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$B_c^{\pm}$ decays into tetraquarks

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The recent observation by the D0 collaboration of a narrow structure $X(5568)$ consisting of four different quark flavors $bdus$, has not been confirmed by LHCb. In the tightly bound diquark model, we estimate the lightest $bdus$, 0^+ tetraquark at a mass of about 5770 MeV, approximately 200 MeV above the reported $X(5568)$, and just 7 MeV below the $B\bar{K}$ threshold. The charged tetraquark is accompanied by $I = 1$ and $I = 0$ neutral partners almost degenerate in mass. A $bdus$, S-wave, 1^+ quartet at 5820 MeV is implied as well. In the charm sector, $cdus$, 0^+ and 1^+ tetraquarks are predicted at 2365 and 2501 MeV, about 40–50 MeV heavier than $D_{s0}(2317)$ and $D_{s1}(2460)$. The $bdus$ tetraquarks can be searched in the hadronic debris of a jet initiated by a b. However, some of them may also be produced in B_c decays, $B_c \to X_{b0} + \pi$ with the subsequent decays $X_{b0} \to B_s + \pi$, giving rise to final states such as $B_s\pi^+\pi^0$. We also emphasize the importance of B_c decays as a source of bound hidden charm tetraquarks, such as $B_c \to X(3872) + \pi$.

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I. INTRODUCTION

Recently, the D0 experiment reported the observation of a new narrow structure in the $B_s^0\pi^+$ invariant mass[1] [1], which promptly attracted considerable attention, see [2] (but skepticism has been raised in [3]). Based on 10.4 fb^{-1} of $p\bar{p}$ collision data at $\sqrt{s} = 1.96$ TeV, this candidate resonance, dubbed $X(5568)$, has a mass and width given by $M = 5568$ MeV and $\Gamma = 22$ MeV, respectively. A state such as $X(5568)$ would be distinct in that a charged light quark pair cannot be created from the vacuum, leading to the unambiguous composition in terms of four valence quarks with different flavors—$\bar{b}\,\bar{d}\,su$ (tetraquarks with flavored quantum numbers have also been discussed in [4]).

As exciting a discovery as it would have been, $X(5568)$ has not been confirmed by the LHCb experiment. Their analysis has been reported recently, based on 3 fb^{-1} of pp collision data at $\sqrt{s} = 7$ and 8 TeV, yielding a data sample of B_s^0 mesons 20 times higher than that of the D0 collaboration. Adding then a charged pion, the $B_s^0\pi^+$ invariant mass shows no structure from the $B_s^0\pi^+$ threshold up to $M_{B_s^0\pi^+} \leq 5700$ MeV and an upper limit on the ratio $\rho(X(5568)/B_s^0) < 0.016(0.018)$ @ 90 (95)% C.L. is set for $p_T(B_s^0) > 10$ GeV [5].

The valence quark composition of $X(5568)$ fits into a diquarkonium interpretation [6–10]. In this framework, the constituents are arranged in a tightly bound diquark-antidiquark pair, $[\bar{b}\,\bar{d}]_{\mathbf{3}_c}[su]_{\bar{\mathbf{3}}_c}$, both of them transforming nontrivially under color SU(3). However, as outlined below, our computation of the tetraquark mass spectrum with the quark flavors $\bar{b}\,\bar{d}\,su$ yields significantly higher values. The lightest in this sector is the S-state, X_{b0}^+, whose mass is estimated by us to be about 5770 MeV—approximately 200 MeV heavier than the X(5568), and below the $B^+\bar{K}^0$ threshold by about 7 MeV.

The tetraquark mass spectrum is calculable up to a theoretical error which we estimate to be of the order of ± 30 MeV, judging from the discrepancies of constituent quark masses obtained from baryons and mesons (see e.g. Table I in Ref. [7]). Thus, X_{b0}^+ and X_{b0}^0 may lie somewhat above the $B^+\bar{K}^0$ threshold, in which case X_{b0}^+ will decay, perhaps mostly, in the $B^+\bar{K}^0$ mode, and the $B_s^0\pi^+$ resonance signal would be reduced.[2] An analysis of the B^+K^- final state has been published by LHCb, based on a limited sample of 1 fb^{-1} [11]. However, it is also within the margin of errors that the actual masses of these tetraquark S-states are couple of tens of MeV below our estimates, in which case the $B^+\bar{K}^0$ mode is not available, and it is logical to anticipate X_{b0}^+ and X_{b0}^0 as resonant $B_s\pi$ states. We pursue this possibility here. An alternative description is found in [12].

[1]Hereafter, adding the charged conjugated modes—e.g. $\bar{B}_s^0\pi^-$—is understood.

[2]This would be similar to the case of $X(3278)$, which decays predominantly in DD^* and also, appreciably, in $J/\psi + \rho/\omega$.

