PHYSICAL REVIEW D **93,** 093005 (2016)

Rare Z decays and neutrino flavor universality

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(Received 2 February 2016; published 9 May 2016)

We study rare four-body decays of the Z-boson involving at least one neutrino and one charged lepton. Large destructive interferences make these decays very sensitive to the Z couplings to neutrinos. As the identified charged leptons can determine the neutrino flavors, these decays probe the universality of the Z couplings to neutrinos. The rare four-body processes could be accurately measured at future lepton colliders, leading to percent level precision.

DOI: 10.1103/PhysRevD.93.093005

I. INTRODUCTION

In the 1990s, the electron-positron colliders working at the Z pole, LEP and SLC, provided critical tests of the Standard Model (SM). The precision measurements of many electroweak observables still set strong bounds on physics beyond the SM. Future circular lepton colliders, like the Circular Electron Positron Collider (CEPC) or Future Circular Collider (FCC-ee, formerly TLEP), with anticipated yields of about 10^{12} Z-bosons [1], could test the SM further by measuring rarer Z decays.

In this work, we focus on the four-body $Z \to jjl\nu_l$, $Z \to ll\nu\nu$, and $Z \to ll'\nu_l\nu_{l'}$ decays, where j stands for a jet. In the SM, these rather clean decays have branching fractions of order of 10^{-8}. Future colliders could bring the precision on their measurements down to the one-percent level. In the first and third channels listed above, the neutrino flavor matches the charged lepton flavor, allowing for identification of the neutrino flavor. Separate sensitivity to the couplings of the Z-boson to each flavor of neutrino is therefore obtained.

Instead of relying on a specific new-physics scenario, we adopt a simplified approach and study the dependence of the decay rates on the relevant couplings. This allows for the identification of the important interference effects which lead to sensitivities comparable to, or better than, the ones obtained from other processes. Existing constraints on the flavor universality of the neutrino neutral currents derive from various other sources and are discussed in detail below. We stress that each observable depends on a different combination of couplings, and that the complementarity of several observables needs to be exploited for constraining the couplings individually. The Z decays we study constitute a fairly direct probe as their dependence on the couplings is often rather simple. For instance, each $Z \to jjl\nu_l$ decay rate probes directly one single coupling of the Z to neutrinos, while two of them enter the $Z \to ll'\nu_l\nu_{l'}$ rate.

While our study is done in a model-independent way, it is still relevant to ask how deviations from the SM could be generated. Since the universality of the couplings of the Z to neutrinos is a consequence of gauge symmetry, it rests on rather robust theoretical grounds. To some extent, it is constrained experimentally by the observed universality of the Z couplings to charged leptons. New-physics scenarios featuring mixings of the Z or neutrinos to new states could alter the Z couplings to neutrinos. The exploration of such models would require dedicated studies that are beyond the scope of the current paper. Thus, the Z decays considered here can be thought of in two ways. First, taking into account our theoretical model-building philosophy, these modes could be treated as "SM candles" to be compared with other probes, like those coming from neutrino experiments. On the other hand, we can look at these decays as a probe of unknown physics. We stress that these decays should be measured experimentally, regardless of theoretical prejudices.

Experimental searches for Z decays to similar final states have been carried out at LEP [2,3]. They focused on specific kinematical regions (with a displaced secondary vertex, or a boosted subsystem) that are populated in the presence of massive sterile neutrinos. A study of the probing power of the FCC-ee on such scenarios has been presented in Refs. [4,5].

II. EXISTING BOUNDS

In this section, we set up the framework for studying neutrino-Z couplings and review the existing constraints.

A. Notations

For the purpose of showcasing the potential of the rare $Z \to jjl\nu_l$, $Z \to ll\nu\nu$, and $Z \to ll'\nu_l\nu_{l'}$ decays to measure the Z-boson couplings to neutrinos, and to compare to existing experimental constraints, we consider minimal modifications of the SM interactions of the Z-boson to neutrinos, rescaling them by a real number,

$$\mathcal{L}_{Z\nu\nu} = -\sum_{l=e,\mu,\tau} C_{\nu_l} \frac{g}{2\cos\theta_W} \bar{\nu}_l \gamma_\rho P_L \nu_l Z^\rho. \tag{1}$$

The SM is recovered when $C_{\nu_l} = 1$. Note that we set the lepton flavor violating off-diagonal couplings to zero, as is the case in the SM. These couplings are highly constrained, for instance, by neutrino oscillations in matter, as will be discussed in Sec. II D. The Hermiticity of the Lagrangian allows phases to be present only in the flavor off-diagonal couplings. Only the known left-handed neutrinos are considered. Therefore, the Lagrangian introduces three new parameters: C_{ν_e}, C_{ν_μ}, and C_{ν_τ}.

