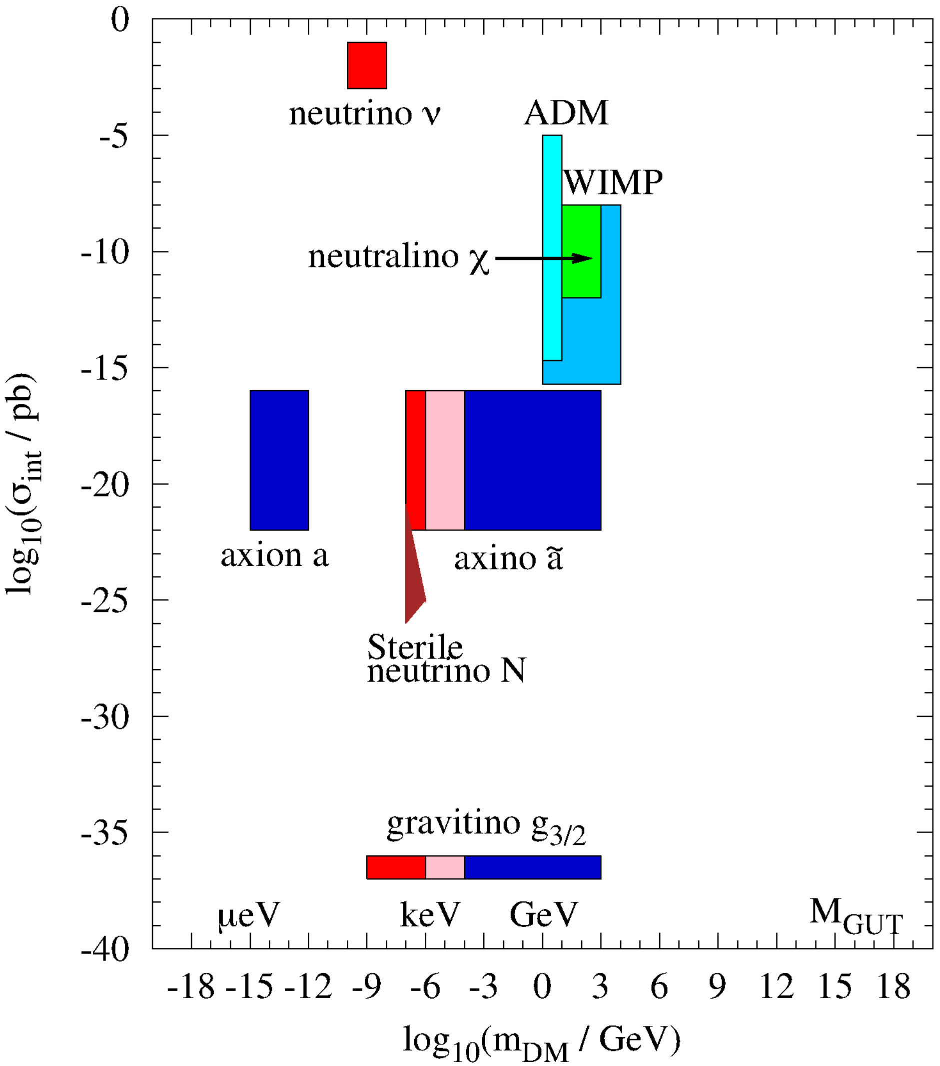
Introduction

- Many cold dark matter candidates



[Baer,Choi,Kim,Roszkowski 10]

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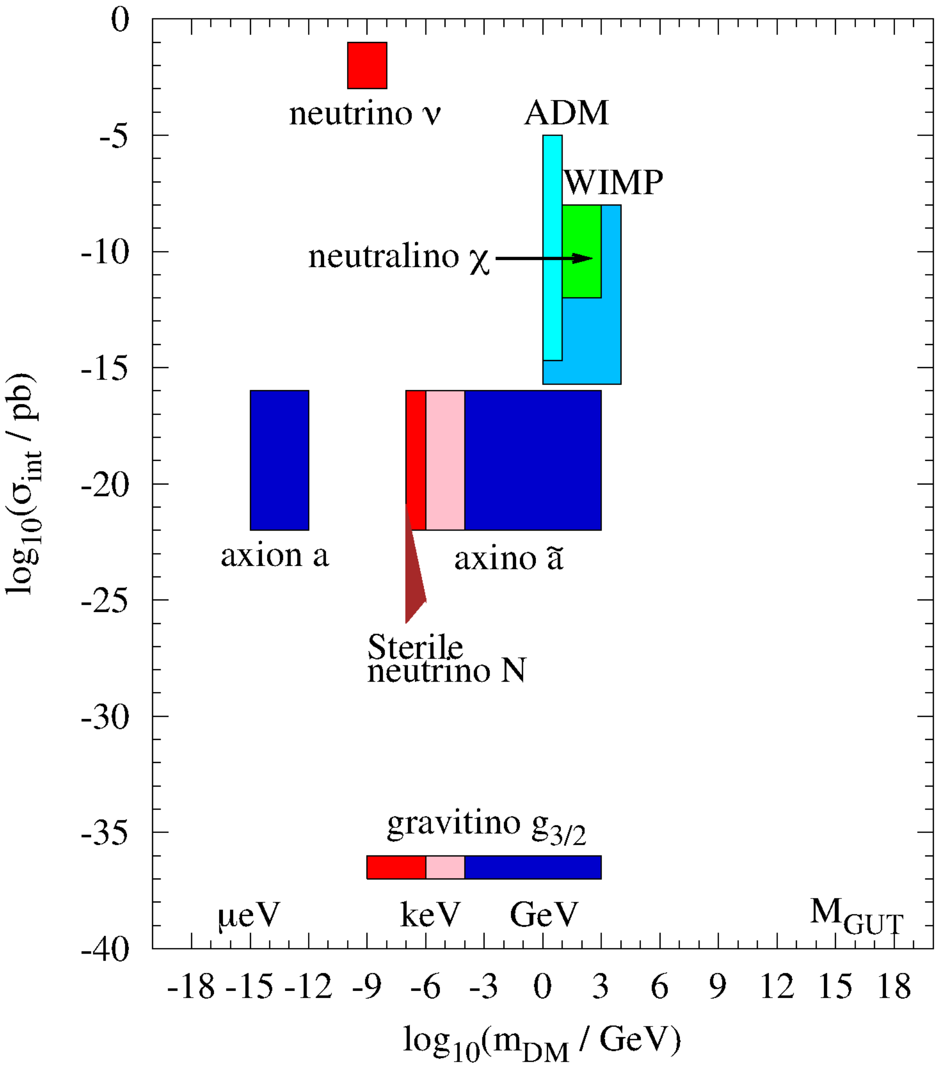
- Best motivated are those that occur in UV completions of the Standard Model which solve also other problems



