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MEASUREMENT OF CKM ELEMENTS AND THE UNITARITY TRIANGLE

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ABSTRACT

An overview of experimental determinations of different CKM elements is given with an emphasis on $|V_{\rm ub}|$ and $|V_{\rm cb}|$ extraction. Measurements are compared to the Standard Model predictions and constraints on the unitarity triangle, arising from combination of measurements, are presented.

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1 Introduction

The general form of the Cabbibo-Kobayashi-Maskawa (CKM) [1] matrix can be described using the generalized Wolfenstein parametrization[1] [2] as

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ A\lambda^3(1-\overline{\rho}-i\overline{\eta}) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4) \ . \tag{1}$$

Product of the first and the last column of the CKM matrix under the unitarity requirement yields the standard form of the unitarity triangle (UT), presented in the complex $(\overline{\rho}, \overline{\eta})$ plane in Fig. 1 (left)[2]. Measurements of various observables,

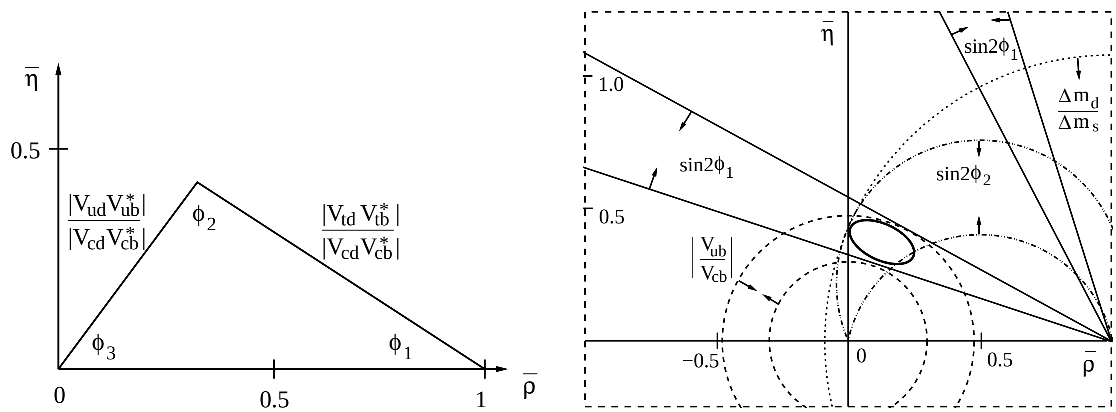


Figure 1: *Presentation of CKM matrix unitarity in the form of the unitarity triangle (left). Marked regions illustrate the constraints imposed on the sides and angles of the triangle by different measurements in the B meson system (right).*

from which individual CKM elements can be determined, impose constraints on the sides and angles of the UT, shown in Fig. 1 (right). The overlap area determines an allowed region for the position of the UT apex. Comparison of individual measurements, described below, enable a consistency check of the CKM description of quark mixing and can give hints of yet unobserved phenomena.

2 $|V_{us}|$, $|V_{ud}|$

The $|V_{us}|$ element of the CKM matrix has traditionally been obtained from the measured branching fractions of $K_{\ell 3}$ decays, the semileptonic decays of charged and neutral kaons $K \to \pi \ell \nu$. Recently, a new measurement of the K^+ branching fraction

[1] $\overline{\rho} = \rho(1-\lambda^2/2)$, $\overline{\eta} = \eta(1-\lambda^2/2)$

[2] Angles ϕ_1, ϕ_2, ϕ_3 are frequently denoted as β, α and γ, respectively.

