Breakdown of Coherence ?

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Abstract

In a recent paper [1], Albrecht Kyrieleis, Jeff Forshaw and I discovered a new tower of *super-leading* logarithms in gaps between jets cross sections. After discussions with the referee of our paper and further investigation, we have come to view this as a breakdown of naïve coherence for initial state radiation. In this talk I illustrate this statement in a simple way, and show how it results in the super-leading logarithms.

1 Introduction and The Bottom Line

I begin by illustrating, in a simple pictorial way, what I mean by naïve coherence. Consider an arbitrary hard process that produces a hard parton, which then fragments into a system of hard collinear partons, as shown in Fig. 1a. To be precise, by hard collinear I mean that the plus components of all the partons are of the same order as that of the originating parton, and all their transverse momenta are much smaller, with the originating parton defining the plus direction. Consider calculating the first correction to this amplitude coming from a soft wide-angle gluon. Again, to be precise, by soft wide-angle, I mean that its transverse momentum is much smaller than the relative transverse momenta of all collinear partons in the jet, and that its plus momentum is at most of order its transverse momentum. As illustrated in Fig. 1b, this amplitude is obtained from the first one by inserting the soft wide-angle gluon onto each of the external partons, summing over those partons. Studying the integral over the momentum of the soft wide-angle gluon, it is straightforward to see that the momentum-dependent parts of all these insertions are identical and they only differ by colour algebra. It is also straightforward to show, for example using the diagrammatic technique of [2], that the contributions are simply additive in colour space. The final result is therefore, as illustrated in Fig. 1c, that the amplitude can be calculated *as if* the soft wide angle gluon was emitted by an *on-shell* parton with the same plus momentum and colour as the initiating parton. This is the usual statement of naïve colour coherence: soft wide angle gluons are emitted by the jet as a whole, imagined to be on shell.

Now I turn to the case of an initial-state parton, Fig. 2. Consider an arbitrary hard process initiated by a hard parton, which fragments into a system of hard collinear partons and its correction coming from a soft wide-angle gluon, as shown in Figs. 2a and c. At first sight it looks the same as the final-state case and, in fact, if the soft wide-angle gluon is real, it is, so it is as if the soft wide-angle gluon was emitted by the internal line, imagined to be on shell, Fig. 2b. However, if the soft wide-angle gluon is virtual, one has to consider the momentum structure of the loop integral more carefully. Performing one integration by contour, we generally pick up poles from either the soft gluon propagator or the hard parton propagators. The former gives a real part that has an identical form in all cases. The problem then reduces to colour algebra again and, just like