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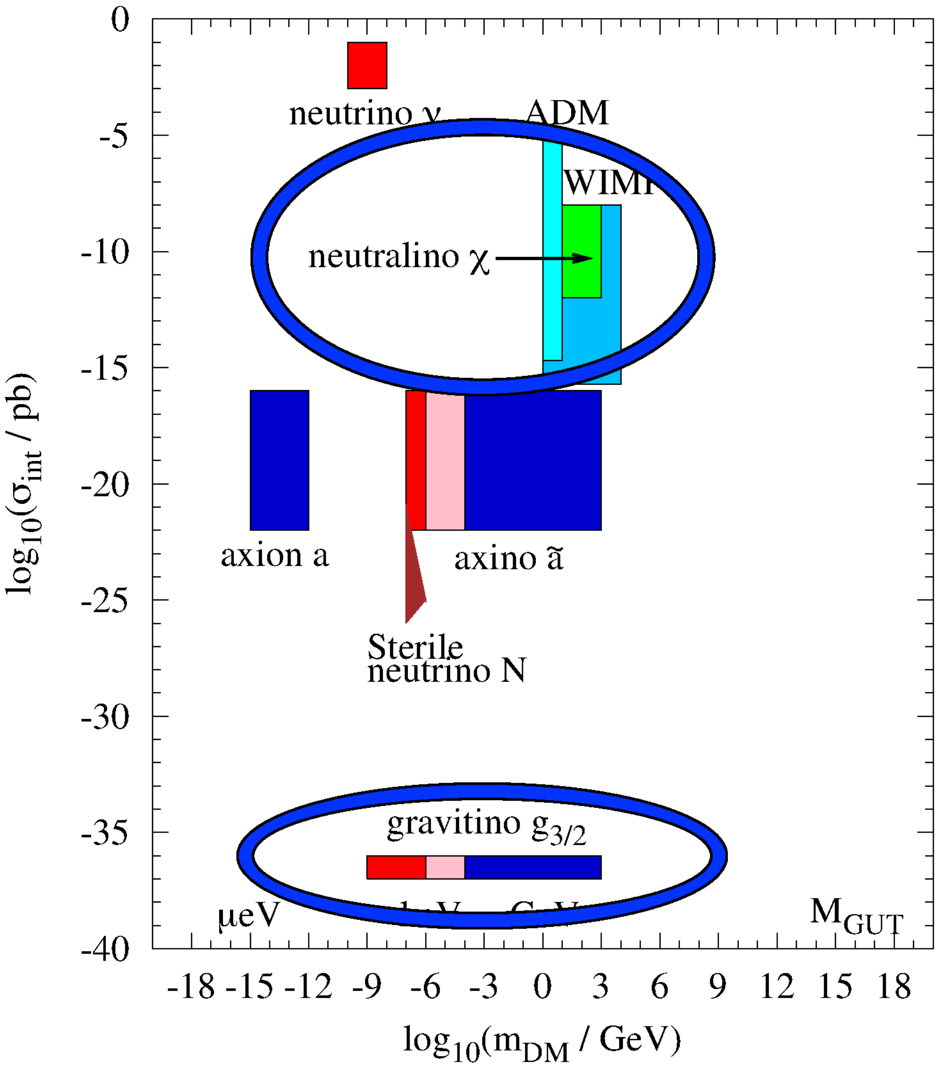
  - Hierarchy Problem



[Baer,Choi,Kim,Roszkowski 10]

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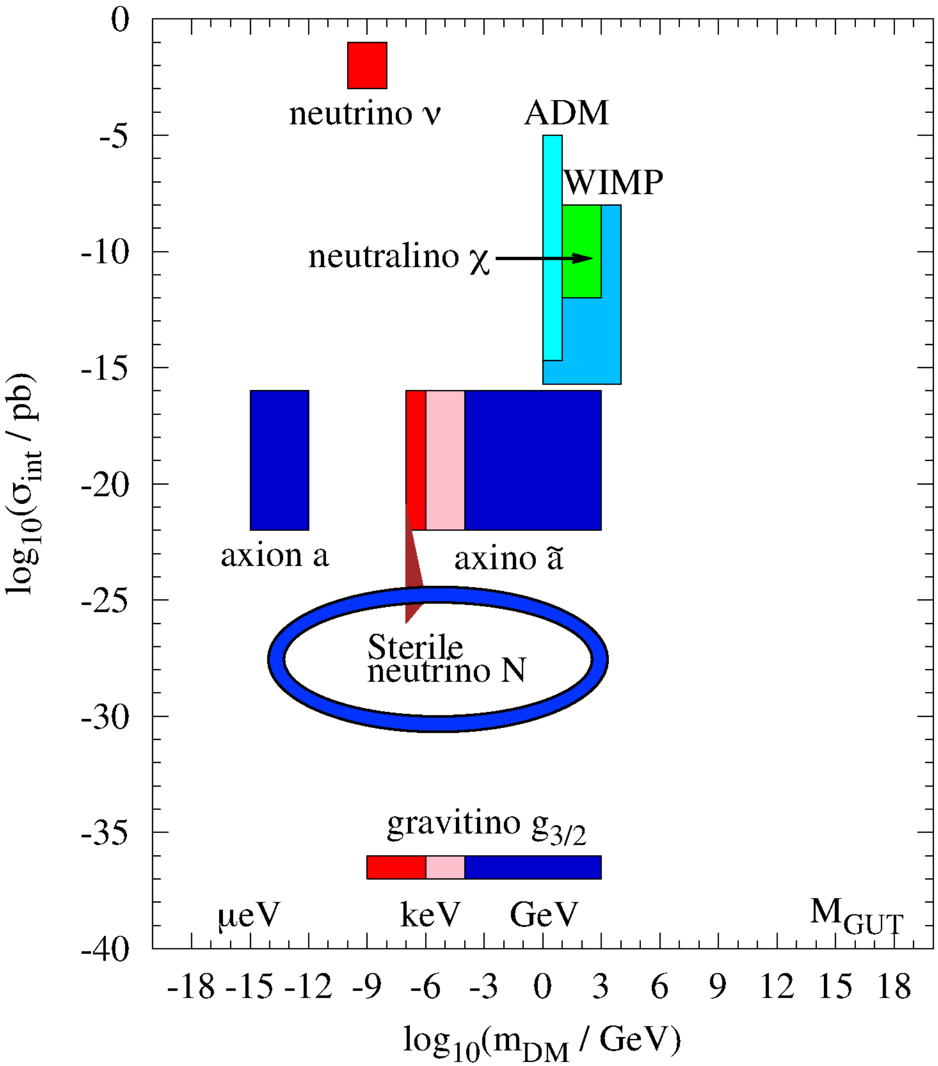
  - Neutrino Masses and Mixing
  - Baryon Asymmetry

- MSSM: Neutralino or Gravitino
- nuMSM: Lightest sterile neutrino



[Baer,Choi,Kim,Roszkowski 10]

Introduction

- Many cold dark matter candidates
- Best motivated are those that occur in UV completions of the Standard Model which solve also other problems
  - Hierarchy Problem
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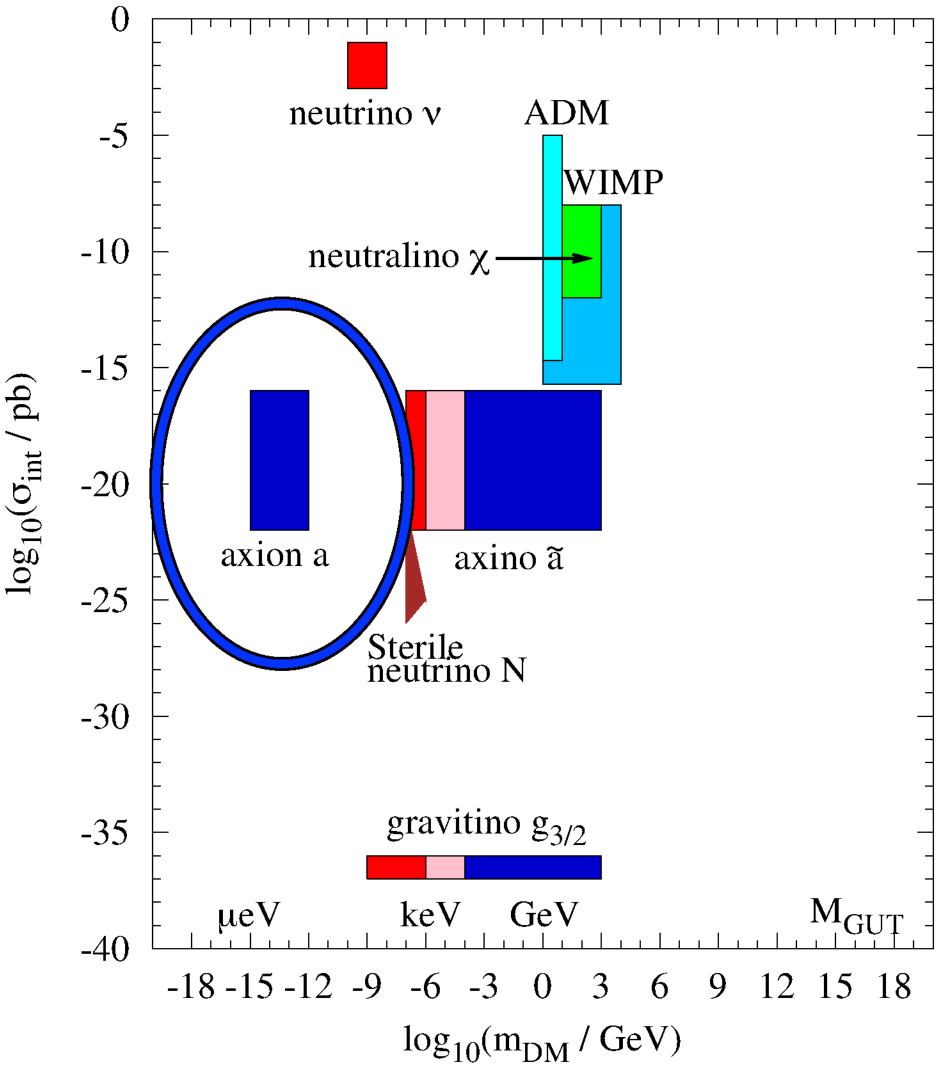
  - Strong CP Problem
- MSSM: Neutralino or Gravitino
- nuMSM: Lightest sterile neutrino
- PQSM: Axion



