

# Non-singlet coefficient functions for charged-current deep-inelastic scattering to the third order in QCD

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We have calculated the coefficient functions for the structure functions  $F_2$ ,  $F_L$  and  $F_3$  in  $\nu - \bar{\nu}$  charged-current deep-inelastic scattering (DIS) at the third order in the strong coupling  $\alpha_s$ , thus completing the description of unpolarized inclusive  $W^\pm$ -exchange DIS to this order of massless perturbative QCD. In this brief note, our new results are presented in terms of compact approximate expressions that are sufficiently accurate for phenomenological analyses. For the benefit of such analyses we also collect, in a unified notation, the corresponding lower-order contributions and the flavour non-singlet coefficient functions for  $\nu + \bar{\nu}$  charged-current DIS. The behaviour of all six third-order coefficient functions at small Bjorken- $x$  is briefly discussed.

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## 1. Introduction

With six structure functions,  $F_2$ ,  $F_L$  and  $F_3$  for  $W^+$  and  $W^-$  exchange [1, 2], inclusive charged-current DIS is an important source of information on the parton structure of nucleons and nuclei and on Standard Model parameters such as the strong coupling  $\alpha_s$  and the weak mixing  $\sin^2 \theta_W$  [3]. Future facilities, e.g., the LHeC [4], will be required to fully realize its phenomenological potential. Charged-current structure functions are, at very small values of the Bjorken variable  $x$ , also of interest for the scattering of high-energy cosmic neutrinos, see Ref. [5] and references therein.

Here we address the corresponding coefficient functions (mass-factorized partonic cross sections) in massless perturbative QCD. These functions are relevant also beyond DIS, e.g., for Higgs production via vector boson fusion [6, 7]. For recent progress on heavy-quark effects see Ref. [8].

## 2. First-order and $\nu + \bar{\nu}$ non-singlet coefficient functions

The first-order coefficient functions for unpolarized inclusive DIS were derived in the early days of QCD, see Refs. [1, 9]. The second- and third-order contributions for the  $\nu + \bar{\nu}$  charged-current case have been calculated in Refs. [10–17]. Here we collect the corresponding flavour non-singlet results in the  $\overline{\text{MS}}$  scheme for the standard choice  $\mu_F = \mu_R = Q$  of the renormalization and factorization scales. The additional contributions at other values of  $\mu_F$  and/or  $\mu_R$  are determined by these results and the corresponding splitting functions [18, 19]; see, e.g., Eqs. (2.16) – (2.18) in Ref. [20].

We denote the non-singlet quark coefficient functions for the charged-current structure functions  $F_{2,3,L}^{\nu p \pm \bar{\nu} p}(x, Q^2)$  in neutrino-proton DIS by  $C_{a,\pm}$ , and write their perturbative expansion as

$$C_{a,\pm}(x, Q^2) = \sum_{n=0} a_s^n c_{a,\pm}^{(n)}(x) \quad \text{with} \quad a_s \equiv \alpha_s(Q^2)/(4\pi). \quad (2.1)$$

In this notation the zeroth- and first-order coefficient functions are given by

$$c_{2,\pm}^{(0)}(x) = c_{3,\pm}^{(0)}(x) = \delta(x_1), \quad c_{L,\pm}^{(0)}(x) = 0, \quad c_{L,\pm}^{(1)}(x) = 4C_F x, \quad (2.2)$$

$$c_{2,\pm}^{(1)}(x) = C_F \{4\mathcal{D}_1 - 3\mathcal{D}_0 - (9 + 4\zeta_2)\delta(x_1) - 2(1+x)(L_1 - L_0) - 4x_1^{-1}L_0 + 6 + 4x\}, \quad (2.3)$$

$$c_{3,\pm}^{(1)}(x) = c_{2,\pm}^{(1)}(x) - 2C_F(1+x) \quad (2.4)$$

with  $C_F = (n_c^2 - 1)/(2n_c) = 4/3$  in QCD. Here and below we use the abbreviations

$$x_1 = 1 - x, \quad L_0 = \ln x, \quad L_1 = \ln x_1, \quad \mathcal{D}_k = [x_1^{-1}L_1^k]_+, \quad (2.5)$$

where  $[a(x)]_+$  denotes +-distributions defined via  $\int_0^1 dx [a(x)]_+ f(x) \equiv \int_0^1 dx a(x) \{f(x) - f(1)\}$ .