We next connect our notations to those used in the context of neutrino experiments. Earlier studies of the nonstandard interactions (NSIs) of neutrinos in neutrino scattering and neutrino oscillation experiments typically used an effective Lagrangian relevant at energies much below the Z mass:

$$\mathcal{L}^{\rm eff} = \mathcal{L}^{\rm eff}_{\rm SM} - \epsilon^{fM}_{\alpha\beta} 2\sqrt{2} G_F (\bar{\nu}_\alpha \gamma_\rho P_L \nu_\beta)(\bar{f}\gamma^\rho P_M f), \tag{2}$$

where $\alpha, \beta = e, \mu, \tau$, $f = e, u, d$ and $M = L, R$. Integrating out the Z-boson in $\mathcal{L}_{\rm SM} + \mathcal{L}_{Z\nu\nu}$, the correspondence between the parameters of Eqs. (1) and (2) is

$$\epsilon^{fM}_{\alpha\alpha} = g^f_M (C_{\nu_\alpha} - 1), \tag{3}$$

where

$$\begin{aligned} g^\ell_L &= -\frac{1}{2} + \sin^2\theta_w, & g^\ell_R &= \sin^2\theta_w, \\ g^u_L &= \frac{1}{2} - \frac{2}{3}\sin^2\theta_w, & g^u_R &= -\frac{2}{3}\sin^2\theta_w, \\ g^d_L &= -\frac{1}{2} + \frac{1}{3}\sin^2\theta_w, & g^d_R &= \frac{1}{3}\sin^2\theta_w. \end{aligned} \tag{4}$$

B. Z pole data

We now review some of the more stringent bounds on the C_{ν_l} couplings. First, the measurement of the total and visible width of the Z constrains the number of neutrinos [6]:

$$N_\nu = 2.984 \pm 0.008. \tag{5}$$

In the parametrization of Eq. (1),

$$N_\nu = C^2_{\nu_e} + C^2_{\nu_\mu} + C^2_{\nu_\tau}, \tag{6}$$

and the measurement translates into a tight bound on the sum of the squared C_{ν_l}'s. Note that the invisible width of the Z does not probe the coupling of each neutrino species separately, and is insensitive of the sign of the couplings.

The $e^+e^- \to \gamma\nu\bar{\nu}$ cross section was also used as a neutrino counting observable, giving $N_\nu = 2.92 \pm 0.05$ [7]. Off the cross-section peak there is sizable interference between the s-channel Z and t-channel W exchange contributions. It produces a linear dependence of the cross section on C_{ν_e}. We estimate the relative difference in cross sections for $C_{\nu_e} = +1$ and $C_{\nu_e} = -1$ to be of the order of 10% near the Z pole (see Fig. 1) and conclude that the possibility of a negative $C_{\nu_e} = -1$ is untenable.

Changes in the way the Z couples to neutrinos can also affect the $Z \to l^+l^-$ partial widths at the one-loop level. Without a gauge invariant framework, one cannot derive reliable bounds on the C_ν couplings. Using naive dimensional analysis, the correction to the $Z \to l^+l^-$ is expected to be around $C_{\nu_l}/16\pi^2$. Given the current experimental precision on the $Z \to l^+l^-$ rates [7], the naive expectation is that a bound of $|C_{\nu_l} - 1| \sim 0.1$ can be obtained within specific models. This is in the same range as other existing bounds discussed in this section. In a specific model, however, and with the future experimental precision, these effects may become important.

C. Neutrino scattering data

Neutrino scatting experiments provide strong constraints on neutrino interactions with electrons and quarks of the first generation. They utilize neutrino beams of known flux

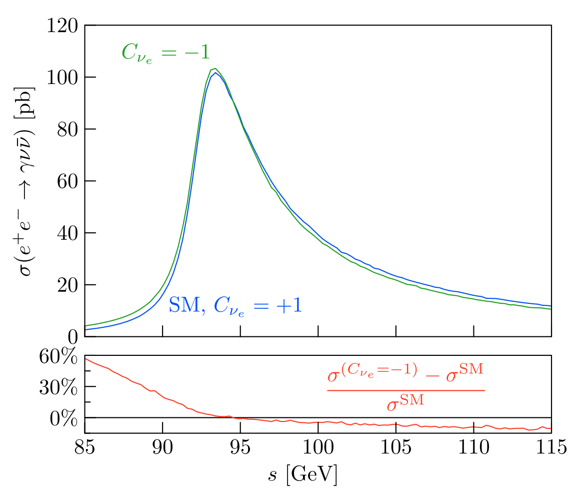

FIG. 1. Cross section of the $e^+e^- \to \gamma\nu\bar{\nu}$ process, as a function of the center-of-mass energy, for $C_{\nu_e} = +1$ and $C_{\nu_e} = -1$. Relative difference between the two distributions obtained with MadGraph5 [8] at leading order for $E_\gamma > 1$ GeV and $45° < \theta_\gamma < 135°$.