[Baer,Choi,Kim,Roszkowski 10]

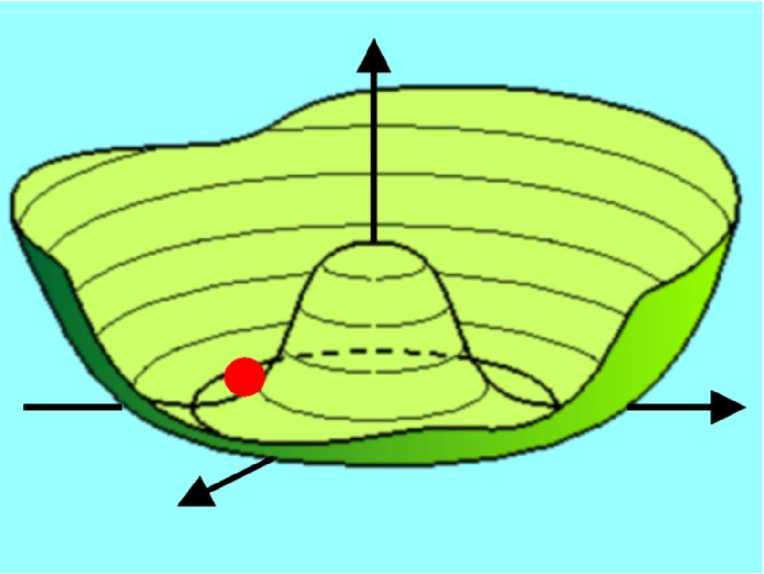
PQSM: Peccei-Quinn Extensions of the Standard Model

- A singlet complex scalar field σ featuring a global $U(1)_{\rm PQ}$ symmetry is added
- Symmetry is broken by vev $\langle\sigma\rangle = v_{\rm PQ}/\sqrt{2}$

$$\sigma(x) = \frac{1}{2}\left(v_{\rm PQ} + \rho(x)\right) {\rm e}^{{\rm i}A(x)/v_{\rm PQ}}$$

  - Excitation of modulus: $m_\rho \sim v_{\rm PQ}$
  - Excitation of angle: NGB $m_A \ll v_{\rm PQ}$
- PQ charges of quarks (SM or extra) are such that $U(1)_{\rm PQ} \times SU(3)_C \times SU(3)_C$ has chiral anomaly: NGB is called axion

[Peccei,Quinn 77; Weinberg 78; Wilczek 78]



[Raffelt]

- No strong CP problem, since axion field acts as x-dependent theta parameter

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi}\frac{A(x)}{f_A} G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu}; \qquad f_A = v_{\rm PQ}/N_{\rm DW}; \qquad N_{\rm DW} = \#\ \text{quarks with PQ charge}$$

QCD dynamics: $\langle A(x)\rangle = 0$

[Kim 79;Shifman,Vainshtein,Zakharov 80;Zhitnitsky 80;Dine,Fischler,Srednicki 81;...]

Axion Potential at High Temperatures

- Cosmological evolution of axion angular field $\theta = A/f_A$ determined by

$$\ddot{\theta} + 3H(T(t))\,\dot{\theta} - \frac{\nabla^2}{a^2(t)}\theta = -\frac{1}{f_A^2}\frac{\partial}{\partial\theta}V(\theta, T(t))$$

- In absence of explicit Peccei-Quinn symmetry breaking: only source of non-trivial axion potential, $\partial V(\theta,T)/\partial\theta \neq 0$, is topological charge density in QCD,

$$\mathcal{L} \supset -\frac{\alpha_s}{8\pi}\theta(x)G^a_{\mu\nu}(x)\tilde{G}^{a,\mu\nu}(x) = -\theta(x)q(x)$$

- Axion potential equals free energy density in QCD as function of θ,

$$V(\theta, T) = -\frac{1}{\mathcal{V}_4}\ln\left[\frac{Z(\theta,T)}{Z(0,T)}\right]$$

Axion Potential at High Temperatures

- QCD partition function can be written in terms of Fourier series of partition functions of sectors with fixed topological charge:

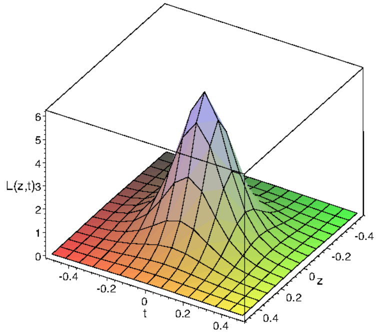
$$Z(\theta,T) = \sum_{Q=-\infty}^{+\infty} e^{i\theta Q} Z_Q(T); \qquad Q = \int d^4x\, q(x)$$

$$Z_Q(T) = \int_Q [dA][d\psi_q][d\bar{\psi}_q] \exp\left\{-S\left([A,\psi_q,\bar{\psi}_q];T\right)\right\}$$

- Axion mass, $m_A^2 f_A^2 \equiv \frac{\partial^2 V}{\partial\theta^2}\Big|_{\theta=0}$, determined by topological susceptibility:

$$m_A^2 f_A^2 = -\frac{1}{\mathcal{V}_4}\frac{1}{Z}\frac{\partial^2 Z}{\partial\theta^2}\Big|_{\theta=0} = \frac{1}{\mathcal{V}_4}\frac{1}{Z}\sum_{Q=-\infty}^{+\infty} Q^2 Z_Q = \frac{\langle Q^2\rangle|_{\theta=0}}{\mathcal{V}_4} = \int d^4x \langle q(x)q(0)\rangle|_{\theta=0} \equiv \chi$$

- Need non-perturbative methods to calculate it:
  - Semiclassical expansion of Euclidean path integral about minima of Euclidean action with fixed topological charge (instantons)
  - Monte Carlo evaluations of lattice-regularized Euclidean path integral



Axion Potential at High Temperatures

- At very large temperatures, far above the quark hadron phase transition, semiclassical expansion expected to be reliable
- In this case, partition function can be described by a dilute gas of instantons and anti-instantons (minima of Euclidean action with $Q = \pm 1$)

$$Z(\theta,T) \simeq \sum_{n,\bar{n}=0}^{\infty} \frac{1}{n!\bar{n}!} \left(Z_1(T)\mathrm{e}^{\mathrm{i}\theta}\right)^n \left(Z_1(T)\mathrm{e}^{-\mathrm{i}\theta}\right)^{\bar{n}} = \exp\left\{2\, Z_1(T)\cos\theta\right\}$$

- Potential has cosine shape: [Callan,Dashen,Gross `78]

$$V(\theta,T) = -\frac{1}{\mathcal{V}_4}\ln\left[\frac{Z(\theta,T)}{Z(0,T)}\right] \simeq \frac{2\,Z_1(T)}{\mathcal{V}_4}\left(1-\cos\theta\right)$$

- All temperature dependence resides in topological susceptibility in terms of contribution of single instanton to path integral:

$$\chi(T) \simeq \frac{2\,Z_1(T)}{\mathcal{V}_4} = 2\int_0^\infty d\rho\, D(\rho)\, G(\pi\rho T)$$