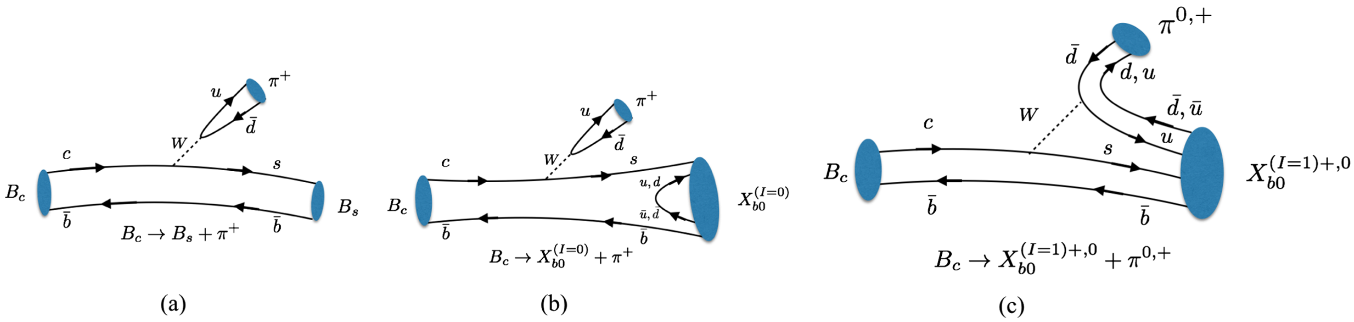

FIG. 1. (a) Leading order Feynman diagram for the decay $B_c^+ \to B_s^0 \pi^+$, (b) $B_c^+ \to X_{b0}^{(I=0)} + \pi^+$, and (c) the corresponding diagram for the decays $B_c^+ \to X_{b0}(5570)^{(I=1)+,0} + \pi^{0,+}$.

With this hindsight, we point out that there are, in principle, two *generic different mechanisms* for producing $X_{b0}(5770)$ in high energy pp and $p\bar{p}$ collisions. These states can be produced as a fragmentation product of a jet initiated by a b-quark, but, subject to phase space, they can also be produced in the decays of the $B_c^\pm$ mesons, $B_c^\pm \to X_{b0}(5770)^{I=1} + \pi$ and $B_c^\pm \to X_{b0}(5770)^{I=0} + \pi^\pm$ as a result of weak ($c \to su\bar{d}$) decays, $q\bar{q}$ excitation, and quark rearrangement (see Fig. 1). With the anticipated decays $X_{b0}^\pm \to B_s^0 \pi^\pm$ and $X_{b0}^0 \to B_s^0 \pi^0$, the decay chains will lead to $B_c^\pm \to B_s^0 \pi^\pm \pi^0$ etc. A resonating structure in the $B_s \pi$ mode can then be fished out by Dalitz analysis. This mechanism is similar to the production mechanism of many multiquark states, seen in B^0 and $B^\pm$ decays, such as $B \to X(3872)(K, K\pi)$, but also for the pentaquarks, such as $P_c(4450)^+$ and $P_c(4380)^+$, in the decays $\Lambda_b^0 \to (P_c(4380)^+, P_c(4450)^+)K^-$. We recall that the dominant two-body decay mode $B_c^\pm \to B_s^0 \pi^\pm$ has been measured by LHCb, with a branching ratio of about 10% [13], and we anticipate that some of the $B_c^\pm$-decays to tetraquarks will be large enough to be measured.

In what follows, we present our estimates of the mass spectrum of the lowest S- and P-states with the flavor quantum numbers of the state $B_s^0 \pi^+ = (\bar{b}s)(\bar{d}u)$, having the angular momentum quantum numbers $J^P = 0^+$, 1^+ together with their counterparts in the charm sector. This is followed by the discussion of the $B_c^\pm$-decays leading to some of these tetraquark states as well as the bound $c\bar{c}$ tetraquark states $X(3872)$ in the decays $B_c^\pm \to X(3872) + \pi^\pm$.

II. SPECTRUM

Within the constituent quark model the color-spin Hamiltonian describing the interaction between the different constituents of a hadron takes the form

$$H = \sum_i m_i + 2\sum_{i<j} \kappa_{ij} \mathbf{S}_i \cdot \mathbf{S}_j, \tag{1}$$

where m_i are the diquark constituent masses, $\mathbf{S}_i$ the quark spins and κ_{ij} some effective, representation-dependent chromomagnetic couplings. The spin-spin interaction is here understood to be a contact one.