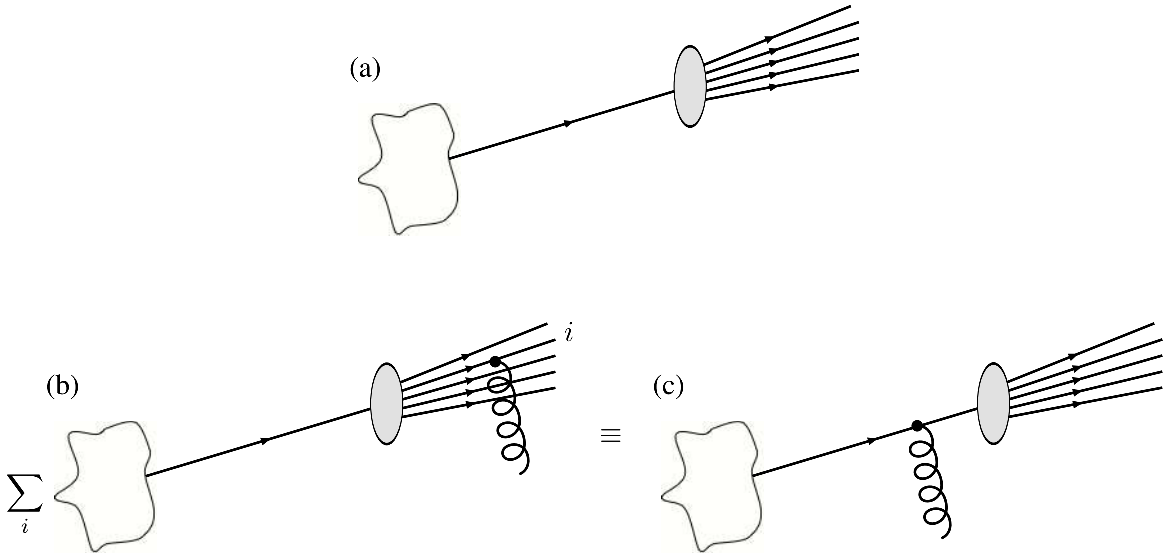

Fig. 1: Illustration of naïve coherence in final-state radiation. A hard parton produced in the hard process fragments into a system of hard collinear partons (a). The amplitude for this system to emit a soft, wide-angle, real or virtual gluon should be calculated from the insertion of the soft gluon onto each of the external hard partons, summed over these partons (b). Colour coherence implies that this can be calculated *as if* the soft gluon were emitted by the original hard parton, i.e. by the total colour charge of the jet (c).

for real emission, it is as if the soft wide-angle gluon was emitted by the internal line, imagined to be on shell, Fig. 2b. However, for the other pole, coming from hard parton propagators, its causal structure depends on whether the hard partons the gluon is attached to are in the final state or the initial state. In particular, the imaginary part is zero if the gluon connects an initial-state parton to a final-state parton[1], and non-zero for initial-initial and final-final connections. Therefore there is a mismatch between the different diagrams in Fig. 2c and they do not correspond to the contribution from a single on-shell parton, Fig. 2d. It is this statement that we describe as a *breakdown of naïve coherence for initial-state radiation*.

We 'discovered' this breakdown of coherence in calculating corrections to the conventional calculations of gaps-between-jets cross sections from one gluon emitted outside the gap accompanied by any number of soft wide-angle gluons. It was a great surprise to us, but we soon learnt that it was actually well known to the early pioneers of QCD. In particular, there are lengthy discussions in the literature of whether or not these known effects (coming from "Coulomb gluons") lead to violations of the Bloch–Nordsiek theorem (see for example Ref. [3]). These issues were eventually settled, at least for massless partons, by Collins, Soper and Sterman's proof of factorization [4]. The hard collinear, and soft real, corrections are quickly dealt with in their paper, and most of the subtlety of their proof is related to gluons with plus and minus momenta much smaller than their transverse momenta (the "Glauber region"), which are exactly the ones that give the imaginary parts we are discussing. They showed that these do lead to violations of

[1] I am working in Feynman gauge.