Axion Potential at High Temperatures

- Zero temperature instanton size distribution: ['t Hooft `76]

$$D(\rho) = \frac{d}{\rho^5}\left(\frac{2\pi}{\alpha_s(\mu_r)}\right)^{2\,N_c} \exp\left(-\frac{2\pi}{\alpha_s(\mu_r)}\right)(\rho\,\mu_r)^{\beta_0}\prod_{i=1}^{n_f}(\rho\,m_i(\mu_r))$$

- At large temperature, color-electric Debye screening prohibits existence of large-scale coherent fields in plasma, leading to [Pisarski,Yaffe `80]

$$G(\pi\rho T) = \exp\left\{-\frac{6+n_f}{3}(\pi\rho T)^2 - 18\left[1-\frac{n_f}{9}\right]A(\pi\rho T)\right\}$$

$$A(\pi\rho T) = -\frac{1}{12}\ln\left[1+(\pi\rho T)^2/3\right] + \alpha\left[1+\gamma(\pi\rho T)^{-2/3}\right]^{-8}$$

- Cuts off integration at $\rho \sim 1/(\pi T)$ and ensures validity of dilute instanton gas approximation (DIGA) at large temperatures, at which $\alpha_s(\pi T) \ll 1$
- Axion mass predicted to decrease power-like with temperature in DIGA:

$$m_A^2(T) f_A^2 = \chi(T) \simeq 2\int_0^\infty d\rho\, D(\rho)\, G(\pi\rho T) \propto T^{-(7+\frac{1}{3}n_f)}$$

Axion Potential at High Temperatures

> How good is DIGA quantitatively? Check with lattice QCD!

> Topological susceptibility notoriously difficult to calculate on lattice

1. Large cutoff effects when exploiting action with non-chiral quarks to calculate topological observables
2. Tiny topological susceptibility needs extremely long simulation threads to observe enough changes of topological sectors

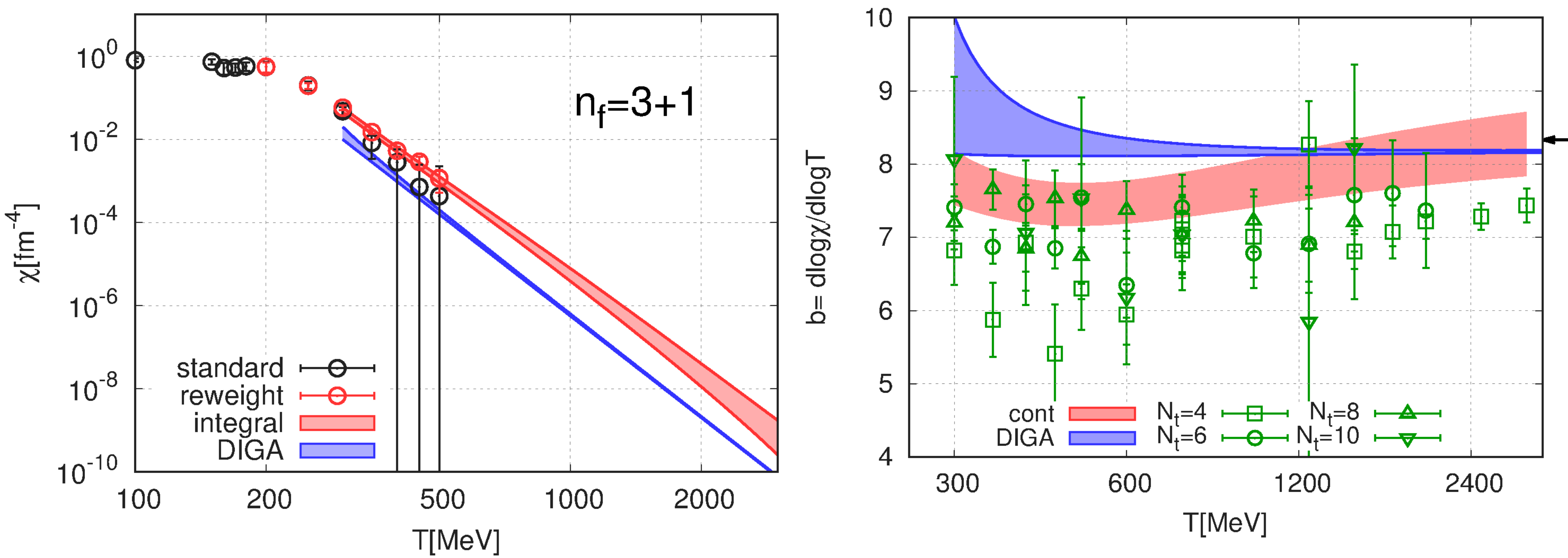
> Solutions of these problems: [Borsanyi et al. `16]

1. Eigenvalue reweighting technique: Substitute topology related eigenvalues of non-chiral quark Dirac operator with its corresponding eigenvalues in continuum
2. Fixed sector integral technique: Measure logarithmic differential of topological susceptibility which is related to quantities to be measured in fixed topological sectors. Then integrate.

Axion Potential at High Temperatures

- Results:

[Borsanyi et al. `16]



- Temperature slope remarkably close to DIGA prediction
- DIGA underestimates topological susceptibility by overall normalization „K factor“ of order ten (should be improved in two-loop DIGA)

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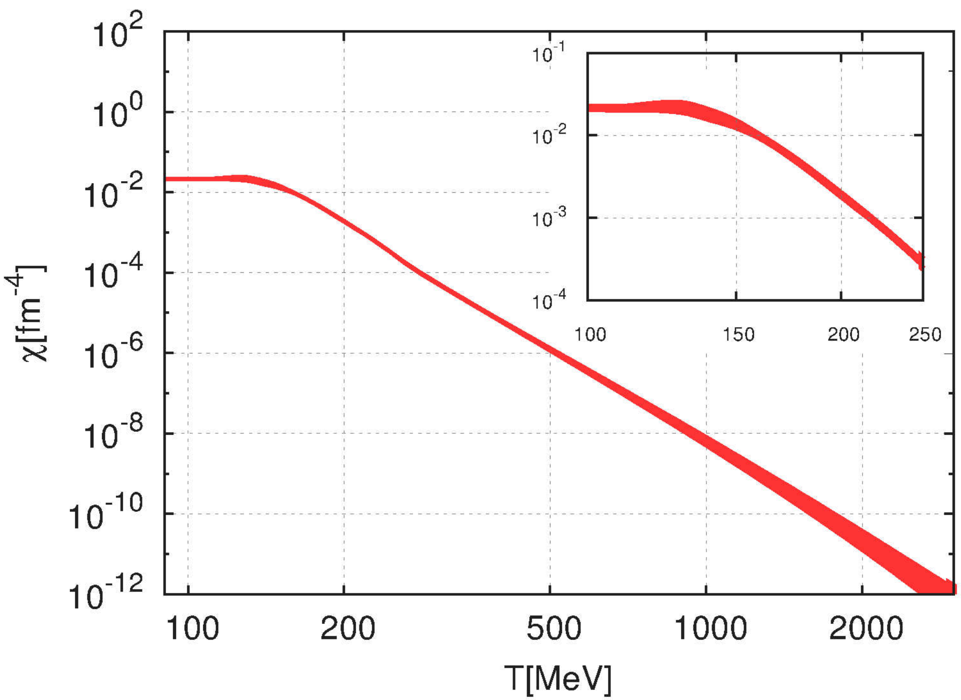