The coefficient functions  $c_{a,+}^{(n)}$  and  $c_{a,-}^{(n)}$  differ at  $n > 1$ . The 2<sup>nd</sup>- and 3<sup>rd</sup>-order contributions to the former quantities read, in an approximate but sufficiently accurate form given in Refs. [15–17],

$$\begin{aligned} c_{2,+}^{(2)}(x) \cong & 128/9 \mathcal{D}_3 - 184/3 \mathcal{D}_2 - 31.1052 \mathcal{D}_1 + 188.641 \mathcal{D}_0 - 338.513 \delta(x_1) - 17.74 L_1^3 \\ & + 72.24 L_1^2 - 628.8 L_1 - 181.0 - 806.7x + L_0 L_1 (37.75 L_0 - 147.1 L_1) \\ & + 0.719 x L_0^4 - 28.384 L_0 - 20.70 L_0^2 - 80/27 L_0^3 \end{aligned} \quad (2.6)$$

$$\begin{aligned} & + n_f \{ 16/9 \mathcal{D}_2 - 232/27 \mathcal{D}_1 + 6.34888 \mathcal{D}_0 + 46.8531 \delta(x_1) - 1.500 L_1^2 + 24.87 L_1 \\ & - 7.8109 - 17.82x - 12.97x^2 + 8.113 L_0 L_1 - 0.185 x L_0^3 + 16/3 L_0 + 20/9 L_0^2 \}, \\ c_{L,+}^{(2)}(x) \cong & -37.338 + 89.53x + 33.82x^2 + 128/9 x L_1^2 - 46.50 x L_1 + x L_0 (32.90 + 18.41 L_0) \\ & - 84.094 L_0 L_1 - 128/9 L_0 - 0.012 \delta(x_1) + 16/27 n_f \{ 6x L_1 - 12x L_0 - 25x + 6 \}, \end{aligned} \quad (2.7)$$

$$\begin{aligned}
c_{3,+}^{(2)}(x) \cong & 128/9 \mathcal{D}_3 - 184/3 \mathcal{D}_2 - 31.1052 \mathcal{D}_1 + 188.641 \mathcal{D}_0 - 338.572 \delta(x_1) - 16.40 L_1^3 \\
& + 78.46 L_1^2 - 470.6 L_1 - 149.75 - 693.2x + 0.218x L_0^4 + L_0 L_1 (33.62 L_0 - 117.8 L_1) \\
& - 49.30 L_0 - 94/3 L_0^2 - 104/27 L_0^3 \\
& + n_f \{ 16/9 \mathcal{D}_2 - 232/27 \mathcal{D}_1 + 6.34888 \mathcal{D}_0 + 46.8464 \delta(x_1) + 0.066 L_1^3 - 0.663 L_1^2 \\
& + 24.86 L_1 - 5.738 - 5.845x - 10.235x^2 - 0.190x L_0^3 + 4.265 L_0 L_1 + 20/9 L_0(4 + L_0) \}
\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}
c_{2,+}^{(3)}(x) \cong & 512/27 \mathcal{D}_5 - 5440/27 \mathcal{D}_4 + 501.099 \mathcal{D}_3 + 1171.54 \mathcal{D}_2 - 7328.45 \mathcal{D}_1 + 4442.76 \mathcal{D}_0 \\
& - 9170.38 \delta(x_1) - 512/27 L_1^5 + 704/3 L_1^4 - 3368 L_1^3 - 2978 L_1^2 + 18832 L_1 - 4926 \\
& + 7725x + 57256x^2 + 12898x^3 - 56000x_1 L_1^2 - L_0 L_1 (6158 + 1836 L_0) + 4.719x L_0^5 \\
& - 775.8 L_0 - 899.6 L_0^2 - 309.1 L_0^3 - 2932/81 L_0^4 - 32/27 L_0^5 \\
& + n_f \{ 640/81 \mathcal{D}_4 - 6592/81 \mathcal{D}_3 + 220.573 \mathcal{D}_2 + 294.906 \mathcal{D}_1 - 729.359 \mathcal{D}_0 \\
& + 2574.687 \delta(x_1) - 640/81 L_1^4 + 153.5 L_1^3 - 828.7 L_1^2 - 501.1 L_1 + 831.6 - 6752x \\
& - 2778x^2 + 171.0x_1 L_1^4 + L_0 L_1 (4365 + 716.2 L_0 - 5983 L_1) + 4.102x L_0^4 + 275.6 L_0 \\
& + 187.3 L_0^2 + 12224/243 L_0^3 + 728/243 L_0^4 \} \\
& + n_f^2 \{ 64/81 \mathcal{D}_3 - 464/81 \mathcal{D}_2 + 7.67505 \mathcal{D}_1 + 1.00830 \mathcal{D}_0 - 103.2366 \delta(x_1) - 64/81 L_1^3 \\
& + 18.21 L_1^2 - 19.09 L_1 + 129.2x + 102.5x^2 + L_0 L_1 (-96.07 - 12.46 L_0 + 85.88 L_1) \\
& - 8.042 L_0 - 1984/243 L_0^2 - 368/243 L_0^3 \} ,
\end{aligned} \tag{2.9}$$