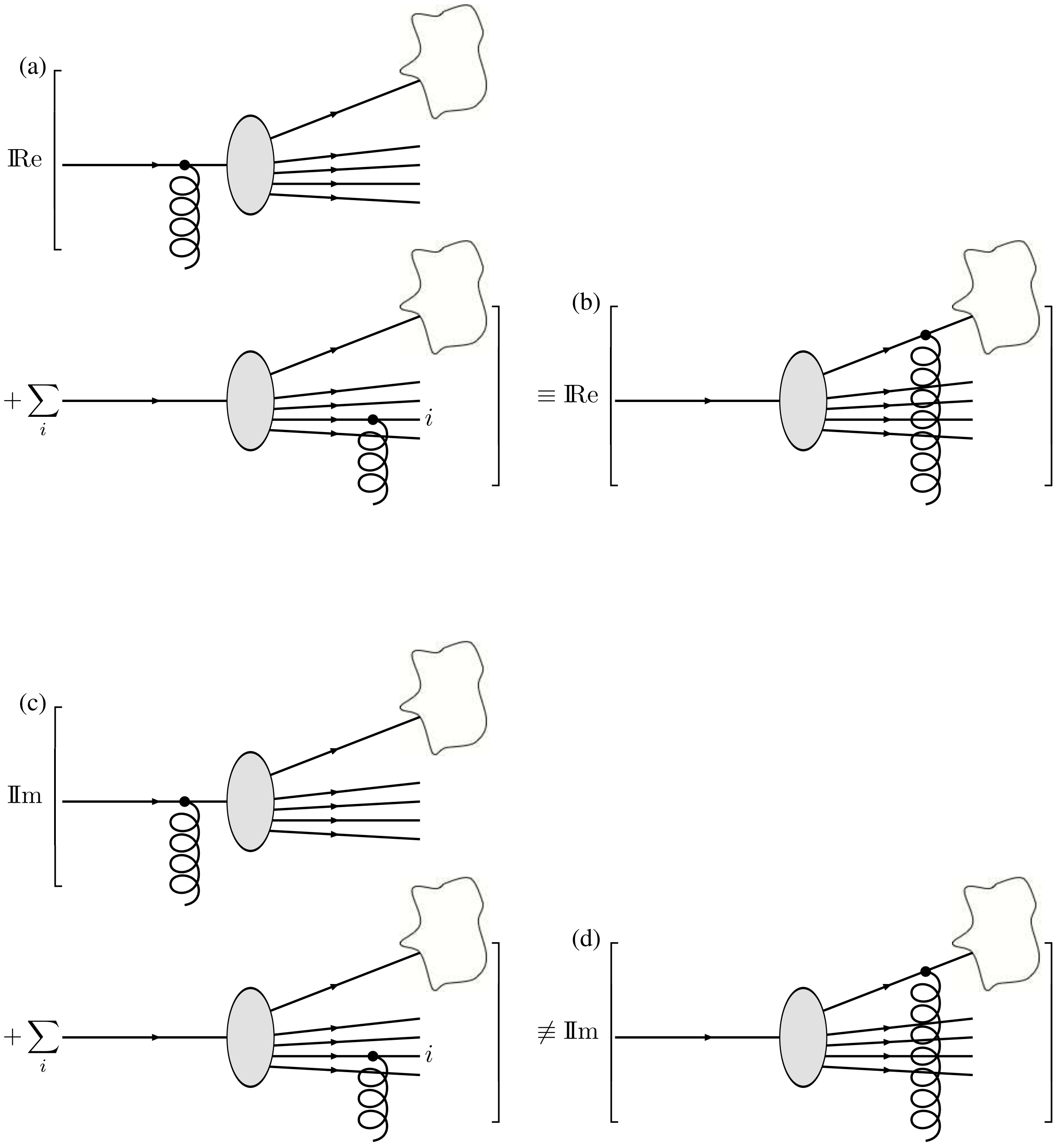

Fig. 2: The analogue of Fig. 1 for initial-state radiation. A hard initial-state parton entering the hard process fragments into a system of hard final-state collinear partons. The amplitude for this system to emit a soft, wide-angle, real or virtual gluon should again be calculated from the insertion of the soft gluon onto each of the external hard partons, summed over these partons (a,c). Because when the soft gluon is virtual the imaginary part of the loop correction is sensitive to the direction of the momentum flow, the colour coherence argument can only be used for real emission and the real part of the loop (b) but not for the imaginary part (d).

factorization in individual diagrams, but that, eventually, these violations cancel each other after summing over all diagrams *for the scattering of colour-singlet incoming hadrons*. Diagrams in which the gluons are attached to the outgoing hadron remnants are essential for this cancelation.

However, in calculating perturbatively-exclusive cross sections, for example the gaps-between-jets cross section defined below, one can perform factorization at the perturbative scale defined by the scale below which the observable is inclusive, and one can calculate the cross section perturbatively using incoming partons defined at this scale. Therefore one cannot appeal to the hadron remnants, and these effects really remain in the cross section.