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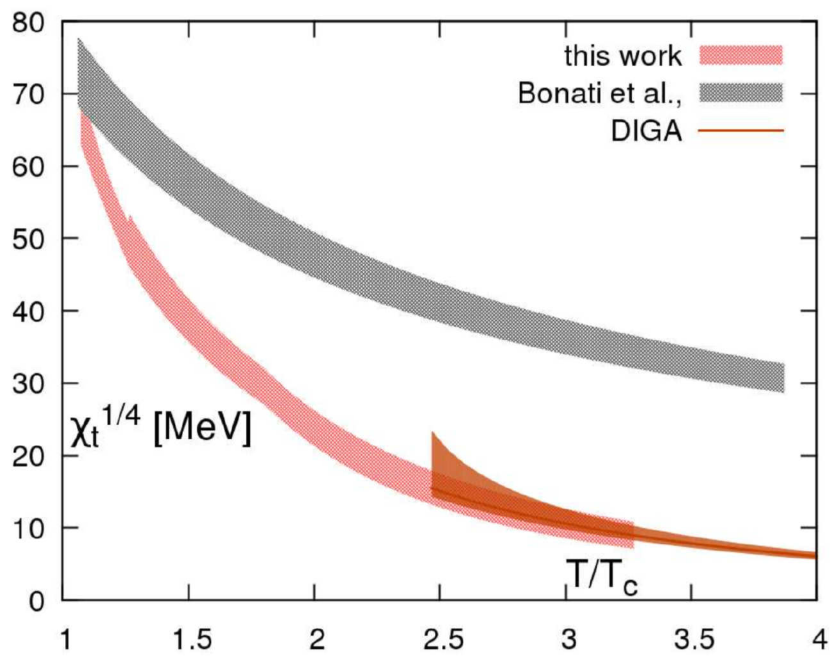


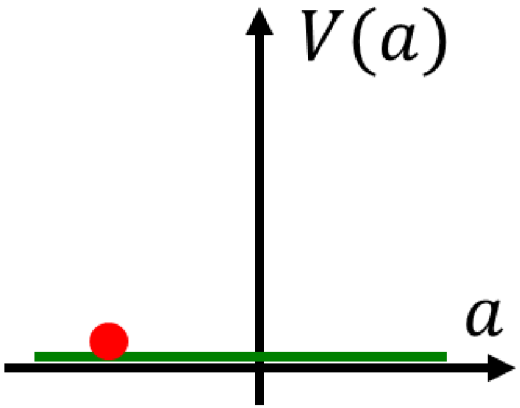
[Petreczky et al. `16]

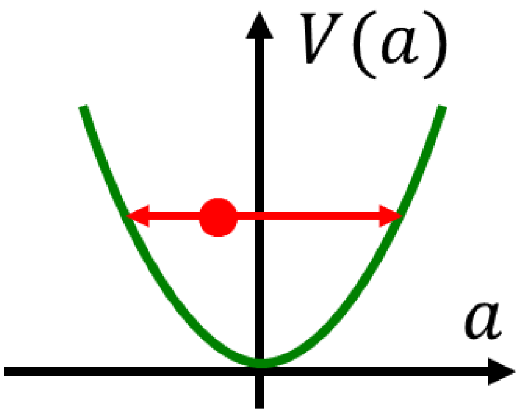
Axion Cold Dark Matter

> DM from vacuum realignment:

[Preskill,Wise,Wilczek 83; Abbott,Sikivie 83; Dine,Fischler 83,....]

- In early universe, axion frozen at random initial value
- Later, field feels pull of mass towards zero and oscillates around it





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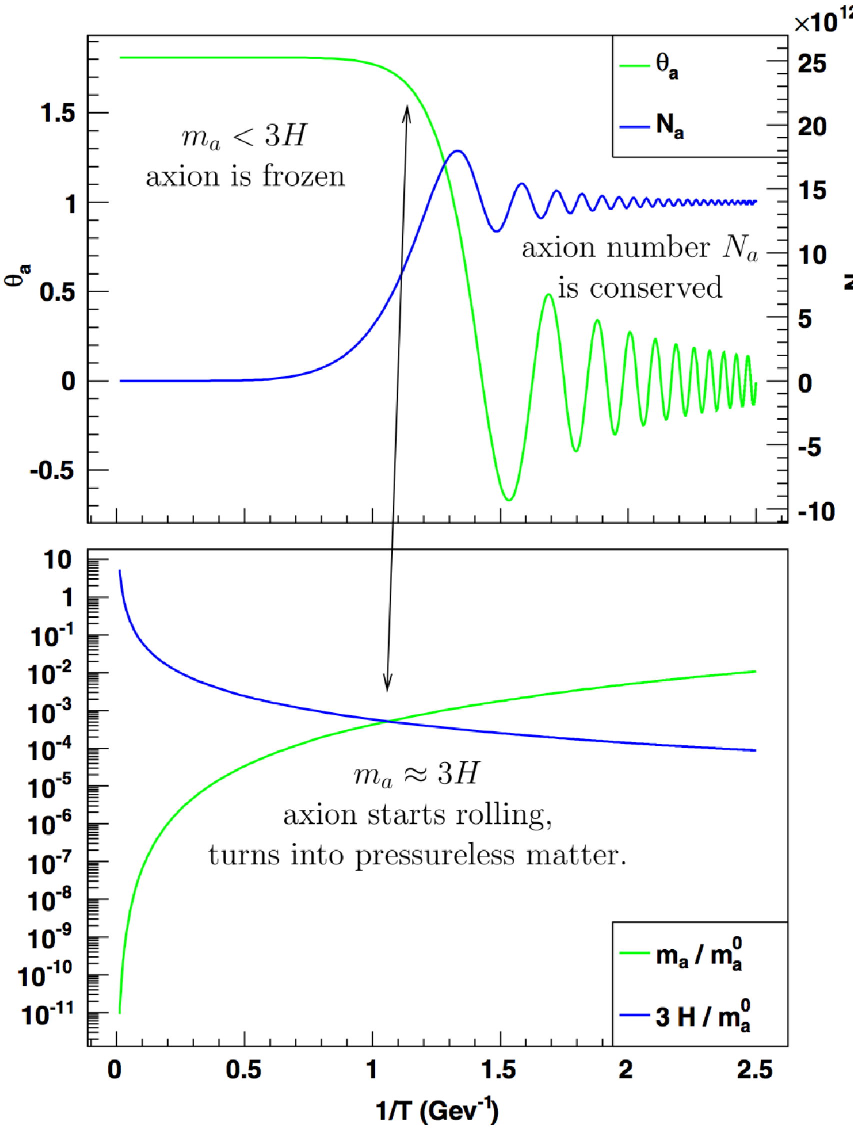

[Wantz,Shellard `09]

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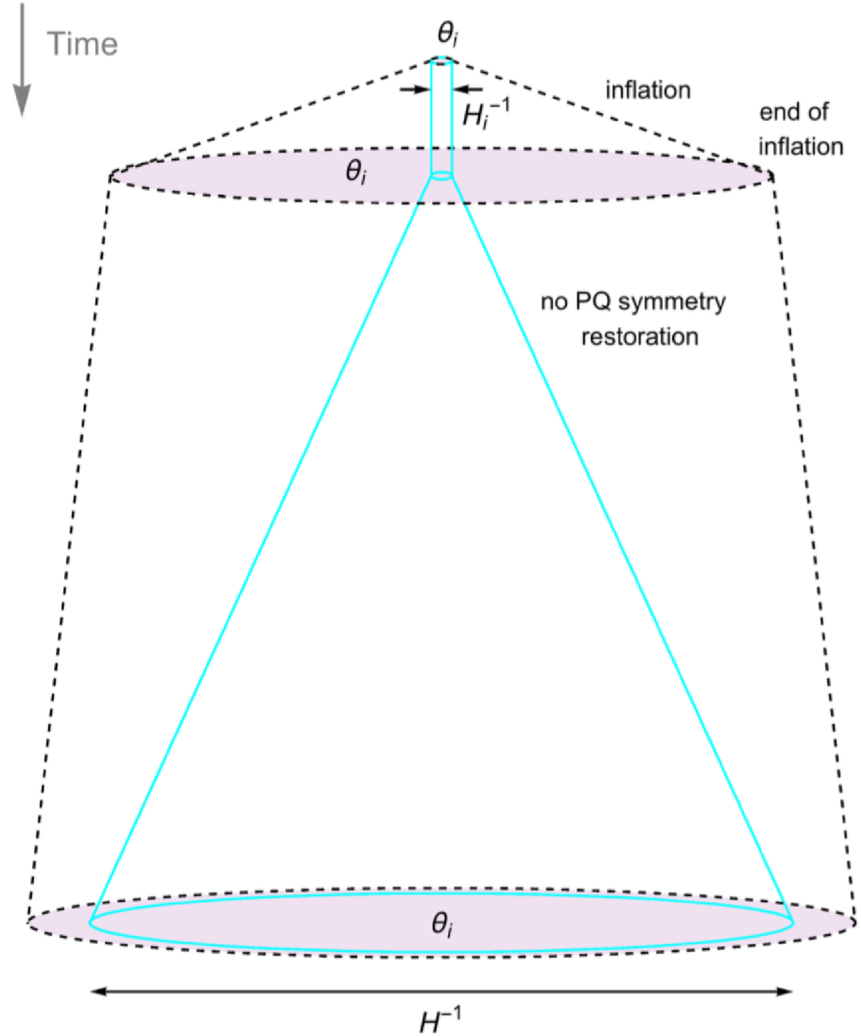
- Axion CDM density depends on single initial angle during inflation and f_A