In the most recent and most successful type-II tetraquark model [8,10], the dominant interactions are assumed to be the spin-spin interactions between quarks (antiquarks) inside the same tightly bound diquark (antidiquark). With the composition, $[\bar{b}\,\bar{q}]_{\mathbf{3}_c}[sq']_{\bar{\mathbf{3}}_c}$ with $q \neq q' = d$, u, this means retaining only κ_{bq} and $\kappa_{sq'}$ and the lightest states will correspond to the heavy-light diquark spins: $S_{[bq]} = 0$, 1 and $S_{[sq]} = 0$. The latter case corresponds to the so-called "good diquark" [14], and the two resulting states have $J^P = 0^+$ or 1^+, the lightest being the 0^+ one. To indicate these particles, we use the notations

$$X_{b0} = |0_{\bar{b}\,\bar{q}}, 0_{sq'}\rangle \qquad X_{b1} = |1_{\bar{b}\,\bar{q}}, 0_{sq'}\rangle. \tag{2}$$

In the above approximation, the resulting mass formula for S-wave, $[\bar{b}\,\bar{q}][sq']$ states is *additive in diquark energies*,

$$\begin{aligned} M(X_{bS}) &= m_{[bq]} + 2\kappa_{bq} \mathbf{S}_{\bar{b}} \cdot \mathbf{S}_{\bar{q}} + m_{[sq]} + 2\kappa_{sq} \mathbf{S}_s \cdot \mathbf{S}_{q'} \\ &= m_{[bq]} + \kappa_{bq}\left(S(S+1) - \frac{3}{2}\right) + m_{[sq]} - \frac{3}{2}\kappa_{sq}, \end{aligned} \tag{3}$$

where $S \equiv S_{[bq]}$.

We may compare (3) with the mass formulas of the related tetraquarks $a_0(980)$ [6], $Z_b(10610)$, $Z_b'(10650)$ [9], obtained with the substitutions: $b\bar{s} \to s\bar{s}$ and $b\bar{s} \to b\bar{b}$

$$\begin{aligned} a_0(980) &= |0_{\bar{s}\,\bar{q}}, 0_{sq'}\rangle \\ M_{a_0} &= 2\left(m_{[sq]} - \frac{3}{2}\kappa_{sq}\right) \end{aligned} \tag{4}$$

$$\begin{aligned} Z_b &= \frac{1}{\sqrt{2}}(|1_{\bar{b}\,\bar{q}}, 0_{bq'}\rangle - |0_{\bar{b}\,\bar{q}}, 1_{bq'}\rangle) \\ M_{Z_b} &= 2m_{[bq]} - \kappa_{bq} \end{aligned} \tag{5}$$

$$\begin{aligned} Z_b' &= |1_{\bar{b}\,\bar{q}}, 1_{bq'}\rangle_{J=1} \\ M_{Z_b'} &= 2m_{[bq]} + \kappa_{bq}. \end{aligned} \tag{6}$$

From Eqs. (5) and (6) and the known masses [15], we derive

$$m_{[bq]} = \frac{M(Z'_b) + M(Z_b)}{4} \simeq 5315 \text{ MeV} \tag{7a}$$

$$\kappa_{bq} = \frac{M(Z'_b) - M(Z_b)}{2} \simeq 22.5 \text{ MeV.} \tag{7b}$$

In the approximation where tetraquark masses are additive in diquark energies, one finds

$$M(X_{b0}) = \left(m_{[bq]} - \frac{3}{2}\kappa_{bq}\right)_{Z_b} + \left(m_{[sq]} - \frac{3}{2}\kappa_{sq}\right)_{a_0} = \simeq 5770 \text{ MeV}(J^P = 0^+) \tag{8}$$

about 200 MeV more than the X(5568) mass and just 7 MeV below the $B^+\bar{K}^0$.

To be seen as resonant $B_s\pi$ states, their masses should lie below the BK threshold. A good part of the $B_s\pi$ invariant mass spectrum is excluded by the LHCb, but still there is a window of opportunity left unexplored so far.

As a side remark, we note that in Ref. [7] the value $m_{[sq]} = 590$ MeV was obtained using the value $\kappa_{sq} \simeq 64$ MeV obtained from a fit to the baryon masses, which however may be different from the spin-spin coupling inside a diquark. On the other hand, κ_{ij} are expected to scale inversely to the constituent quark masses and this relation is approximately verified by κ_{bq} and κ_{cq} [10] estimated from $Z_{b,c}$ and $Z'_{b,c}$ masses, Eqs. (7b) and (12b) below. If we scale κ_{sq} from κ_{cq} using the strange and charm constituent quark masses, we obtain

$$\kappa_{sq} \simeq 200 \text{ MeV} \tag{9}$$

leading to

$$m_{[sq]} \simeq 800 \text{ MeV.} \tag{10}$$

The diquark mass thus obtained is close to the sum of constituent light and strange quark masses, 330 and 520 MeV, respectively.