in the electron decay mode was presented by the Brookhaven E865 experiment [3]. The result $Br(K^+ \to \pi^0 e^+ \nu) = (5.13 \pm 0.02_{\text{stat.}} \pm 0.09_{\text{syst.}} \pm 0.04_{\text{norm.}})\%$ is about 2.3 standard deviations higher than the world average of previous measurements of this quantity [4]. The measured branching ratio, where the last error comes from the uncertainty in the normalization decay modes used in the experiment, can be converted into the value of $|V_{us}| = 0.2272 \pm 0.0020_{\text{exp.}} \pm 0.0018_{\text{th.}}$. The error on this value receives a significant contribution from the theoretical estimate of the $K^\pm$ decay form factor [5]. Using the value of $|V_{ud}| = 0.9740 \pm 0.0005$, evaluated most precisely from the super-allowed nuclear Fermi beta decays [6], and the above result for $|V_{us}|$, the unitarity requirement in the first row of the CKM matrix is perfectly satisfied[3]. A possible disagreement between the world averages of $Br(K^+ \to \pi^0 e^+ \nu)$ and $Br(K^0 \to \pi^- e^+ \nu)$[4] will be addressed by the forthcoming precision results from KLOE and NA48 experiments.

3 $|V_{cb}|$, $|V_{ub}|$

Due to a reasonably good theoretical understanding and a satisfactory statistical power of the sample, the determination of $|V_{cb}|$ and $|V_{ub}|$ elements relies on the measurements of semileptonic B meson decays, using either exclusive or inclusive methods. Within the exclusive methods, the $B \to D^* \ell \nu$ decay channel is the most prominent one for the $b \to c$ transitions. Decays into π, ρ and ω mesons and a lepton pair were used to probe the $b \to u$ transitions. Inclusive methods comprise a determination of the total B meson semileptonic width combined with measurements of various moments of distributions that reduce the theoretical uncertainties entering the CKM element extraction.

The differential decay rate for the $B \to D^* \ell \nu$ can be expressed in terms of a four-velocity transfer $w = v_B v_{D^*}$ using a single B meson decay form factor $F(w)$, which coincides with the Isgur-Wise function up to the heavy quark symmetry breaking terms [7]. Experimentally, the efficiency corrected differential decay rate $d\Gamma/dw$ is measured and extrapolated to the maximum momentum transfer ($w = 1$). Measurement uncertainties are dominated by the systematic errors as seen in Fig. 2 (left) [8]. At the Z^0 energies, the w resolution is influenced by a spread of B meson momenta. Another important source of the systematic error is the modeling of the D^{**} background. At the $\Upsilon(4S)$ energies the kinematic constraints enable a better

[3]The value of $|V_{ub}|$ can be safely neglected.

[4]In the following the notations include the charge conjugate modes, unless explicitly stated otherwise.

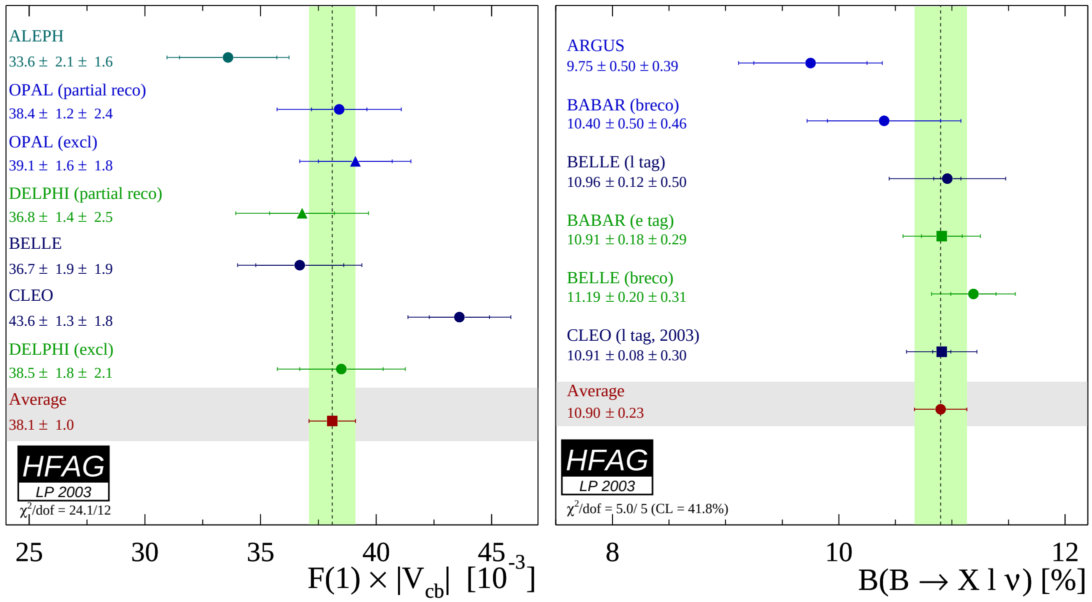


Figure 2: *Summary of $|V_{cb}|$ determination using $B \to D^* \ell \nu$ (left) and inclusive semileptonic B meson branching fraction (right) as given by [8].*

w resolution and an effective D^{**} rejection, but the efficiency for the reconstruction of low momentum pions from $D^{*+} \to D^0 \pi^+$ falls rapidly at low values of w. Using the interval of values $F(1) = 0.91 \pm 0.04$ [4] and the average value of $F(1)|V_{cb}|$ one obtains

$$|V_{cb}| = (41.9 \pm 1.1_{exp.} \pm 1.8_{F(1)}) \times 10^{-3} \ , \tag{2}$$

where the first error represents the experimental and the second the theoretical uncertainty.

The relation between the semileptonic decay width of B mesons and the $|V_{cb}|$ can be written as $Br(b \to c\ell\nu)/\tau_B = K_{th}|V_{cb}|^2$. While the average of measurements of the left hand side of equation contributes about 1% to the relative error on the $|V_{cb}|$, the theoretical factor K_{th} introduces an error of about 5% [4]. The solution is provided in the framework of operator product expansion, where the semileptonic width is expressed to order $O(1/m_b^2)$ using two parameters and heavy quark masses. The same parameters enter the expressions for differential distributions of several observables like the energy of the lepton in semileptonic decays, the invariant mass of the produced hadronic system or the energy of the photon in radiative B meson decays [9]. Measurements of moments of those distributions enable the relevant parameters determination and hence help reducing the theoretical uncertainty in the extraction of $|V_{cb}|$ from the measured semileptonic width. A summary of $Br(B \to c\ell\nu)$ measurements at $\Upsilon(4S)$ energies is shown in Fig. 2 (right) [8]. The precision is almost identical to the measurements performed by the LEP experiments, which benefit from the fact that the total lepton energy spectrum can be used in analyzes. Combination of results together with the measured B meson lifetime yields a value of $\Gamma(b \to c\ell\nu) = (0.43 \pm 0.01) \times 10^{-10}$ MeV [4].

Cleo and Delphi experiments measured the moments (up to the third) of distributions mentioned above [10]. Taking as an input the value of the semileptonic width, $|V_{cb}|$ evaluations [11] can be summarized as [12]

$$|V_{cb}| = (41.2 \pm 0.7_{exp.} \pm 0.6_{th.}) \times 10^{-3} \; . \tag{3}$$

The experimental error includes contributions from moments and semileptonic width measurements, while the theoretical error reflects the uncertainties due to the perturbative QCD and $1/m_b$ series truncation.

The exclusive methods of $|V_{ub}|$ determination consist of $Br(B \to \pi(\rho,\omega)\ell\nu)$ measurements over a limited interval of lepton pair invariant mass (q^2), lepton momentum or lepton energy, in order to suppress the dominant $b \to c$ background. One or more dimensional fits to distributions of kinematical variables, separating the $b \to u$ transitions from the background, make use of isospin relations between the charged and neutral π and ρ decay modes. In order to obtain the full $Br(B \to X_u\ell\nu)$, where X_u represents a light meson, an extrapolation to the full range of fitted observables is necessary, introducing a model dependence. The results are shown in Fig. 3 (left). Error bars mark the statistical uncertainty, while the total line represents the quadratic sum of statistical and experimental systematic errors; for the latter the largest contributions arise from detector simulation used in neutrino reconstruction and $b \to u\ell\nu$ background modeling. Lines below the results show the theoretical uncertainties. They are obtained using various form factors, calculated from lattice QCD, light cone sum rules or quark models, with different methods and are thus not directly comparable. The Belle experiment made a first measurement of $Br(B \to \omega\ell\nu)$ [13], however the value of $|V_{ub}|$ is not quoted. In order to reduce the theoretical uncertainty, the Cleo experiment has performed a measurement of $Br(B \to \pi\ (\rho)\ell\nu)$ (last line of Fig. 3 (left)) in distinct q^2 bins [14]. The advantage of such an approach is a lower systematic error arising from the q^2 dependence of the efficiency, and the discriminating power of the q^2 distribution for different form factor models. For the exclusive $|V_{ub}|$ determination no official average is provided by the Heavy Flavor Averaging Group [8] up to date. Using the first four results of Fig. 3 (left), and treating half of the systematic error as completely correlated (due

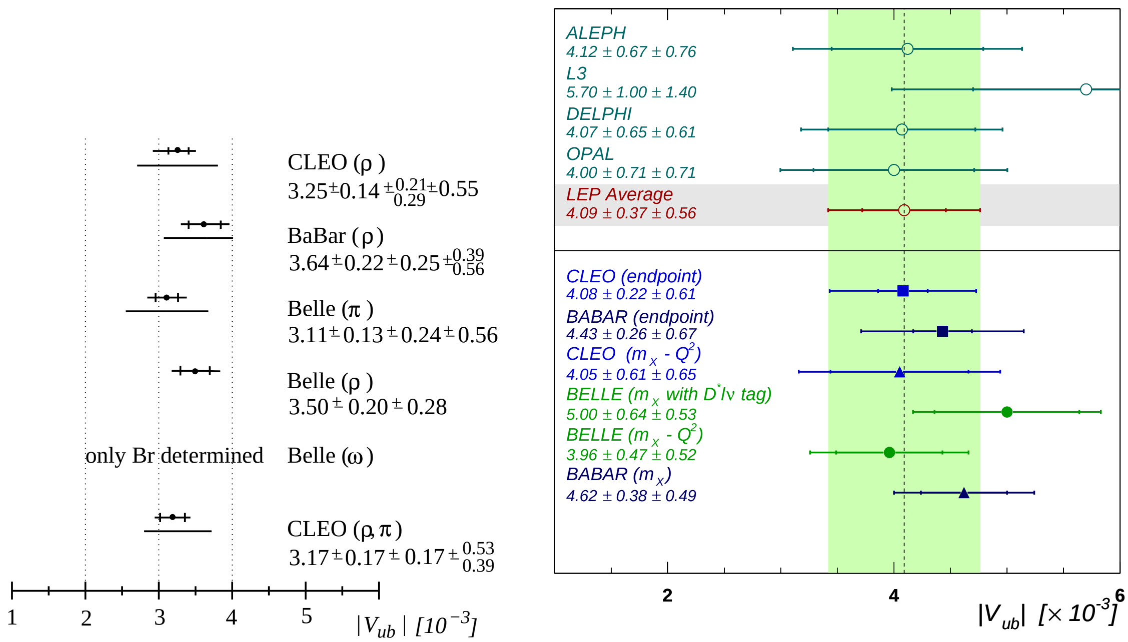


Figure 3: *Summary of $|V_{ub}|$ determinations using $B \to \pi(\rho,\, \omega)\ell\nu$ (left) and inclusive $B \to X_u\ell\nu$ branching fraction (right) [8].*

to $b \to u\ell\nu$ background), one obtains the value

$$|V_{ub}| = (3.32 \pm 0.22_{exp.} \pm 0.54_{th.}) \times 10^{-3} \;, \tag{4}$$

illustrating the uncertainty dominated by the theory.