Pre–inflationary PQ symmetry breaking scenario



[Saikawa]

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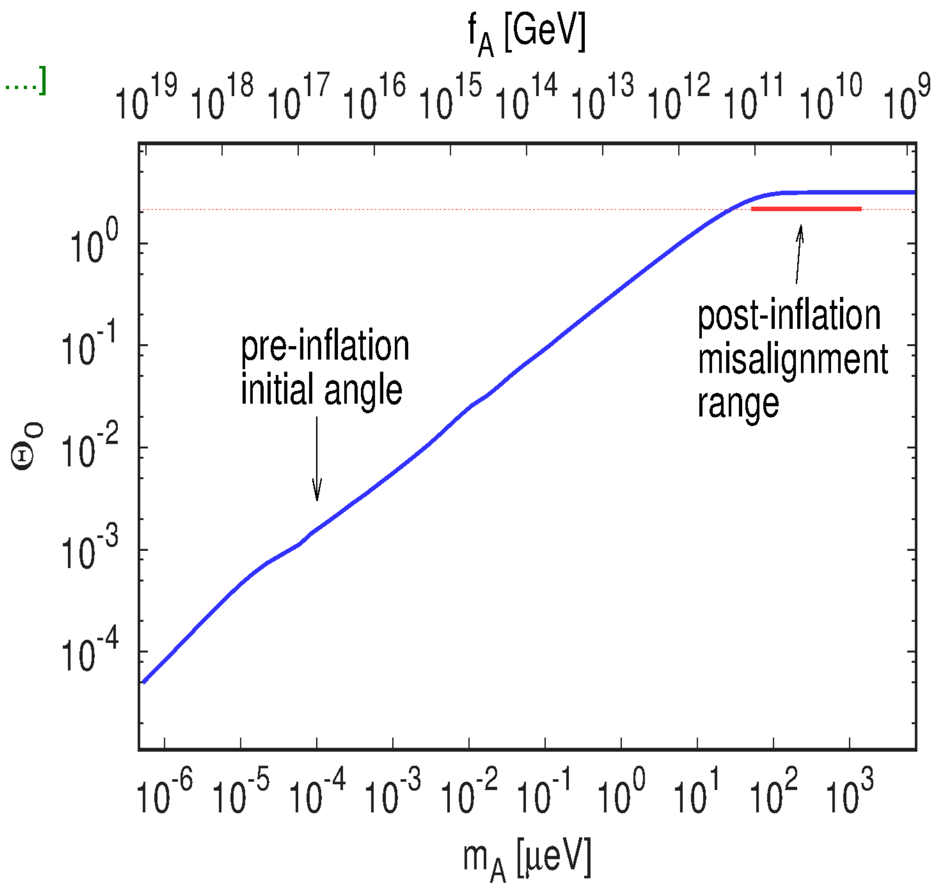

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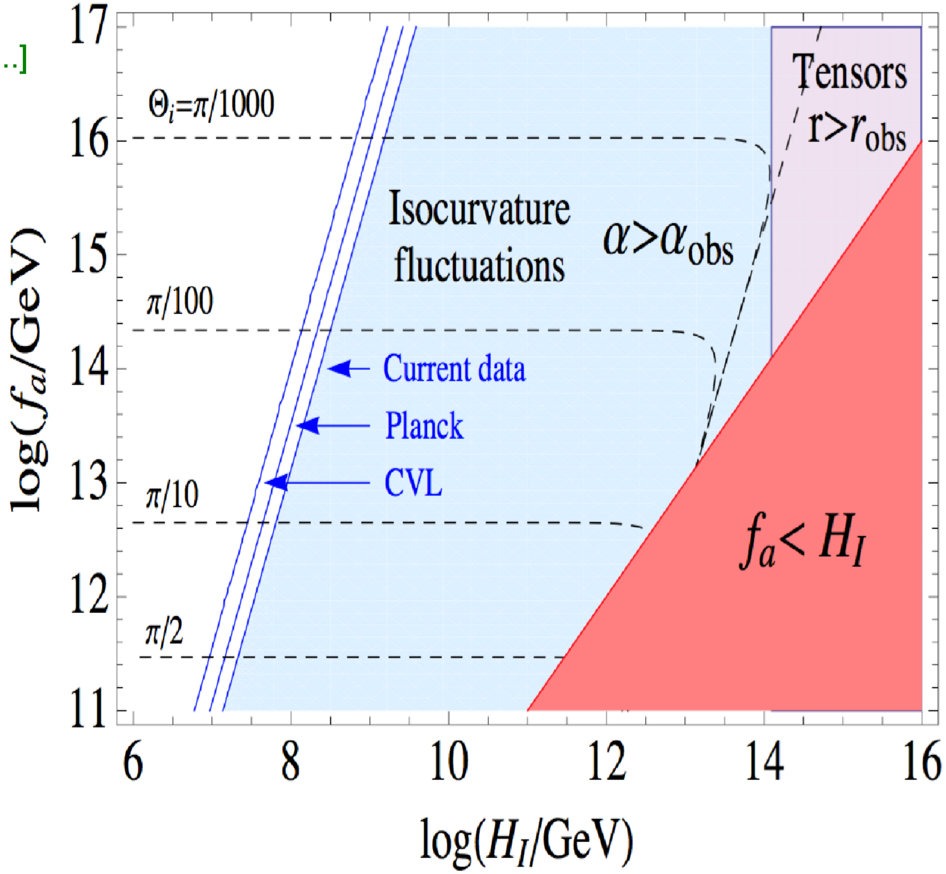
  - Axion is present during inflation and creates isocurvature fluctuations which are not erased after inflation



[Hamann et al. `09]

[Wilczek,Turner `91; Beltran et al. 06; Hertzberg,Tegmark,Wilczek 08; Visinelli,Gondolo 09; Hamann et al. 09; Wantz,Shellard 09]

Axion Cold Dark Matter

- If Peccei-Quinn symmetry restored after inflation (post-inflationary PQ breaking scenario)
  - Vacuum realignment contribution depends on spatially averaged initial misalignment angle and f_A

$$\Omega_{A,\mathrm{real}} h^2 = (3.8 \pm 0.6) \times 10^{-3} \times \left(\frac{f_A}{10^{10}\,\mathrm{GeV}}\right)^{1.165}$$