The $J^P = 1^+$ exotic states lies close by. From Eq. (3) we find

$$M(X_{b1}) \simeq 5820 \text{ MeV}(J^P = 1^+). \tag{11}$$

The X_{b1} state is expected to decay into $B_s^{*0}\pi^+$ followed by $B_s^{*0} \to B_s^0\gamma$, with a photon energy of 48 MeV in the B_s^* rest frame, escaping detection at hadron colliders. The observed peak of the X_{b1} would be shifted towards lower invariant masses, but essentially coincide with the X_{b0} peak.

In the type-II model [8], we estimate the parameters $m_{[cq]}$ and κ_{cq}, from the masses of $Z_c(3900)$, $Z'_c(4020)$ [15], obtaining

$$m_{[cq]} = \frac{M(Z'_c) + M(Z_c)}{4} \simeq 1978 \text{ MeV} \tag{12a}$$

$$\kappa_{cq} = \frac{M(Z'_c) - M(Z_c)}{2} \simeq 67 \text{ MeV.} \tag{12b}$$

One might use the previous results to estimate the mass of the analogous $X_{cS}^{\pm}$ expected in the charm sector and decaying into $D_s\pi$:

$$M(X_{c0}) = m_{[cu]} + m_{[sd]} - 3/2\kappa_{sq} - 3/2\kappa_{cq} \simeq 2367 \text{ MeV} \tag{13}$$

$$M(X_{c1}) = m_{[cu]} + m_{[sd]} - 3/2\kappa_{sq} + 1/2\kappa_{cq} \simeq 2501 \text{ MeV.} \tag{14}$$

The estimates in Eqs. (13) and (14) set the exotic candidates $X_{c0}^{\pm}$ just above the DK and D^*K thresholds (2363 and 2504 MeV, respectively), so that it could be useful to search also in these decay channels.

If the light diquark is in the $S = 0$ configuration, i.e. it is antisymmetric in spin and color, it must also be antisymmetric in $\mathrm{SU(3)_F}$ (F for flavor), therefore the tetraquarks $[\bar{Q}\,\bar{q}][q'q'']$, with $Q = b, c$ and $q, q', q'' = u, d, s$ belong to the $\mathrm{SU(3)_F}$ representation: $\bar{\mathbf{3}} \otimes \bar{\mathbf{3}} = \mathbf{3} \oplus \bar{\mathbf{6}}$.

In the charm sector, one doubly charged state is present, belonging to the $\bar{\mathbf{6}}$, e.g. with the flavor content $[\bar{c}\,\bar{u}][sd] \to D_s^-\pi^-$. In the beauty sector, doubly charged states lie in the symmetric $\mathbf{15}$ representation of $SU(3)_\mathrm{F}$ (see, He and Ko in [2]), originating from the product: $\bar{\mathbf{3}} \otimes \mathbf{6} = \mathbf{3} \oplus \mathbf{15}$. This requires a light diquark with $S = 1$, the so-called "bad diquarks," which may be argued to have little binding [14].

At present, upper limits on the production at lepton colliders of charmed-strange doubly charged resonances have been given [16] in the $D_s^+\pi^+$ channel, for masses between 2.25 and 2.61 GeV.

We close this discussion by considering the flavor multiplicity of the states $X_{b0} = [\bar{b}\,\bar{q}][sq']$, with $q, q' = u$, d, and their decay modes. These states are obviously organized in an isospin triplet and singlet, similar in structure to the scalar light tetraquarks $a_0(980)$ and $f_0(980)$. The neutral X_{b0} states are similarly expected to be nearly degenerate in mass.

The isoscalar state should decay as $X_{b0}^{(I=0)} \to B_s + \eta$ which is most likely phase space forbidden, leaving the possibility of the strong decay $X_{b0}^{(I=0)} \to B + \bar{K}$, a situation very similar to the decay $f_0 \to K\bar{K}$. Should also the latter mode be forbidden by phase space, $X_{b0}^{(I=0)}$ has to decay by isospin violating interactions: $X_{b0}^{(I=0)} \to B_s + \pi^0$, which may occur due to isospin violating mixing with $X_{b0}^{(I=1)}$ or via $\eta - \pi^0$ mixing, similarly to η decay.

Similar considerations apply to the $I = 0$ $X^{\pm}_{cS}$ states which estimates in Eqs. (13) and (14) place only 40–50 MeV above the well-known $D_{s0}(2317)$ and $D_{s1}(2460)$. The mass difference is quite close to the theoretical error so as to suggest X_{cS} to be identified with the latter resonances, the decays into $D_s^+\pi^0$ arising also from isospin breaking interactions, either due to the mixing with the $I = 1, I_3 = 0$ component or via $\eta - \pi^0$ mixing.

III. TETRAQUARK PRODUCTION IN WEAK DECAYS OF $B_c^{\pm}$ MESONS

Motivated by the observation of a large number of exotic XYZ mesons in the decays of the $B^{\pm}$- and B^0-mesons, as well as the pentaquark states $P_c(4450)^+$ and $P_c(4380)^+$ in the decays of the Λ_b-baryons, $\Lambda_b^0 \to (P_c(4380)^+, P_c(4450)^+)K^-$, we anticipate production of the charged $X_{b0}(5570)^{\pm}$ and neutral $X_{b0}(5570)^0$ tetraquark states in weak decays of the $B_c^{\pm}$ mesons. We also emphasize that $B_c^{\pm}$-decays are a copious, as yet unexplored, source of hidden $c\bar{c}$ tetraquark states, via decay modes such as $B_c^{\pm} \to X(3872)\pi^{\pm}$. Other candidate tetraquark states in the same family but having different J^{PC} quantum numbers are, likewise, anticipated in $B_c^{\pm}$ decays.

For the weak decays of $B_c^{\pm} \to B_s^0\pi^{\pm}$, $B_c^{\pm} \to X_{b0}^0\pi^{\pm}$, and $B_c^{\pm} \to X_{b0}^{\pm}\pi^0$, the active decay at the quark level is $c \to su\bar{d}$, with the $\bar{b}$ decay treated as a spectator. This accounts for approximately 50% of the $B_c^{\pm}$ decays [17].

The effective Hamiltonian for such nonleptonic decays is

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}^*[C^{(-)}\mathcal{O}^{(-)} + C^{(+)}\mathcal{O}^{(+)}]$$
$$\mathcal{O}^{(\pm)} = (\bar{s}^{\alpha}c_{\alpha})_{V-A}(\bar{u}^{\beta}d_{\beta})_{V-A} \pm (\bar{s}^{\alpha}d_{\alpha})_{V-A}(\bar{u}^{\beta}c_{\beta})_{V-A}, \quad (15)$$

where G_F is the Fermi coupling constant, V_{ij} are the Cabibbo-Kobayashi-Maskawa matrix elements, α and β are the color indices and $C^{(\pm)} = (C_1 \pm C_2)/2$, $C_{1,2}(\mu)$ being the Wilson coefficients at scale $\mu = m_c$, m_b. We have dropped QCD penguin contributions and $C^{(\pm)}$ are QCD renormalization factors [18] computed at a momentum scale equal to the b-quark mass, with [19]

$$2C^{(-)} \simeq 1.4 \qquad 2C^{(+)} \simeq 0.85. \quad (16)$$

The amplitude for $B_c^+ \to B_s\pi^+$ can be written in the factorized form, see Fig. 1(a):

$$\mathcal{M}(B_c^+ \to B_s^0\pi^+) = \frac{G_F}{\sqrt{2}} V_{cs}V_{ud}^*(C^{(-)} + C^{(+)})\tilde{M} \quad (17)$$

with $(C^{(-)} + C^{(+)}) = C_1$

$$\tilde{M} = \frac{f_{\pi}}{m_{\pi}^2} q^{\mu}\langle B_s|\bar{s}\gamma_{\mu}P_L c|B_c^+\rangle$$
$$= \frac{f_{\pi}}{m_{\pi}^2}[f_+(m_{\pi}^2)(m_{B_c}^2 - m_{B_s}^2) + f_-(m_{\pi}^2)m_{\pi}^2]. \quad (18)$$

Here, $f_{\pm}(q^2)$ are the vector current form factors, evaluated at $q^2 = m_{\pi}^2$, which have been studied in a number of models (see, for example [20] for a comparative evaluation), f_{π} is the pion decay constant, $f_{\pi} = 140$ MeV [15], C_1 is the (QCD renormalized) effective Wilson coefficient, estimated to be $C_1 \approx 1.1$, and the second term above can be neglected, as it is multiplied by m_{π}^2. With this, the decay width can be evaluated straightforwardly,

$$\Gamma(B_c^{\pm} \to B_s^0\pi^{\pm}) = |\mathcal{M}|^2 \frac{|\boldsymbol{p}_{\pi}|}{8\pi m_{B_c}^2}, \quad (19)$$

where $|\boldsymbol{p}_{\pi}|$ is the $\pi^{\pm}$ 3-momentum in the rest frame of the $B_c^{\pm}$-meson.

The branching ratio for $B_c^{\pm} \to B_s^0\pi^{\pm}$ has been measured by LHCb:

$$\mathcal{B}(B_c^+ \to B_s^0\pi^+)\frac{P(\bar{b} \to B_c^+)}{P(\bar{b} \to B_s)} = (2.37^{+0.37}_{-0.35}) \times 10^{-3}. \quad (20)$$

Here, $P(\bar{b} \to B_s)$ and $P(\bar{b} \to B_c^+)$ are the fragmentation probabilities. The ratio of the two probabilities, i.e., the ratio of the production rates of B_c^+ mesons and B_s mesons in a b-quark jet is estimated to be about 0.02, yielding a 10% branching ratio for $B_c^+ \to B_s\pi^+$. This is the largest branching ratio of any B-meson observed in a single channel.

The decay $B_c^{\pm} \to X_{b0}^{I=0}(5770)\pi^{\pm}$ is expected to have a large branching ratio, as this decay amplitude, like $B_c^{\pm} \to B_s^0\pi^{\pm}$, is factorizable [see Fig. 1(b)]. The relevant matrix element can be written down in an analogous way to that of $B_c^{\pm} \to B_s^0\pi^{\pm}$. One now needs to know the hadronic matrix element $\langle X_{b0}^{I=0}|\bar{s}\gamma_{\mu}P_L c|B_c^+\rangle$. Recalling that $X_{b0}^{I=0}$ has $J^P = 0^+$, the transition goes via the axial-vector part of the charged current, yielding an expression similar to the one for $\mathcal{M}(B_c^{\pm} \to B_s^0\pi^{\pm})$ obtained above. However, in this case, the corresponding hadronic quantity, which we denote by $f_+(m_{\pi}^2)^{B_cX_{b0}}$, is unknown. This can be calculated using QCD sum rules or lattice QCD, as it involves the axial-current matrix element of a single hadron → single hadron transition. In the diquark model at hand, it is expected to be not too different from $f_+(m_{\pi}^2)^{B_cB_s^0}$, as the heavy flavor content of the X_{b0}^0 and B_s^0 is the same, namely $\bar{b}s$.

Denoting the ratio of the two form factors as $F(X_{b0}/B_s) \equiv f_+(m_{\pi}^2)^{B_cX_{b0}}/f_+(m_{\pi}^2)^{B_cB_s^0}$, the relative branching ratios can be expressed as

$$\frac{\mathcal{B}(B_c^{\pm}\to X_{b0}^{I=0}\pi^{\pm})}{\mathcal{B}(B_c^{\pm}\to B_s^0\pi^{\pm})}=F(X_{b0}/B_s)^2\frac{(m_{B_c}^2-m_{X_{b0}}^2)^2}{(m_{B_c}^2-m_{B_s}^2)^2}\frac{|\boldsymbol{p}_\pi|^{B_c\to X_{b0}\pi}}{|\boldsymbol{p}_\pi|^{B_c\to B_s\pi}}. \tag{21}$$

With the known masses, and using our estimate $m(X_{b0}^{I=0}) = 5.770$ GeV, we get a branching ratio of 1(2)% for the decay $B_c^{\pm} \to X_{b0}^{I=0}\pi^{\pm}$ for an assumed value of $F(X_{b0}/B_s)^2 = 0.5(1)$. Given the large sample of $B_c^{\pm}$ already available and in forthcoming LHC runs, this branching ratio is measurable in the decay mode $B_c^{\pm} \to (B_s^0\pi^0)\pi^{\pm}$, assuming a good π^0 detection efficiency.

We expect the corresponding branching ratio for the decay $B_c^{\pm} \to X_{b0}^{\pm}\pi^0 \to (B_s^0\pi^{\pm})\pi^0$ to be multiplied by a factor $C^{(-)2}/(C^{(-)} + C^{(+)})^2 \simeq 0.62$. In fact, only $\mathcal{O}^{(-)}$ contributes to the decay into $X_{b0}^{I=1}$, due to color antisymmetry of the final us pair, Fig. 1(c), as in the Pati and Woo argument [21] about $\Delta I = 1/2$ rule in nonleptonic baryon decays. This pattern could be modified by nonperturbative effects, as also seen in a number of similar $B^{\pm}$ and B^0 decays [15].

We now discuss the B_c^{+} decays leading to the bound $c\bar{c}$ tetraquarks. This requires the quark decay $\bar{b} \to \bar{c}u\bar{d}$, with the c-quark in B_c^{+} acting as an spectator quark. The benchmark decay for this class is $B_c^{\pm} \to J/\psi\pi^{\pm}$. Requiring now the excitation of a $q\bar{q}$ pair, followed by quark recombination, leads to decays such as $B_c^{\pm} \to X(3872)^{I=0}\pi^{\pm}$ and $B_c^{\pm} \to X(3872)^{I=1\pm,0}\pi^{0,\pm}$. These diagrams allow access to both the $I = 0$ (isosinglet) and the $I = 1$ (isotriplet) partners of the $X(3872)$, decaying, respectively, to $J/\psi\omega$ and $J/\psi\rho^0$, as well as the decay of the charged partner $X(3872)^{\pm} \to J/\psi\rho^{\pm}$, in addition, possibly, to $\bar{D}^*D$ decays. There would be enough phase space to observe the corresponding P-states as well.

