# Bootstrapping Superconformal Field Theories

Madalena Lemos

Classical and quantum symmetries in mathematics and physics
Jena, Jul. 28 2016

Based on:
1312.5344 w/ C. Beem, P. Liendo, W. Peelaers, L. Rastelli and B. van Rees

1511.07449 w/ P. Liendo

# Outline

1. **The (Super)conformal Bootstrap Program**
   - Conformal bootstrap
   - Superconformal bootstrap
2. **A solvable subsector**
3. **Constraining the space of $\mathcal{N} = 2$ SCFTs**
4. **Summary and Outlook**

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## Can we bootstrap specific theories?

$\rightarrow$ Particularly helpful if theory is uniquely fixed by a set of discrete data

$\rightarrow$ Only tool available for finite $N$ non-Lagrangian theories, such as the $6d$ $(2, 0)$ theory

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Set of local operators and their correlation functions

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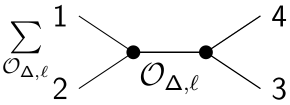
**CFT data strongly constrained**

- Unitarity
- Associativity of the operator product algebra

# Conformal Bootstrap

## Crossing Symmetry

$\langle(\mathcal{O}_1(x_1)\ \mathcal{O}_2(x_2))\mathcal{O}_3(x_3)\ \mathcal{O}_4(x_4)\rangle =$



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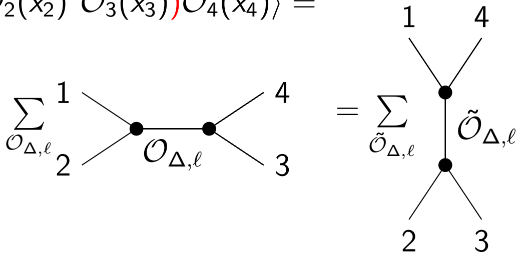

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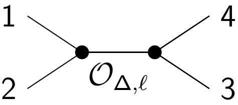
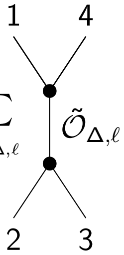
$\sum_{\mathcal{O}_{\Delta,\ell}}$ 

$= \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}}$ 


$$\frac{1}{x_{12}^{2\Delta_{\mathcal{O}}} x_{34}^{2\Delta_{\mathcal{O}}}} \sum_{\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_2\mathcal{O}_{\Delta,\ell}} \lambda_{\mathcal{O}_3\mathcal{O}_4\mathcal{O}_{\Delta,\ell}} g_{\Delta,\ell}(u,v) =$$

where $\Delta_{\mathcal{O}_i} = \Delta_{\mathcal{O}}$, $u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} = z\bar{z}$, $v = \frac{x_{23}^2 x_{14}^2}{x_{13}^2 x_{24}^2} = (1-z)(1-\bar{z})$

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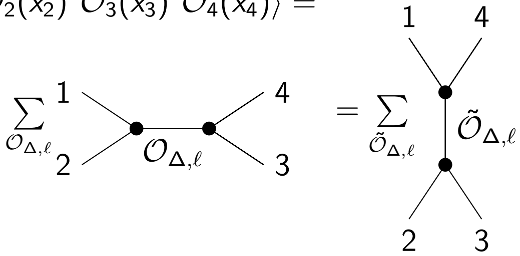


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$$\frac{1}{x_{14}^{2\Delta_{\mathcal{O}}} x_{23}^{2\Delta_{\mathcal{O}}}} \sum_{\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda_{\mathcal{O}_1\mathcal{O}_4\tilde{\mathcal{O}}_{\Delta,\ell}} \lambda_{\mathcal{O}_2\mathcal{O}_3\tilde{\mathcal{O}}_{\Delta,\ell}} g_{\Delta,\ell}(v,u)$$

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## Sum rule: identical scalars $\phi$

$\rightarrow$ Identity operator $\lambda_{\mathcal{O}\mathcal{O}\mathbb{1}} = 1$

$$\boxed{1 = \sum_{\substack{\mathcal{O}_{\Delta_\ell} \neq \mathbb{1} \\ \mathcal{O} \in \phi\phi}} \lambda^2_{\phi\phi\mathcal{O}} \underbrace{\frac{u^{\Delta_\phi} g_{\Delta,\ell}(v,u) - v^{\Delta_\phi} g_{\Delta,\ell}(u,v)}{v^{\Delta_\phi} - u^{\Delta_\phi}}}_{F_{\Delta,\ell}}}$$

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$\rightarrow$ Always true bounds

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$6d$ $\mathcal{N} = (2, 0)$ and $2d$ $\mathcal{N} = (0, 4)$ [Beem Rastelli van Rees]

- Step 1: Solve this subsector
- Step 2: Full blown numerics for the rest

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→ Meromorphic!

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$\rightarrow$ $h = R + j_1 + j_2$

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**Example: free hypermultiplet**

Complex scalars in hypermultiplet are in the cohomology

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Complex scalars in hypermultiplet are in the cohomology

$$Q^I = \begin{bmatrix} Q \\ \tilde{Q}^\star \end{bmatrix} , \qquad \tilde{Q}^I = \begin{bmatrix} \tilde{Q} \\ -Q^\star \end{bmatrix}$$

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$$u_I = (1, \bar{z})$$

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$\hookrightarrow$ $\boxed{c_{2d} = -12c_{4d}}$

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$$J^a(z)J^b(0) \sim -\frac{k_{4d}/2\delta^{ab}}{z^2} + if^{abc}\frac{J^c(0)}{z} + \dots ,$$

$\hookrightarrow$ $\boxed{k_{2d} = -\frac{k_{4d}}{2}}$

$\rightarrow$ ...

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**$4d$ $\mathcal{N}=2$ SCFTs with a flavor symmetry**

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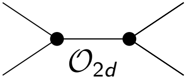
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 

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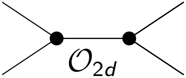
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 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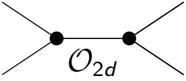
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 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(with some assumptions, interacting theory, unique stress tensor)

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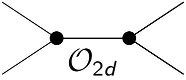
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 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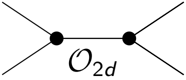
$\sum_{\mathcal{O}_{2d}} \lambda^2_{\mathcal{O}_{2d}}$ 


$\rightarrow \; \lambda^2_{\mathcal{O}_{2d}} \rightsquigarrow \lambda^2_{\mathcal{O}_{4d}} \underbrace{\geqslant}_{4d \text{ unitarity}} 0 \; \Rightarrow$ New unitarity bounds

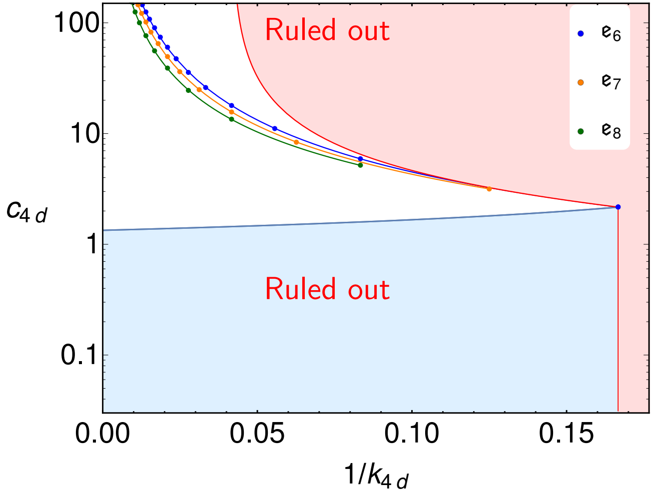
(with some assumptions, interacting theory, unique stress tensor)

# Outline

1. **The (Super)conformal Bootstrap Program**
   - Conformal bootstrap
   - Superconformal bootstrap
2. **A solvable subsector**
3. **Constraining the space of $\mathcal{N} = 2$ SCFTs**
4. **Summary and Outlook**

# $4d$ $\mathcal{N} = 2$ SCFTs with $E_6$ flavor symmetry

$\langle TTTT \rangle$, $\langle J^a J^b J^c J^d \rangle$, $\langle TTJ^a J^b \rangle$




[Beem ML Liendo Peelaers Rastelli van Rees, ML Liendo]

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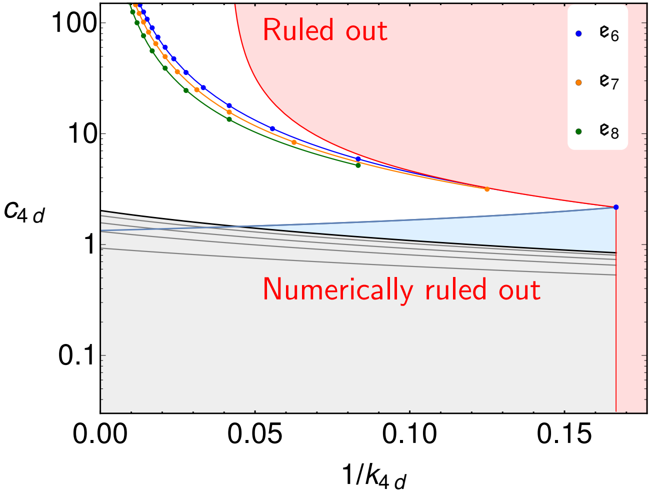


[Beem ML Liendo Rastelli van Rees]

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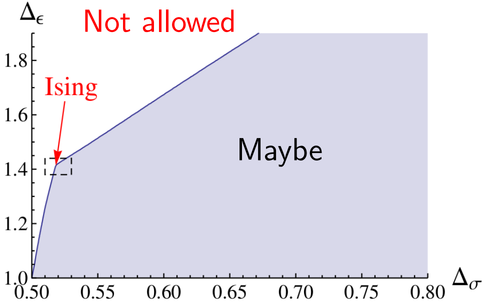
$\rightarrow$ Is $c$ bounded by $k$ for large $k$?

**Thank you!**

# Backup slides

# $3d$ Ising Model

[El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, Vichi, PRD 86 025022]



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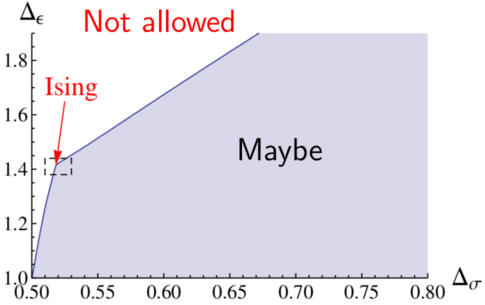


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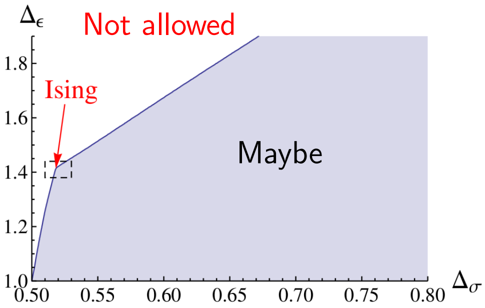


$\rightarrow$ Saturated by 3$d$ Ising model

$\rightarrow$ 3$d$ Ising lives at "kink"

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