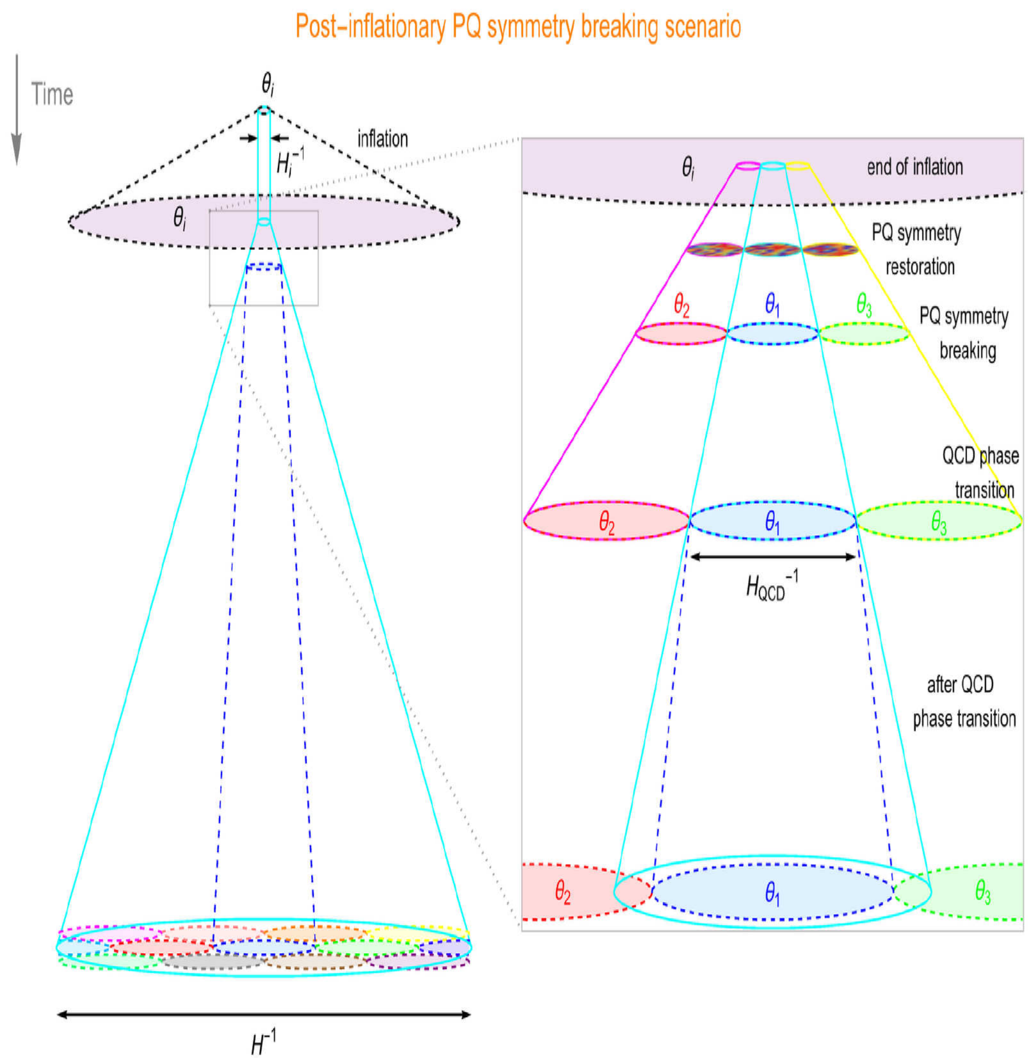
  - Upper limit on f_A from requirement that realignment contribution should not exceed DM abundance gives lower limit on axion mass:

$$m_A > 28(2)\ \mu\mathrm{eV}$$

[Borsanyi et al. `16]

using lattice result on $\chi(0)$,

$$m_A = 57.0(7) \left(\frac{10^{11}\mathrm{GeV}}{f_A}\right) \mu\mathrm{eV}$$

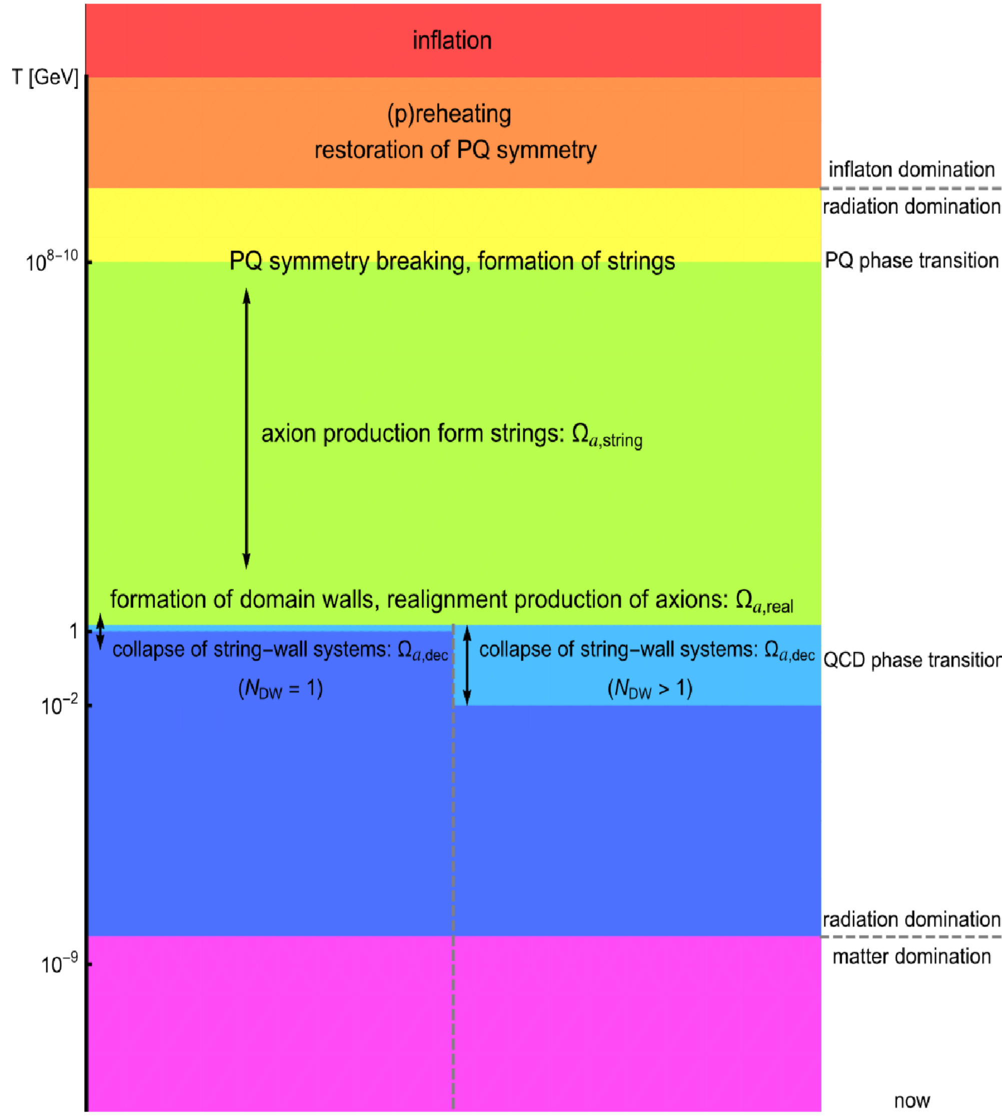


[Saikawa]

Axion Cold Dark Matter

- If Peccei-Quinn symmetry restored after inflation (post-inflationary PQ breaking scenario)
  - Additional contributions arise from decay of topological defects (axion strings and axion domain walls)

$$\Omega_{A,\text{tot}}h^2 = \Omega_{A,\text{real}}h^2 + \Omega_{A,\text{string}}h^2 + \Omega_{A,\text{wall}}h^2$$



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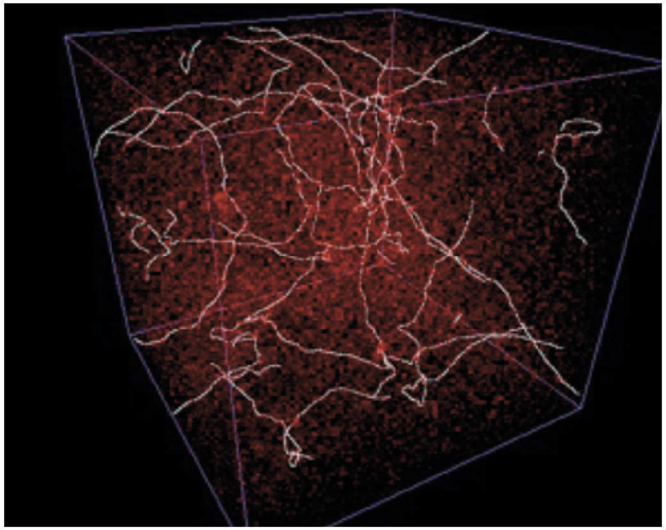
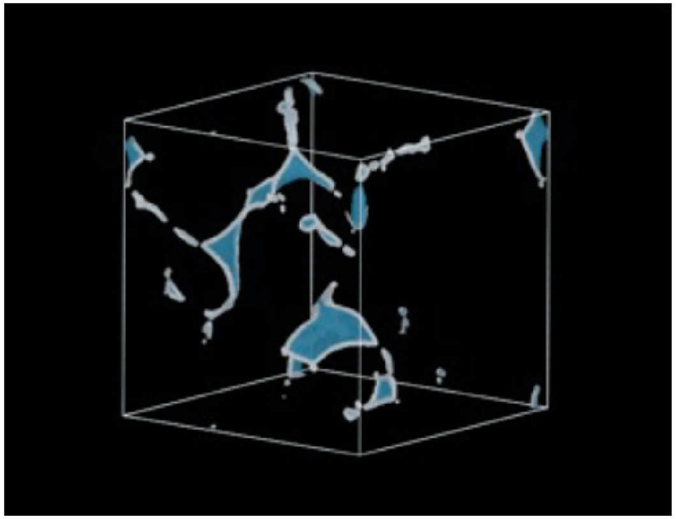
  - Latter determined by lattice simulations of cosmic string and wall networks