2 Consequences: Super-Leading Logarithms in Gaps Between Jets Cross Sections

In the remainder of the talk, I discuss the consequences of this breakdown of naïve coherence and, in particular, the appearance of super-leading logarithms in the gaps-between-jets cross section. Here I am simply recapping the results of Ref. [1], so I can be brief.

To define the gap cross section, and the kinematic variables I use to describe it, consider two-jet production at lowest order in hadron collisions. Since I am interested in the soft or collinear corrections, the lowest-order kinematics are sufficient. I define the jets to have transverse momenta Q and to be separated by a (reasonably large) rapidity interval Δy. I define a 'gap' event sample by summing up the total scalar transverse momentum in a rapidity interval of length $Y < \Delta y$ in the region between the two jets and only accepting events in which this summed transverse momentum is less than $Q_0 \leq Q$. Provided Q_0 is well above the confinement scale, this gap cross section is perturbatively calculable. For $Q_0 \ll Q$ it develops large logarithmic corrections at every order that must be summed to all orders to yield a reliable result.

The conventional wisdom for such calculations is that the logarithmic series is $\alpha_s^n \log^n$, which define the leading logs for this process, that these leading logs can be calculated by considering only soft wide-angle virtual gluons stretched between the hard external partons, and that for every real emission outside the gap there is an equal and opposite virtual correction. Our findings contradict all of these points: we find super-leading[2] logarithms $\alpha_s^n \log^{n+1}$. We already expected, based on the work of Dasgupta and Salam [5] contributions from emission from gluons outside the gap, but in distinction to their result which is an edge effect: emission just outside the gap produces radiation just inside, the effect we find comes from emission arbitrarily far outside the gap. These results are directly related to a real–virtual mis-cancellation due to Coulomb gluon effects and ultimately due to the breakdown of naïve coherence for initial-state radiation.

To illustrate how these effects ultimately give rise to the super-leading logarithms, I briefly recap the ingredients of the 'conventional' calculations for gaps-between-jets developed by Sterman and others over many years [6], first in the simpler setting of e^+e^- annihilation.

[2] I should clear up one possible point of confusion: by super-leading we do not mean that there are more than the two logarithms per order expected from QCD, but only that this observable, which is expected to have only soft contributions, so one logarithm per power of α_s, actually develops additional collinear logarithms at high orders, which we call super-leading since they are beyond the expected soft-only tower. More precisely, one power of Y that appears in the coefficient of the leading logarithm, and is the remnant of the collinear logarithm, gets promoted to become a logarithm of Q/Q_0.

2.1 Gaps in e^+e^- annihilation

The lowest order process produces a quark of momentum p_1 and an antiquark of momentum p_2. Its amplitude is defined to be $\mathcal{A}$. The one-loop correction in the Feynman gauge is given by the single diagram with a gluon of momentum k stretched between p_1 and p_2. In order to extract the leading logarithms, the eikonal approximation is sufficient. Performing the loop integral over k by contour, one picks up poles at $k^2 = 0$ and at $p_1^2 = p_2^2 = 0$. The former gives a contribution that has exactly the form of a phase-space integral for a real gluon emission and leads to a term in the cross section that is exactly equal and opposite to the real-emission cross section. The conventional calculation uses this fact, by assuming that the real–virtual cancellation is perfect for transverse momenta below Q_0 and rapidities outside the gap, so that the entire first-order correction can be calculated from the loop diagram integrated over the disallowed region of phase space,

$$\mathcal{A}_1 = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F\left(Y - i\pi\right)\mathcal{A}_0, \tag{1}$$

where the Y term is the integral of the $k^2 = 0$ pole over the gap region and the $i\pi$ term comes from the $p_1^2 = p_2^2 = 0$ pole. To obtain the leading logarithmic contribution at nth order, one can simply nest the k_t integral n times and obtain

$$\mathcal{A} = e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0. \tag{2}$$

The gap cross section is then given by

$$\sigma = \mathcal{A}^\star\mathcal{A} = \mathcal{A}_0^\star e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y+i\pi)}e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}C_F(Y-i\pi)}\mathcal{A}_0. \tag{3}$$

It is easy to see that the Coulomb phase terms in the amplitude and its conjugate cancel, having no physical effect.