$$\begin{aligned}
c_{L,+}^{(3)}(x) \cong & 512/27 L_1^4 - 177.40 L_1^3 + 650.6 L_1^2 - 2729 L_1 - 2220.5 - 7884x + 4168x^2 \\
& - (844.7 L_0 + 517.3 L_1) L_0 L_1 + (195.6 L_1 - 125.3) x_1 L_1^3 + 208.3x L_0^3 - 1355.7 L_0 \\
& - 7456/27 L_0^2 - 1280/81 L_0^3 + 0.113 \delta(x_1) \\
& + n_f \{ 1024/81 L_1^3 - 112.35 L_1^2 + 344.1 L_1 + 408.4 - 9.345x - 919.3x^2 \\
& + (239.7 + 20.63 L_1) x_1 L_1^2 + (887.3 + 294.5 L_0 - 59.14 L_1) L_0 L_1 - 1792/81 x L_0^3 \\
& + 200.73 L_0 + 64/3 L_0^2 + 0.006 \delta(x_1) \} \\
& + n_f^2 \{ 3x L_1^2 + (6 - 25x) L_1 - 19 + (317/6 - 12 \zeta_2)x - 6x L_0 L_1 + 6x \text{Li}_2(x) + 9x L_0^2 \\
& - (6 - 50x) L_0 \} 64/81 ,
\end{aligned} \tag{2.10}$$

$$\begin{aligned}
c_{3,+}^{(3)}(x) \cong & 512/27 \mathcal{D}_5 - 5440/27 \mathcal{D}_4 + 501.099 \mathcal{D}_3 + 1171.54 \mathcal{D}_2 - 7328.45 \mathcal{D}_1 + 4442.76 \mathcal{D}_0 \\
& - 9172.68 \delta(x_1) - 512/27 L_1^5 + 8896/27 L_1^4 - 1396 L_1^3 + 3990 L_1^2 + 14363 L_1 \\
& - 1853 - 5709x + x x_1 (5600 - 1432x) - L_0 L_1 (4007 + 1312 L_0) - 0.463x L_0^6 \\
& - 293.3 L_0 - 1488 L_0^2 - 496.95 L_0^3 - 4036/81 L_0^4 - 536/405 L_0^5 \\
& + n_f \{ 640/81 \mathcal{D}_4 - 6592/81 \mathcal{D}_3 + 220.573 \mathcal{D}_2 + 294.906 \mathcal{D}_1 - 729.359 \mathcal{D}_0 \\
& + 2575.46 \delta(x_1) - 640/81 L_1^4 + 32576/243 L_1^3 - 660.7 L_1^2 + 959.1 L_1 + 31.95x_1 L_1^4 \\
& + 516.1 - 465.2x + x x_1 (635.3 + 310.4x) + L_0 L_1 (1496 + 270.1 L_0 - 1191 L_1) \\
& - 1.200x L_0^4 + 366.9 L_0 + 305.32 L_0^2 + 48512/729 L_0^3 + 304/81 L_0^4 \} \\
& + n_f^2 \{ 64/81 \mathcal{D}_3 - 464/81 \mathcal{D}_2 + 7.67505 \mathcal{D}_1 + 1.00830 \mathcal{D}_0 - 103.2602 \delta(x_1) - 64/81 L_1^3 \\
& + 992/81 L_1^2 - 49.65 L_1 + 11.32 - x x_1 (44.52 + 11.05x) + 51.94x + 0.0647x L_0^4 \\
& - L_0 L_1 (39.99 + 5.103 L_0 - 16.30 L_1) - 16.00 L_0 - 2848/243 L_0^2 - 368/243 L_0^3 \} \\
& + f l_{02} n_f \{ 2.147 L_1^2 - 24.57 L_1 + 48.79 - x_1 (242.4 - 150.7x) - L_0 L_1 (81.70 + 9.412 L_1) \\
& + x L_0 (218.1 + 82.27 L_0^2) - 477.0 L_0 - 113.4 L_0^2 + 17.26 L_0^3 - 16/27 L_0^5 \} x_1 .
\end{aligned} \tag{2.11}$$

### 3. $\nu - \bar{\nu}$ non-singlet coefficient functions

The differences between corresponding  $\nu + \bar{\nu}$  and  $\nu - \bar{\nu}$  coefficient functions are, as conjectured in Ref. [23], suppressed at large  $x$  by two powers of  $1-x$ . Hence it is convenient to present the coefficient functions for  $\nu - \bar{\nu}$  charged-current DIS in terms of differences which we define as

$$\delta C_{2,L} \equiv C_{2,L}^{\nu p + \bar{\nu} p} - C_{2,L}^{\nu p - \bar{\nu} p}, \quad \delta C_3 \equiv C_3^{\nu p - \bar{\nu} p} - C_3^{\nu p + \bar{\nu} p}. \quad (3.1)$$

The flavour class  $fl_{02}$ , see Fig. 1 of Ref. [16], does not contribute to the flavour asymmetries probed in the  $\nu - \bar{\nu}$  combinations, hence it is understood that the corresponding part of Eq. (2.11) is removed before the difference for  $F_3$  is formed. The  $\delta C_a$  can be perturbatively expanded as

$$\delta C_a = \sum_{n=2} a_s^n \delta c_a^{(n)}, \quad (3.2)$$

where  $a_s$  is defined in Eq. (2.1) above. The second-order results were already given in Ref. [21] in exact and parametrized form. The later results, written in terms of the abbreviations (2.5), read

$$\begin{aligned} \delta c_2^{(2)}(x) \cong & \{-9.1587 - 57.70x + 72.29x^2 - 5.689x^3 - xL_0(68.804 + 24.40L_0 \\ & + 2.958L_0^2) + 0.249L_0 + 8/9L_0^2(2 + L_0)\}x_1, \end{aligned} \quad (3.3)$$

$$\begin{aligned} \delta c_L^{(2)}(x) \cong & \{10.663 - 5.248x - 7.500x^2 + 0.823x^3 + xL_0(11.10 + 2.225L_0 \\ & - 0.128L_0^2) + 64/9L_0\}x_1^2, \end{aligned} \quad (3.4)$$

$$\begin{aligned} \delta c_3^{(2)}(x) \cong & \{-29.65 + 116.05x - 71.74x^2 - 16.18x^3 + xL_0(14.60 + 69.90x \\ & - 0.378L_0^2) - 8.560L_0 + 8/9L_0^2(4 + L_0)\}x_1. \end{aligned} \quad (3.5)$$

The corresponding third-order corrections are the new results of the present contribution. They supersede the previous approximate expressions in Eqs. (3.7) – (3.9) of Ref. [21], which were based on the lowest five even- integer and odd-integer Mellin moments of  $C_{3,-}$  and  $C_{a,-}$ ,  $a = 2, L$ , respectively, computed in Ref. [22]. Our new exact results can be parametrized as

$$\begin{aligned} \delta c_2^{(3)} \cong & \{273.59 - 44.95x - 73.56x^2 + 40.68x^3 + 0.1356L_0^5 + 8.483L_0^4 + 55.90L_0^3 \\ & + 120.67L_0^2 + 388.0L_0 - 329.8L_0L_1 - xL_0(316.2 + 71.63L_0) + 46.30L_1 \\ & + 5.447L_1^2\}x_1 - 0.0008\delta(x_1) \end{aligned} \quad (3.6)$$

$$+ n_f \{(-19.093 + 12.97x + 36.44x^2 - 29.256x^3 - 0.76L_0^4 - 5.317L_0^3 - 19.82L_0^2 - 38.958L_0 - 13.395L_0L_1 + xL_0(14.44 + 17.74L_0) + 1.395L_1)x_1 + 0.0001\delta(x_1)\},$$

$$\begin{aligned} \delta c_L^{(3)} \cong & \{-620.53 - 394.5x + 1609x^2 - 596.2x^3 + 0.217L_0^3 + 62.18L_0^2 + 208.47L_0 \\ & - 482.5L_0L_1 - xL_0(1751 - 197.5L_0) + 105.5L_1 + 0.442L_1^2\}x_1^2 \end{aligned} \quad (3.7)$$

$$+ n_f \{-6.500 - 12.435x + 23.66x^2 + 0.914x^3 + 0.015L_0^3 - 6.627L_0^2 - 31.91L_0 - xL_0(5.711 + 28.635L_0)\}x_1^2,$$

$$\begin{aligned} \delta c_3^{(3)} \cong & \{-553.5 + 1412.5x - 990.3x^2 + 361.1x^3 + 0.1458L_0^5 + 9.688L_0^4 + 90.62L_0^3 \\ & + 83.684L_0^2 - 602.32L_0 - 382.5L_0L_1 - xL_0(2.805 + 325.92L_0) + 133.5L_1 \\ & + 10.135L_1^2\}x_1 - 0.0029\delta(x_1) \end{aligned} \quad (3.8)$$

$$+ n_f \{(-16.777 + 77.78x - 24.81x^2 - 28.89x^3 - 0.7714L_0^4 - 7.701L_0^3 - 21.522L_0^2 - 7.897L_0 - 16.17L_0L_1 + xL_0(43.21 + 67.04L_0) + 1.519L_1)x_1 + 0.00006\delta(x_1)\}.$$

#### 4. Discussion

With the exception of the  $n_f$  part of Eq. (2.7) and the  $n_f^2$  part of Eq. (2.10) which are exact, the second- and third-order expressions in sections 2 and 3 have been obtained by fitting the coefficients not written as fractions in the non-distribution parts to the exact coefficient functions at  $x \geq 10^{-6}$ . Where useful, the coefficients of  $\delta(1-x)$  have been adjusted (even from zero) to fine-tune the accuracy of Mellin moments and convolutions. The resulting accuracy of Eqs. (2.6) – (2.11) and (3.3) – (3.8) and their convolutions with typical quark distributions of hadrons is 0.1% or better except where the functions are very small. Towards smaller  $x$  the accuracy deteriorates, but the results are still accurate to about 1% and 3% at  $x = 10^{-8}$  and  $x = 10^{-10}$ , respectively.