and flavor. Combined bounds can be found in Refs. [9–12], including the measurements of the following processes: (i) Electron-neutrino scattering off electrons, $\sigma_{\nu_e e\to\nu e}$, by LSND [13], (ii) the ratio of neutral-current to charged-current scatterings of electron-neutrino off nucleons, $\sigma_{\nu_e N\to\nu X}/\sigma_{\nu_e q\to eX}$, by CHARM [14], (iii) muon-neutrino scattering off elections, $\sigma_{\nu_\mu e\to\nu e}$, by CHARM II [15], and (iv) the ratios of neutral-current to charged-current scatterings of muon-neutrino off nucleons, $\sigma_{\nu_\mu N\to\nu X}/\sigma_{\nu_\mu q\to\mu X}$, by NuTeV [16]. Translating the bounds from $\epsilon_{\alpha\alpha}^{fM}$ to C_{ν_l} gives

$$0.72 < C_{\nu_e} < 1.32, \qquad 0.99 < |C_{\nu_\mu}| < 1.01, \tag{7}$$

with no constraint on C_{ν_τ}. The interference between charged and neutral currents provides a handle on the sign of C_{ν_e} and excludes, here again, a negative value.

Global fits of the electron-neutrino NSI parameters to scattering rate measurements were also performed in Refs. [17–19]. Their inclusion of additional data from MUMU [20], Rovno [21], LAMPF [22], Krasnoyarsk [23] and Texono [24] resulted in tighter bound on C_{ν_e}:

$$0.94 < C_{\nu_e} < 1.07. \tag{8}$$

Note that the constraints on NSI parameters in the above analyses derived from multidimensional fits for the $\epsilon_{\alpha\alpha}^{fP}$ degrees of freedom. Each C_{ν_l} represents only one degree of freedom in the χ^2-distribution. We did not correct for this in Eqs. (7) and (8) and consider these bounds as estimates to be compared with the prospects at future colliders.

D. Neutrino oscillations

As discussed in Refs. [11,25–28], both flavor-diagonal and flavor-changing NSI parameters are constrained by neutrino oscillation experiments. They can affect neutrino production, detection, and propagation through matter.

Atmospheric neutrinos are very sensitive to matter NSIs as they travel a long distance through the Earth. Since the Earth is made up of an approximately equal number of protons, neutrons and electrons, the atmospheric neutrino oscillation experiments bound the quantity

$$\epsilon_{\alpha\beta}^{\oplus} = \sum_M (3\epsilon_{\alpha\beta}^{uM} + 3\epsilon_{\alpha\beta}^{dM} + \epsilon_{\alpha\beta}^{eM}). \tag{9}$$

Experimental studies of the matter NSIs with the Super-Kamiokande atmospheric neutrino data [29,30] result in

$$|\epsilon_{\tau\tau}^{\oplus} - \epsilon_{\mu\mu}^{\oplus}| < 0.147, \tag{10}$$

where the analysis in Ref. [30] assumed that neutrinos interact only with the d-quarks inside the Earth to fix the normalization [31]. Considering the complete Earth contribution, as given in Eq. (9), and using Eq. (4), the bound on C_{ν_l} becomes

$$|C_{\nu_\tau} - C_{\nu_\mu}| < 0.294, \tag{11}$$

which is much weaker than the constraints on C_{ν_l} discussed in Sec. II C.

In our study we do not consider flavor-changing NSI parameters, but briefly discuss the bounds from neutrino oscillations. Super-Kamiokande atmospheric neutrino data yields

$$\epsilon_{\mu\tau}^{\oplus} < 0.033, \tag{12}$$

which was obtained with a normalization that assumes neutrinos interact only with d-quarks [29,30] inside the Earth. While earlier studies [32–34] demanded that reactor experiments bound the NSI parameters $\epsilon_{e\mu}$ and $\epsilon_{e\tau}$, recent analysis [35,36] showed that Daya Bay experiment cannot constrain the flavor-changing NSI parameters because of their strong correlation with the reactor angle θ_{13}. It can, however, bound $|\epsilon_{ee}| < \mathcal{O}(10^{-2})$, when the parameter is considered to be real, and a normalization error in the neutrino flux is taken into account. No bound can be put if an arbitrary phase is allowed. The accelerator neutrino experiments like K2K [37], MINOS [38–43], T2K [44], OPERA [45–47] and NOνA [48] also bound the flavor-changing NSI parameter $\epsilon_{e\tau}$. Future neutrino-factory experiments could test the off-diagonal NSI parameters down to the 10^{-3} level, whereas diagonal NSI parameter combinations such as $(\epsilon_{ee} - \epsilon_{\tau\tau})$ and $(\epsilon_{\mu\mu} - \epsilon_{\tau\tau})$ could only be tested down to 10^{-1} and 10^{-2}, respectively [49].