2.2 Gaps in $2 \to 2$ scattering

In $2 \to 2$ scattering, one can make an exactly analogous calculation, with the one-loop result nesting and exponentiating to give the all-order result. The only difference is that for a hard process involving more than three partons there can be more than one colour structure, so the amplitude becomes a vector in colour space and the loop correction (the $C_F(Y - i\pi)$ in the e^+e^- case) becomes a matrix,

$$\sigma = \mathcal{A}_0^\dagger e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}\Gamma^\dagger}S\,e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}\Gamma}\mathcal{A}_0, \tag{4}$$

where S is the metric of the colour space. The simplest case is quark scattering, in which the colour space is 2 dimensional, and the anomalous dimension matrix Γ is given by[3]

$$\Gamma = \begin{pmatrix} 0 & \frac{N_c^2-1}{4N_c^2}i\pi \\ i\pi & \frac{N_c}{2}Y - \frac{1}{N_c}i\pi \end{pmatrix}. \tag{5}$$

The important point is that Γ and $\Gamma^\dagger$ do not commute, so the Coulomb phase terms do not cancel. Instead, they are responsible for important physical effects, giving rise to the ‘BFKL’-type logarithms in the limit of large Y [8].

[3] I am grateful to Lev Lipatov for pointing out that this matrix was first calculated in Ref. [7].

2.3 Emission outside the gap

The main point of Ref. [1] was to check whether emission outside the gap really cancels to all orders, as is observed in the lowest order case, and as is assumed in the structure of the all-order calculation. To do this, we explicitly calculated the cross section for one (real or virtual) gluon outside the gap, summed over any number of soft virtual gluons integrated inside the gap and any number of Coulomb gluons. The result is simply the sum of the all-order corrected virtual and real terms, integrated over the out-of-gap phase space,

$$\sigma_1 = -\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t}{k_t}\int_{out}\frac{dy\,d\phi}{2\pi}\left(\Omega_V+\Omega_R\right). \tag{6}$$

Ω_V corresponds to one virtual emission outside the gap and its all-order evolution. It has a very similar structure to the conventional gap cross section,

$$\Omega_V = \mathcal{A}_0^\dagger e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{Q}\frac{dk_t'}{k_t'}\Gamma^\dagger} S_V e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_t}\frac{dk_t'}{k_t'}\Gamma}\gamma\, e^{-\frac{2\alpha_s}{\pi}\int_{k_t}^{Q}\frac{dk_t'}{k_t'}\Gamma}\mathcal{A}_0 + c.c., \tag{7}$$

where γ describes the virtual emission (roughly speaking it is the differential of Γ) and I have just renamed S to S_V for a reason that will be seen shortly. The real part has a more complicated structure, because it involves the evolution of a five-parton system at scales below k_t (the soft wide-angle gluon can be attached to the real out-of-gap gluon, in addition to the original four partons). The five-parton colour structure has a different (higher) dimensionality (four for the simplest case, $qq \to qqg$ for which the anomalous dimension matrix, Λ, was calculated in Ref. [9]) so the real emission matrix element, D^μ, is a rectangular matrix acting on the colour space of the four-parton process on the right and of the five-parton process on the left. The structure is then

$$\Omega_R = \mathcal{A}_0^\dagger e^{-\frac{2\alpha_s}{\pi}\int_{k_t}^{Q}\frac{dk_t'}{k_t'}\Gamma^\dagger} D_\mu^\dagger e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_t}\frac{dk_t'}{k_t'}\Lambda^\dagger} S_R e^{-\frac{2\alpha_s}{\pi}\int_{Q_0}^{k_t}\frac{dk_t'}{k_t'}\Lambda} D^\mu\, e^{-\frac{2\alpha_s}{\pi}\int_{k_t}^{Q}\frac{dk_t'}{k_t'}\Gamma}\mathcal{A}_0, \tag{8}$$

where S_R is the metric of the five-parton colour space.