[Hiramatsu et al. 12; Kawasaki et al. 15]





[Hiramatsu et al. 12]

Axion Cold Dark Matter

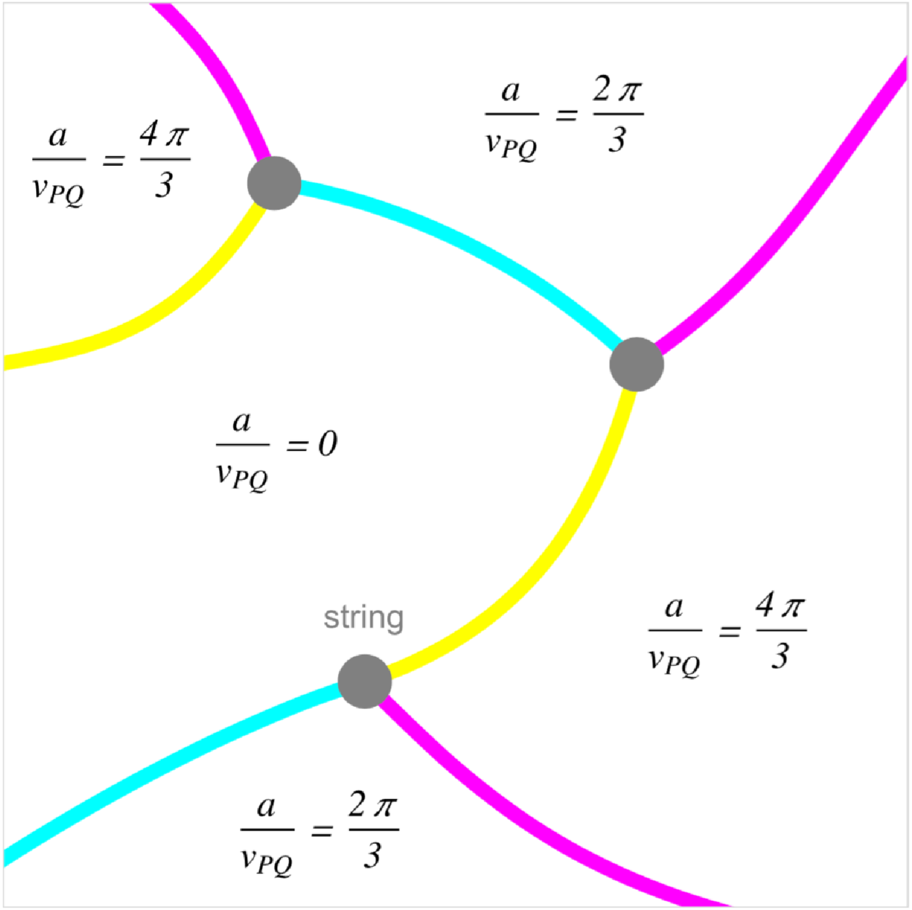
- If Peccei-Quinn symmetry restored after inflation (post-inflationary PQ breaking scenario)
  - For $N_{\rm DW} = 1$, string-wall systems short lived, leading to

$$\Omega_{A,{\rm tot}} h^2 \approx 1.6^{+1.0}_{-0.7} \times 10^{-2} \times \left(\frac{f_A}{10^{10}\,{\rm GeV}} \right)^{1.165}$$

    CDM explained for

$$m_A \approx (50\text{–}200)\,\mu{\rm eV}$$

  - For $N_{\rm DW} > 1$, string-wall systems absolutely stable, eventually overclosing Universe, unless PQ symmetry explicitely broken e.g. by Planck suppressed operators $\Delta V = g\sigma^N / M_{\rm P}^{N-4}$



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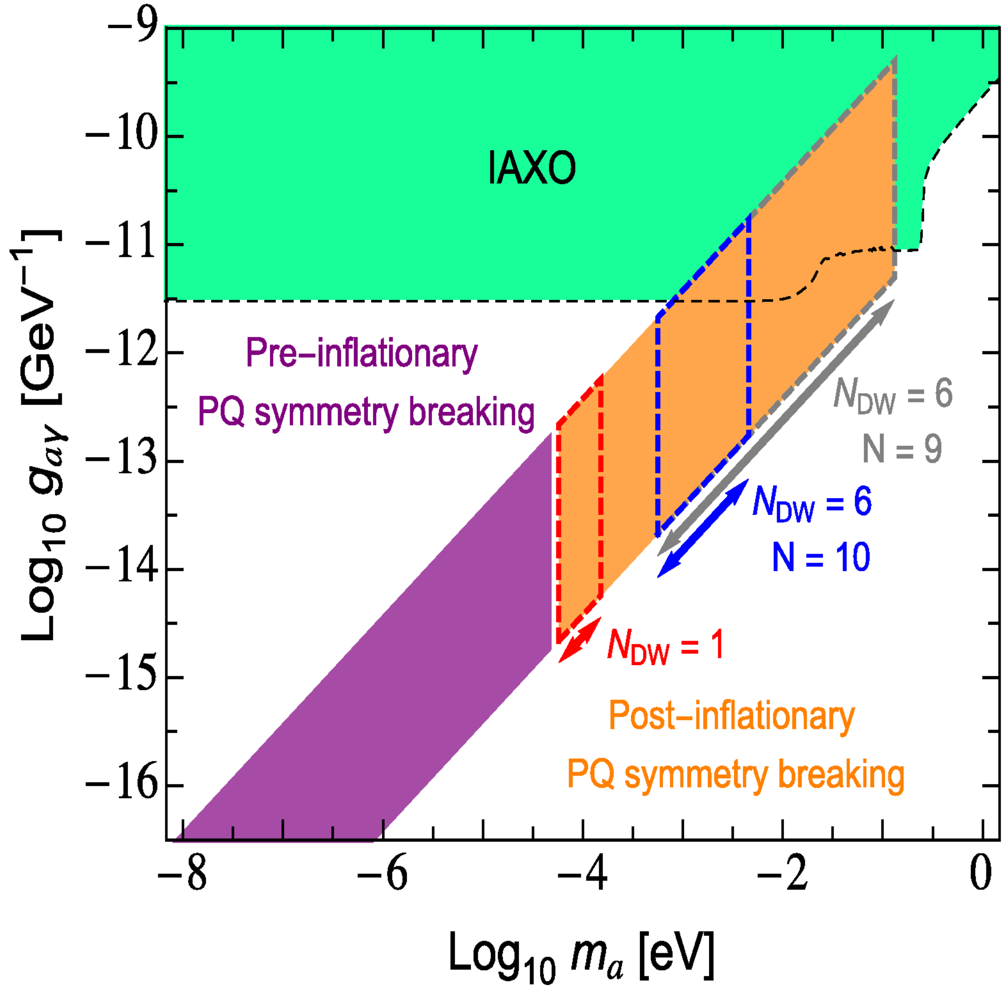

[Saikawa]

Summary

- Lattice results on topological susceptibility: Vacuum realignment contribution to axion cold dark matter abundance can be predicted now with O(10%) accuracy
- Lower limit on axion mass in post-inflationary PQ breaking scenario:

$$m_A > 28(2)\ \mu\mathrm{eV}$$

- Lot of search space for axion cold dark matter: