

**NLO SCALE DEPENDENCE OF SEMI-INCLUSIVE PROCESSES.**

A. Daleo, C. A. García Canal

*Laboratorio de Física Teórica Departamento de Física,  
Facultad de Ciencias Exactas Universidad Nacional de La Plata  
C.C. 67 - 1900 La Plata, Argentina*

R. Sassot

*Departamento de Física, F.C.E.N, Universidad de Buenos Aires  
Ciudad Universitaria, Pab.1 (1428) Buenos Aires, Argentina*

**ABSTRACT**

We discuss the order- $\alpha_s^2$  gluon initiated QCD corrections to one particle inclusive deep inelastic processes. We focus in the the NLO evolution kernels relevant for the non homogeneous QCD scale dependence of these cross sections and factorization.

**1 Introduction**

In recent years there has been an increasing interest in semi-inclusive deep inelastic scattering (SIDIS), driven both by crucial breakthroughs in the QCD description of these processes and also by an incipient availability of data [1].

Although QCD corrections to SIDIS are well known at LO [2, 3], until Ref. [1] no computations had been done up to NLO accuracy, nor assessments of how relevant the non homogeneous scale dependence might be. In LO, non homogeneous evolution effects are restricted to a relatively small kinematic region. This suggested to neglect these effects in many phenomenological analyses of polarized SIDIS, leading baryon production, and diffractive DIS [4].

In NLO the above mentioned kinematical restrictions are no longer present, which in principle may lead to important corrections. At variance with the totally inclusive case, for the computation of the SIDIS NLO corrections it is necessary to keep additional variables unintegrated. This leads to entangled singularities in more than one variable which requires special prescription techniques [1].

## 2 $\mathcal{O}(\alpha_s^2)$ corrections.

The cross section for a one-particle inclusive process in which a lepton scatters off a nucleon and a hadron is tagged in the final state can be written as [2]

$$\begin{aligned} \frac{d\sigma}{dx_B dy dv_h dw_h} = & \sum_{i,j=q,\bar{q},g} \int_{x_B}^1 \frac{du}{u} \int_{v_h}^1 \frac{dv_j}{v_j} \int_0^1 dw f_{i/P} \left( \frac{x_B}{u} \right) D_{h/j} \left( \frac{v_h}{v_j} \right) \frac{d\hat{\sigma}_{ij}}{dx_B dy dv_j dw_j} \delta(w_h - w_j) \\ & + \sum_i \int_{\frac{x_B}{1-(1-x_B)v_h}}^1 \frac{du}{u} M_{i,h/P} \left( \frac{x_B}{u}, (1-x_B)v_h \right) (1-x_B) \frac{d\hat{\sigma}_i}{dx_B dy} \delta(1-w_h), \quad (1) \end{aligned}$$

where in addition to the usual DIS variables  $x_B$  and  $y$ , we introduce energy and angular variables  $v_h = E_h/E_0(1-x_B)$  and  $w_h = \frac{1-\cos\theta_h}{2}$ .  $E_h$  and  $E_0$  are the energies of the final state hadron and of the incoming nucleon in the  $\vec{P} + \vec{q} = 0$  frame, respectively.  $\theta_h$  is the angle between the momenta of the hadron and the virtual photon in the same frame.

The first term in eq.(1), contain the partonic cross section which develop forward collinear singularities ( $w_h = 1$ ) that can not be factorized in the usual partonic densities and fragmentation functions  $f_{i/P}$  and  $D_{h/j}$ , respectively. This divergences are factorized into fracture functions  $M_{i,h/P}$  and lead to their non homogeneous scale dependence.

$$\begin{aligned} \frac{\partial M_{i,h/P}(\xi, \zeta, Q^2)}{\partial \log Q^2} = & \frac{\alpha_s(Q^2)}{2\pi} \int_{\frac{\xi}{1-\zeta}}^1 \frac{du}{u} P_{i \leftarrow j}(u) M_{j,h/P} \left( \frac{\xi}{u}, \zeta, Q^2 \right) \quad (2) \\ & + \frac{\alpha_s(Q^2)}{2\pi} \frac{1}{\xi} \int_{\xi}^{\frac{\xi}{\xi+\zeta}} \frac{du}{u} \int_{\frac{\zeta}{\xi}}^{\frac{1-u}{\xi}} \frac{dv}{v} P_{ki \leftarrow j}(u, v) f_{j/P} \left( \frac{\xi}{u}, Q^2 \right) D_{h/k} \left( \frac{\zeta}{\xi v}, Q^2 \right) \end{aligned}$$