The $b \to u\ell\nu$ semileptonic width is sensitive to $|V_{ub}|$ in an analog way as the $b \to c\ell\nu$ is sensitive to $|V_{cb}|$. However, in isolating the $b \to u$ transitions, one is restricted to a limited interval of kinematic variables, separating the signal from the major $b \to c$ background. While stringent requirements on the lepton energy, hadronic or lepton pair invariant mass in semileptonic decays lead to a better purity of the selected sample, the selection of events in a vicinity of the phase space boundaries introduces large non-perturbative corrections to theoretical predictions and hence a larger theoretical error [15]. A number of measurements using individual or several of the above mentioned observables for $b \to u\ell\nu$ separation were reported. For the reconstruction of the invariant mass of the hadronic system (m_X), produced in a semileptonic B meson decay at B factories, one needs to separate the decay products of individual B mesons. A large number of recorded B decays enables the use of a sample, where one of the B mesons is fully reconstructed through its $b \to c$ decay modes. Using this method, BaBar collaboration determines the yield of $B \to X_u\ell\nu$ events at low values of m_X [16]. The corresponding $|V_{ub}|$ value is shown in the last line of Fig. 3 (right). To separate decay products, Belle collaboration uses a "simulated annealing" technique, based on maximization of a likelihood function for a correct and wrong assignment of particles, by iteratively exchanging particles between the two B mesons [17]. Selection based on the reconstructed value of m_X and q^2 yields the result marked in Fig. 3 (right) as "Belle ($m_x - Q^2$)". Similarly as in the case of the exclusive $|V_{ub}|$ determination, uncertainties of all results shown are dominated by the theoretical error introduced by the extrapolation of $b \to u\ell\nu$ yield to the full interval of kinematic observables. As an illustration, the average of BaBar, Belle and Cleo measurements [18] using the lepton energy for $b \to u/b \to c$ separation, yields

$$|V_{ub}| = (4.15 \pm 0.18_{exp.} \pm 0.61_{th.}) \times 10^{-3} \ , \tag{5}$$

when the theoretical error is taken as completely correlated, while experimental systematic error is treated as uncorrelated.

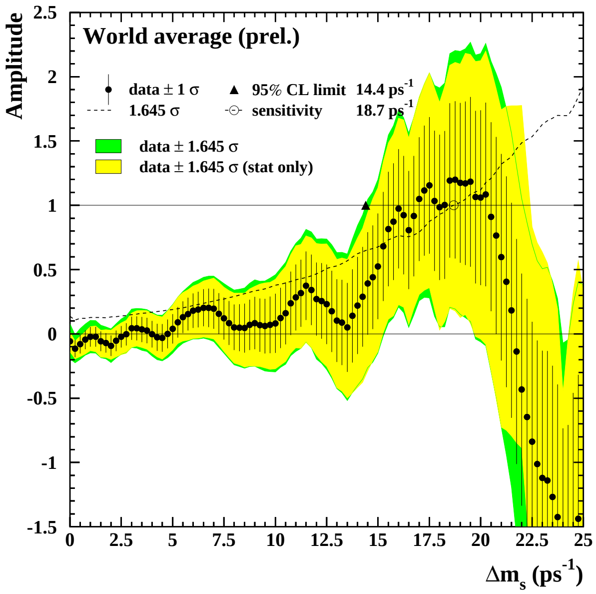
4 $|V_{td}|$, $|V_{ts}|$

The $|V_{td}/V_{ts}|$ value is determined from the ratio of B_d and B_s meson oscillation frequencies, Δm_d and Δm_s. The use of the ratio instead of the individual measurements is motivated by a partial cancellation of the theoretical uncertainties. While the world average of Δm_d has a relative error of 1.2% [8] and is completely dominated by the measurements performed at B factories [19], the Δm_s limits steam from LEP experiments and will until the LHC era remain in the domain of the Tevatron collider. The amplitude method for the B_s oscillations measurements consists of fitting the observed decay time distribution with the amplitude of oscillations as a free parameter. The fitted amplitudes at given values of Δm_s are then converted into the lower limit on the oscillation frequency. The world average of these measurements is shown in Fig. 4 (left), yielding $\Delta m_s > 14.4$ ps^{-1} at 95% C.L. [8]. Taking the $\Delta m_d/\Delta m_s$ ratio and the lattice QCD results for $\sqrt{B_{B_s}}f_{B_s}/\sqrt{B_{B_d}}f_{B_d} = 1.18 \pm 0.04 \pm {}^{0.12}_{0.00}$ [20], one arrives at the limit