Again, we expect the decay $B_c^{\pm} \to X(3872)^{(I=0)}\pi^{\pm}$ to have a large branching ratio, which is similar to $B_c^{\pm} \to J/\psi\pi^{\pm}$, as both are factorizable processes and are proportional to $C^{(-)} + C^{(+)}$. The decays of $B_c^{\pm}$ to the $[cq][\bar{c}\,\bar{q}']$-tetraquarks have the potential to map out a large number of anticipated states in this sector.

IV. CONCLUDING REMARKS

The observation of the $X(5568)$ has not been confirmed by LHCb. It remains to be seen if a state with the quark flavors $b\bar{s}u\bar{d}$ exists in nature, with a different mass, decay pattern, and width. In this paper we have used the diquark-antidiquark picture to give predictions about the mass spectrum of the lowest S-state, X_{b0} and its $J^P = 1^+$ partners, both in charm and bottom sectors. Our estimates set the mass of the lowest such state in the b-quark sector at around 5770 MeV, somewhat below the BK threshold. Within the errors of our approach, X_{b0}^{+} could lie just above this threshold, and one has to look for it in the decay $X_{b0}^{+} \to B^+\bar{K}^0$. However, X_{b0}^{+} may as well reveal itself as a resonating $B_s\pi$ state, or not manifest at all, if below threshold, as discussed in [12].

Here we propose to search tetraquark states in the decays of the $B_c^{\pm}$ mesons, $B_c^{\pm} \to X_{b0}^{0}\pi^{\pm}$ and $B_c^{\pm} \to X_{b0}^{\pm}\pi^0$ and have argued that some of these decay modes may have a large branching ratio, and this requires a good π^0-detection efficiency, which we advocate to improve in hadron collider experiments, such as the LHCb. So far, only a handful of $B_c^{\pm}$ decays have been observed [15], and it is worthwhile to put in a dedicated effort to increase this database. Apart from the possibility of observing the tetraquark states of the $B_s\pi$ variety, we anticipate several bound $c\bar{c}$ tetraquark states, which emerge from the decay $B_c^{+} \to (c\bar{c})u\bar{d}$, followed by a $q\bar{q}$ excitation from the vacuum. These would lead to decays such as $B_c^{\pm} \to X(3872)^0\pi^{\pm}$ and $B_c^{\pm} \to X(3872)^{\pm}\pi^0$, as well as to other related tetraquark states. They should be searched for at the LHC, and also at Belle-II, if the e^+e^- center of mass energies could reach the $B_c^+B_c^-$ threshold.

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[1] V. M. Abazov *et al.* (D0 Collaboration), Observation of a New $B_s^0\pi^{\pm}$ State, Phys. Rev. Lett. **117**, 022003 (2016).

[2] W. Chen, H. X. Chen, X. Liu, T. G. Steele, and S. L. Zhu, Investigation of the $X(5568)$ as a Fully Open-Flavor $su\bar{b}\,\bar{d}$ Tetraquark State, Phys. Rev. Lett. **117**, 022002 (2016); W. Wang and R. Zhu, Can $X(5568)$ be a tetraquark state?, Chin. Phys. C **40**, 093101 (2016); Z. G. Wang, Analysis of the $X(5568)$ as scalar tetraquark state in the diquark-antidiquark model with QCD sum rules, arXiv:1602.08711; S. S. Agaev, K. Azizi, and H. Sundu, Mass and decay constant of the newly observed exotic $X(5568)$ state, Phys. Rev. D **93**, 074024 (2016); C. J. Xiao and D. Y. Chen, Possible $B^{(*)}\bar{K}$ hadronic molecule state, arXiv:1603.00228; S. S. Agaev, K. Azizi, and H. Sundu, Width of the exotic $X_b(5568)$ state through its strong decay to $B_s^0\pi^+$, Phys. Rev. D **93**, 114007 (2016); Y. R. Liu, X. Liu, and S. L. Zhu, $X(5568)$ and and its partner states, Phys. Rev. D **93**, 074023 (2016); C. M.

Zanetti, M. Nielsen, and K. P. Khemchandani, A QCD sum rule study for a charged bottom-strange scalar meson, Phys. Rev. D **93**, 096011 (2016); X. H. Liu and G. Li, Could the observation of $X(5568)$ be resulted by the near threshold rescattering effects?, arXiv:1603.00708; X. G. He and P. Ko, Flavor $SU(3)$ symmetry for the $X(5568)$ state, arXiv:1603.02915; Y. Jin and S. Y. Li, Large production rate of new $B_s^0\pi^\pm$ and $D_s^\pm\pi^\pm$ states in high energy multi-production process, arXiv:1603.03250; L. Tang and C. F. Qiao, $X(5568)$ as tetraquark state with open flavors and its charmed partners, arXiv:1603.04761.