The first order corrections to the one-particle inclusive cross section can be found in [2]. At  $\mathcal{O}(\alpha_s^2)$  the prescription of overlapping divergences in the region B0 =  $\{u \in [x_B, x_u], v \in [v_h, a], w \in [0, 1]\}$  with  $x_u = x_B/(x_B + (1-x_B)v_h)$  and  $w_r = (1-v)(1-u)x_B/v(u-x_B)$ , can be done using the prescription

$$\begin{aligned} & (1-w)^{-1+\epsilon_1} (w_r - w)^{-1+\epsilon_2} \xrightarrow{\text{B0}} \\ & \frac{\Gamma(1+\epsilon_1) \Gamma(1-\epsilon_1-\epsilon_2)}{\epsilon_1 (\epsilon_1 + \epsilon_2) \Gamma(1-\epsilon_2)} \delta(1-w) \delta(a-v) (a-z)^{\epsilon_1+\epsilon_2} (a(1-a))^{1-\epsilon_1-\epsilon_2} \\ & + \frac{1}{\epsilon_1} \delta(1-w) \left( (a-v)^{-1+\epsilon_1+\epsilon_2} \right)_{+v[z,\underline{a}]} (v(1-a))^{1-\epsilon_1-\epsilon_2} w_r^{-\epsilon_1} \\ & \times {}_2F_1 \left[ \epsilon_1, \epsilon_1 + \epsilon_2, 1 + \epsilon_1; \frac{1}{w_r} \right] + \left( (1-w)^{-1+\epsilon_1} (w_r - w)^{-1+\epsilon_2} \right)_{+w[0,\underline{1}]} \quad (3) \end{aligned}$$

and with a similar recipe for B1 =  $\{u \in [x_B, x_u], v \in [a, 1], w \in [0, w_r]\}$  [1]

### 3 Scale Dependence

Figure 1 compares (for different values of  $\xi$  and  $\zeta$ ) the relative size of the LO and NLO contributions to the non homogeneous term in the evolution equation (2) computed with standard sets of parton distributions and fragmentation functions [1] for the case  $i = q$  and  $h = \pi^+$ . The inset plots show the integral over  $Q^2$  of this contributions.

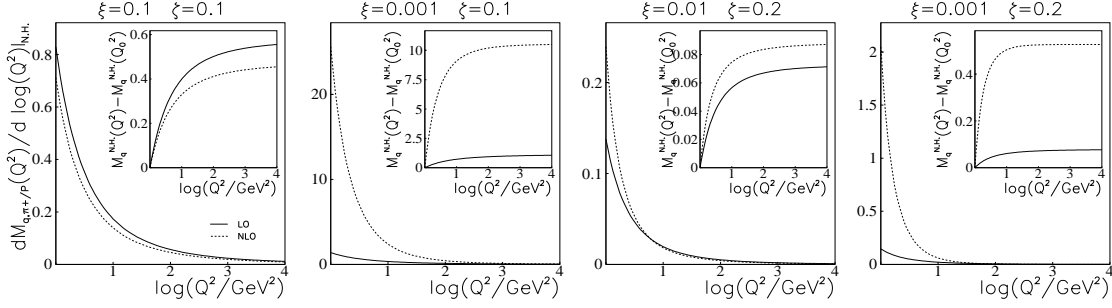


Figure 1: Non homogeneous contributions to the derivative of  $M_q$

The  $\mathcal{O}(\alpha_s^2)$  contributions to the evolution equations are mild in most of the kinematical range, however they are as important or even larger than the  $\alpha_s$  ones for small values of  $x_B$ , where these last contributions are suppressed by the available phase space. This behavior, at variance with the LO case, allows the non homogeneous effects to be sizable even at larger hadron momentum fractions, thus being relevant for the scale dependence of a SIDIS process.

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