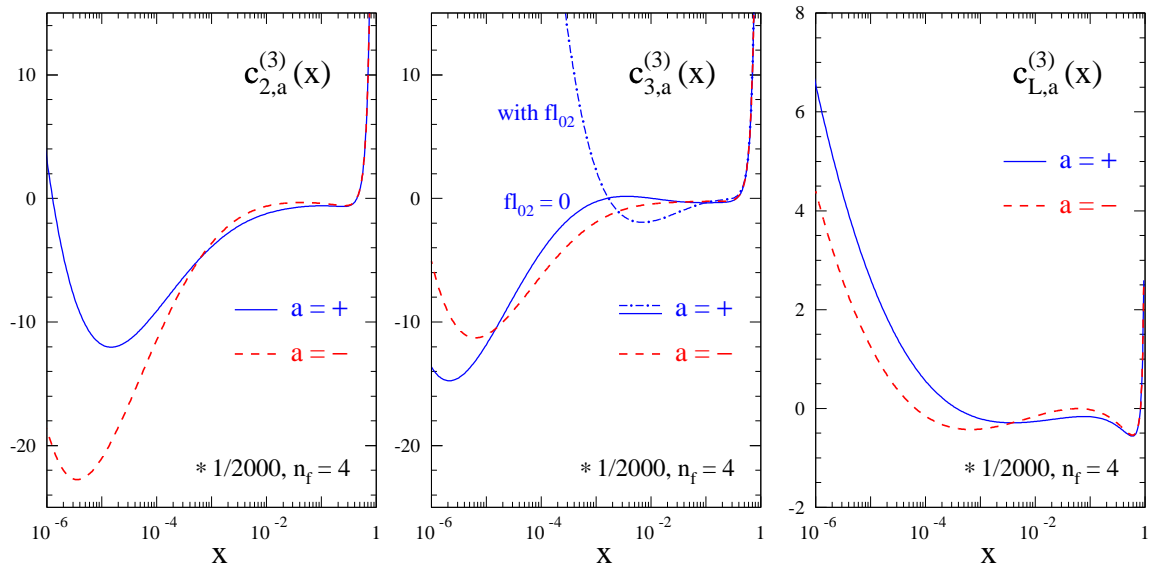
FORTTRAN subroutines of these functions can be obtained from the preprint server ARXIV.ORG by downloading the source of this note. They are also available from the authors upon request.

Analogous parametrizations for the pure-singlet quark and gluon coefficient functions for  $F_2$  and  $F_L$  have been given in Ref. [15] and section 4 of Ref. [16]. The partly very lengthy exact expression corresponding to Eqs. (2.6) – (2.11) and (3.3) – (3.5) can be found in Ref. [16] – where the  $fl_{11}$  contribution has to be disregarded for the present charged-current case – and Refs. [17,21]; those for Eqs. (3.6) – (3.8) will be presented in Ref. [25]. Only the latter expressions allow for the analytical calculation of all integer Mellin moments of the coefficient functions.

The second moments of  $\delta c_2^{(3)}$  and  $\delta c_L^{(3)}$  are of particular relevance, since they enter the QCD corrections to the Paschos-Wolfenstein relation [24] for the determination of  $\sin^2 \theta_W$  from charged-current DIS [3]. The truncated numerical values of these moments for  $n_f$  light flavours are

$$\delta c_2^{(3)}(N=2) = -20.4001 + 0.72202 n_f, \quad \delta c_L^{(3)}(N=2) = -24.7755 + 0.80134 n_f. \quad (4.1)$$

Within their error estimates, the previous approximate results for these moments [21] agree with Eq. (4.1). The corresponding analytical expressions will also be presented in Ref. [25].



**Figure 1:** The third-order non-singlet coefficient functions for  $F_2$ ,  $F_L$  and  $F_3$  in  $\nu p + \bar{\nu} p$  ( $a = +$ ) and  $\nu p - \bar{\nu} p$  ( $a = -$ ) charged-current DIS for four light flavours. The function  $c_{3,+}^{(3)}(x)$  in Eq. (2.11) is shown with and without the  $fl_{02}$  contribution. The factor  $1/2000$  approximately converts the curves to an expansion in  $\alpha_s$ .

The  $a_s^3$  coefficients  $c_{a,\pm}^{(3)}$  in Eq. (2.1) are illustrated in Fig. 1 over a wide range in  $x$ . All six functions exhibit a sharp small- $x$  rise, but only at  $x < 10^{-5}$  for  $F_2$  and  $F_3$  and at  $x \lesssim 10^{-4}$  for  $F_L$ . With the exception of the flavour structure  $fl_{02}$  that occurs at three loops for the first time but dominates  $c_{a,+}^{(3)}$  at small  $x$ , at least four of five  $\ln^k x$  small- $x$  terms are required for a good approximation for  $c_{2,\pm}^{(3)}$  and  $c_{3,\pm}^{(3)}$ , and all three such terms for  $c_{L,\pm}^{(3)}$ , even below the  $x$ -range shown in the figure. Further discussions and illustrations of these coefficient functions will be presented in Ref. [25].

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