[3] T. J. Burns and E. S. Swanson, Interpreting the $X(5568)$, Phys. Lett. B **760**, 627 (2016).

[4] A. Esposito, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo, Doubly charmed tetraquarks in B_c and Ξ_{bc} decays, Phys. Rev. D **88**, 054029 (2013); A. L. Guerrieri, M. Papinutto, A. Pilloni, A. D. Polosa, and N. Tantalo, Flavored tetraquark spectroscopy, *Proc. Sci.*, LATTICE2014 (2015) 106 [arXiv:1411.2247].

[5] V. V. Gligorov and M. Pappagallo (LHCb Collaboration), Search for structure in the $B_s^0\pi^\pm$ invariant mass spectrum, Reports No. LHCb-CONF-2016-004 and No. CERN-LHCb-CONF-2016-004.

[6] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, A New Look at Scalar Mesons, Phys. Rev. Lett. **93**, 212002 (2004).

[7] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, Diquark-antidiquarks with hidden or open charm and the nature of $X(3872)$, Phys. Rev. D **71**, 014028 (2005).

[8] L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, The $Z(4430)$ and a new paradigm for spin interactions in tetraquarks, Phys. Rev. D **89**, 114010 (2014).

[9] A. Ali, C. Hambrock, and W. Wang, Tetraquark interpretation of the charged bottomonium-like states $Z_b^{+-}(10610)$ and $Z_b^{+-}(10650)$ and implications, Phys. Rev. D **85**, 054011 (2012).

[10] A. Ali, L. Maiani, A. D. Polosa, and V. Riquer, Hidden-beauty charged tetraquarks and heavy quark spin conservation, Phys. Rev. D **91**, 017502 (2015).

[11] R. Aaij *et al.* (LHCb Collaboration), First Observation of the Decay $B_{s2}^*(5840)^0 \to B^{*+}K^-$ and Studies of Excited B_s^0 Mesons, Phys. Rev. Lett. **110**, 151803 (2013).

[12] A. Esposito, A. Pilloni, and A. D. Polosa, Hybridized tetraquarks, Phys. Lett. B **758**, 292 (2016).

[13] R. Aaij *et al.* (LHCb Collaboration), Observation of the Decay $B_c^+ \to B_s^0\pi^+$, Phys. Rev. Lett. **111**, 181801 (2013).

[14] R. L. Jaffe, Exotica, Phys. Rep. **409**, 1 (2005); L. Maiani, F. Piccinini, A. D. Polosa, and V. Riquer, A New Look at Scalar Mesons, Phys. Rev. Lett. **93**, 212002 (2004); C. Alexandrou, P. de Forcrand, and B. Lucini, Evidence for Diquarks in Lattice QCD, Phys. Rev. Lett. **97**, 222002 (2006).

[15] K. A. Olive *et al.* (Particle Data Group Collaboration), Review of particle physics, Chin. Phys. C **38**, 090001 (2014).

[16] S. Stone and J. Urheim, Observation of the $D_{sJ}(2463)$ and confirmation of the $D_{sJ}^*(2317)$, AIP Conf. Proc. **687**, 96 (2003); B. Aubert *et al.* (BABAR Collaboration), A Study of the $D_{sJ}^*(2317)$ and $D_{sJ}(2460)$ mesons in inclusive c anti-c production near $\sqrt{s} = 10.6$ GeV, Phys. Rev. D **74**, 032007 (2006); B. Aubert *et al.* (*BABAR* Collaboration), Study of D_{sJ} decays to D^*K in inclusive e^+e^- interactions, Phys. Rev. D **80**, 092003 (2009).

[17] I. P. Gouz, V. V. Kiselev, A. K. Likhoded, V. I. Romanovsky, and O. P. Yushchenko, Prospects for the B_c studies at LHCb, Phys. At. Nucl. **67**, 1559 (2004) [Yad. Fiz. **67**, 1581 (2004)].

[18] M. K. Gaillard and B. W. Lee, $\Delta I = \frac{1}{2}$ Rule for Nonleptonic Decays in Asymptotically Free Field, Phys. Rev. Lett. **33**, 108 (1974); G. Altarelli and L. Maiani, Octet enhancement of non-leptonic weak interactions in asymptotically free gauge theories, Phys. Lett. B **52**, 351 (1974).

[19] See, e.g., A. J. Buras, Weak Hamiltonian, CP violation and rare decays, arXiv:hep-ph/9806471.

[20] W. Wang, Y. L. Shen, and C. D. Lu, Covariant light-front approach for B_c transition form factors, Phys. Rev. D **79**, 054012 (2009).

[21] J. C. Pati and C. H. Woo, Delta $I = 1/2$ rule with fermion quarks, Phys. Rev. D **3**, 2920 (1971).