The out-of-gap gluon must be integrated everywhere outside the gap, including right into the collinear regions, in which Ω_V and Ω_R are separately divergent. It is easy to check that in the final-state collinear region, they indeed become equal and opposite and the singularities cancel. In the initial-state collinear limit[4] however, one finds

$$\Omega_V + \Omega_R \overset{|y|\to\infty}{\longrightarrow} \text{const.} \tag{9}$$

This means that in the pure eikonal theory the cross section is not well-behaved, because the contribution from hard collinear configurations becomes significant. This non-cancellation can be traced to the Coulomb phase terms in the evolution matrices, and ultimately to the breakdown of naïve coherence discussed earlier.

Having made this discovery, it is easy to see how this behaviour leads to the superleading logarithms we observed. To leading approximation, the effect of incorporating the correct splitting functions, energy conservation, etc, in the collinear limit is to introduce an effective cutoff on

[4]This means the rapidity tending to infinity, but at fixed k_t, so the emission never becomes truly collinear.

the rapidity range over which the eikonal result should be integrated, $y_{max} \sim \ln \frac{Q}{k_t}$. The nested integrals over k_t then have one additional log of k_t, leading to one additional log of Q/Q_0,

$$\sigma_1 \sim \sigma_0 \left(\frac{2\alpha_s}{\pi}\right)^4 \pi^2 Y \ln^5 \frac{Q}{Q_0} + \mathcal{O}\left(\alpha_s^n \ln^{n+1} \frac{Q}{Q_0}\right). \tag{10}$$

3 Open Issues

I end this talk by briefly mentioning some of the many open issues that remain.

I stated that we do not need to consider soft gluons attached to the hadron remnants. A simple estimate shows that this has to be the case. Since we are only interested in gluons with transverse momenta above Q_0, even in the Glauber region, and we assume that Q_0 is large relative to the hadronic scale, any such corrections should be suppressed by powers of Q_0. Nevertheless, since a number of objections have been made in this direction, it would be worth working through the first such correction, to shore up this argument.

Once we accept the breakdown of naïve coherence, the choice of ordering variable becomes relevant. Our calculation is based on transverse momentum ordering and different ordering variables might give different coefficients for the super-leading logarithms. Further work, for example by developing a full diagrammatic approach, is needed to be sure that transverse momentum ordering leads to the correct physical results.

Once we have found that one gluon outside the gap gives a tower of terms enhanced by one additional logarithm, it is natural to speculate that n gluons outside the gap will give n additional logarithms [1]. If this is right it would mean that at each order, the leading term is actually $\alpha_s^n \log^{2n-3}$ and it would be imperative to organize and sum these terms to all orders. Performing such a resummation is a daunting task, since, like an exact calculation of non-global logarithms, it would depend on the full colour structure of multi-parton ensembles.

I close by mentioning that I look forward to a critical appraisal of this work. The result came as such a surprise to us that we felt sure it was wrong. Two years of checking has not diminished this feeling. However we have certainly ruled out simple error, since, in addition to the two independent calculations we made, James Keates recently succeeded in constructing an algorithm that generates all possible cut diagrams order by order and evaluates them [10]. Within the same strongly-ordered-in-k_t approximation that we use, his calculation reproduces ours up to fourth order, and confirms the coefficient of the first super-leading logarithm. Once issues of calculational speed have been solved, he will be able to run at fifth order and beyond and check our speculation about the rôle of multiple gluons outside the gap.

In the meantime we are trying to obtain a deeper understanding of our findings, and I welcome any comments that help us in this direction.

Acknowledgements

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