$$|\frac{V_{td}}{V_{ts}}| < 0.24 \ \text{ at 95\% C.L.} \tag{6}$$

5 Angles

A detailed overview of $\sin 2\phi_1$ measurements, most prominently in the $B^0 \to J/\Psi K_s$ decays, is given in [19]. Averaging results of BaBar and Belle collaborations in all charmonium decay modes yields $\sin 2\phi_1 = 0.736 \pm 0.049$ [21], a 7% relative precission



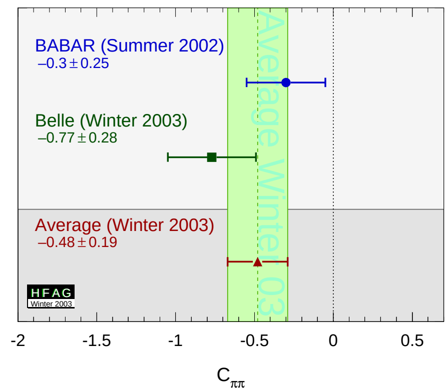


Figure 4: *Summary of Δm_s measurements using the amplitude method (left)[8]. Direct CP violation component in $B^0 \to \pi^+\pi^-$ (right)[8].*

result, where the error is still limited by statistics. An interesting feature evolves in the measurements of the CP violation using the $B^0 \to \phi K_s$ decay mode with only penguin processes contributing to the amplitude. These theoretically clean decays are expected to yield a value of $\sin 2\phi_1$ consistent with the above quoted result. However, the present situation reveals a discrepancy of the two values, the latter being $\sin 2\phi_1 = -0.15 \pm 0.33$ [21]. With an increased statistics one might expect these measurements to reveal new phenomena beyond the SM.

The angle ϕ_2 of the UT can be determined examining the $B^0 \to \pi^+\pi^-$ decays [19]. The amplitude for the process involves a tree digram as well as a significant contribution from the penguin processes, and hence the interpretation of the CP asymmetry measurements is complicated. While the Belle result [22] points to a direct CP violation in these processes (Fig. 4 (right)), the result of the BaBar collaboration [23] is consistent with no direct CP violation. Limits obtained on the angle ϕ_2 from the Belle measurement, taking into account a range of predictions for the ratio of penguin and tree amplitude contributions, are $78° < \phi_2 < 152°$ at 95.5% C.L.

6 UT Constraints

The CKM Fitter group [24] provides a fit to the measured observables and range of theoretical parameters in order to constrain the region in the $(\overline{\rho}, \overline{\eta})$ plane. In the so called Rfit approach one maximizes the $\mathcal{L}(y_{th}) = \mathcal{L}_{exp}(x_{exp} - x_{th}(y_{th})) \cdot \mathcal{L}_{th}(y_{th})$, where measurements x_{exp} and theoretical predictions x_{th}, depending on parameters y_{th}, enter the experimental part $\mathcal{L}_{exp}$. Theoretical part $\mathcal{L}_{th}$ equals unity if the set of parameters is within an allowed range of predictions and vanishes otherwise. Using the set of inputs described at [24] one obtains a 90% C.L. region in $(\overline{\rho}, \overline{\eta})$ plane shown in Fig. 5 (left). At the present, measurements of $\sin 2\phi_1$, $|V_{ub}|$ and Δm_s all impose severe constraints on the position of the UT apex. Fig. 5 (right) includes

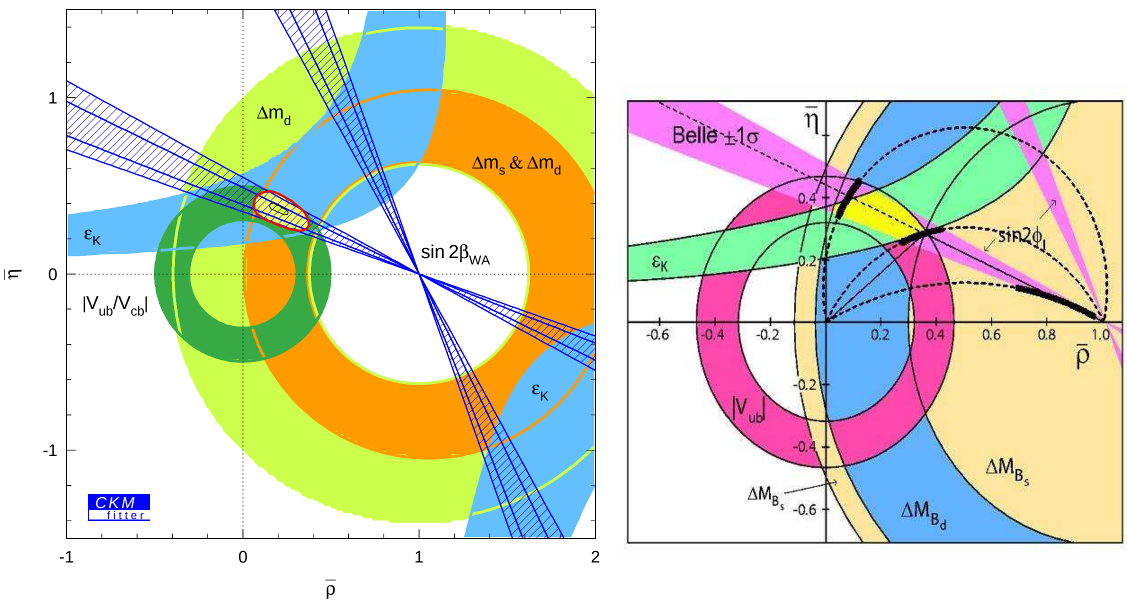


Figure 5: *Constraint on the position of UT apex in the* $(\overline{\rho}, \overline{\eta})$ *plane obtained from the combined fit to CKM elements measurements [24] (left). Illustration of constraints imposed by* $\sin 2\phi_2$ *measurement (right) [25].*

results on $\sin 2\phi_1$ and $\sin 2\phi_2$ by the Belle experiment [25], showing a consistency of measurements.

7 Summary

- $|V_{ud}|$ is measured with a relative precision of 5×10^{-4} in the nuclear super-allowed Fermi transitions. The measurement uncertainty is completely theoretically dominated.

- $|V_{us}|$: a new measurement of K_{e3} branching fraction by E865 experiment resolves the question of the unitarity in the first row of CKM matrix. A possible disagreement between the charged and neutral K_{e3} decay modes will be addressed in the future by KLOE and NA48 experiments.

- $|V_{cb}|$, determined from $B \to D^* \ell \nu$ decays, is limited by the uncertainty in the decay form factor normalization. Inclusive determinations will be improved with new measurements of moments of E_ℓ, q^2 and m_X distributions.

- $|V_{ub}|$: the q^2 dependent measurements have started in exclusive decay channels. New inclusive measurements of $Br(b \to u \ell \nu)$ using m_X and q^2 variables will be performed by BaBar and Belle experiments. Theoretical ambiguities are expected to be resolved through tests of different models (in exclusive channels) and determination of moments of differential distributions ($b \to s\gamma$ transitions).

- $|V_{td}|$, $|V_{ts}|$: while results on Δm_d are already very precise, further important constraints on UT are hoped for from Δm_s measurements by D0 and CDF experiments.

- $\sin 2\phi_1$ has become a precision measurement, new phenomena might arise in the CP asymmetry measurements using the $B^0 \to \phi K_s$ decay channel.

- $\sin 2\phi_2$ determinations have just started and although complicated, these measurements will in the future provide an interesting constraint on the UT.

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