III. INTERFERENCE PATTERNS

Before turning to the four-body decays of interest and their sensitivity to the C_{ν_l} couplings, we detail the interference pattern of the simpler three-body $Z \to Wl\nu_l$ decays.

They receive three contributions, respectively proportional to the Z couplings to neutrinos, to charged leptons, and to the W-boson. The corresponding diagrams are shown in Fig. 2. From the full analytical expressions given in the Appendix, we obtain the tree-level SM branching fraction:

$$\Gamma^{\mathrm{SM}}(Z \to Wl\nu_l) \simeq 1.99 \times 10^{-8}\ \mathrm{GeV} \tag{13}$$

for each lepton flavor. Given the total Z-boson width of 2.50 GeV, this corresponds to a branching ratio of the order of 10^{-8}. The rates for distinct lepton flavors differ by ratios of the charged lepton masses to the Z-boson mass, which we have neglected in this section.

To examine the pattern of interferences, we momentarily introduce rescaling parameters for the Z couplings to the charged leptons and W-boson, C_l and C_W, respectively. We assume that these parameters are real, as is the case for the C_{ν_l}'s. The total decay rate is then

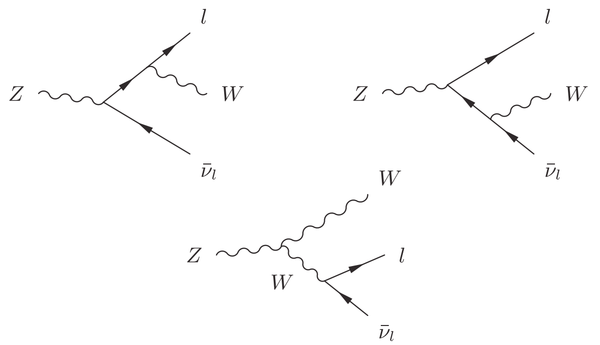

FIG. 2. Tree-level diagrams contributing to $Z \to W l \nu_l$ decays.

$$\frac{\Gamma(Z \to W l \nu_l)}{10^{-8}\ \mathrm{GeV}} \simeq \begin{pmatrix} C_{\nu_l} \\ C_l \\ C_W \end{pmatrix}^T \begin{pmatrix} 1.36 & -0.24 & -1.59 \\ -0.24 & 0.39 & -0.86 \\ -1.59 & -0.86 & 5.63 \end{pmatrix} \begin{pmatrix} C_{\nu_l} \\ C_l \\ C_W \end{pmatrix}. \qquad (14)$$

Since we consider the W as a final-state particle, we have neglected its width here.

The coupling strengths of the Z to neutrinos, charged leptons, and W mainly determine the magnitude of each contribution taken in isolation. Remarkably, all their interferences are destructive and correct the decay rate by a factor of 3.7. Interferences linear in the coupling of the Z to neutrinos alone are responsible for a factor of 2.0. Flipping the sign of C_{ν_l} would increase the partial decay rate by a factor of 4.7. The rate of the $Z \to W l \nu_l$ decay has a high sensitivity to modifications of its magnitude and sign, for each flavor l, individually. Focusing on the neutrino couplings, fixing $C_l = C_W = 1$, the three-body width is

$$\frac{\Gamma(Z \to W l \nu_l)}{10^{-8}\ \mathrm{GeV}} = 4.3 - 3.7 C_{\nu_l} + 1.4 C_{\nu_l}^2. \qquad (15)$$

IV. FOUR-BODY DECAYS

In practice, the observable processes are four-body decays. They receive contributions from diagrams featuring an intermediate $W l \nu_l$ state as well as new contributions that do not derive from the three-body process discussed in the previous section (see Fig. 3). Higher-order corrections could be important given the future experimental accuracy. In this first qualitative study, we only show leading-order results.

We focus on three different channels. In the semileptonic $Z \to jjl\nu_l$ decay, the flavor of the neutrino is fixed by that of the lepton and the coupling of each neutrino to the Z can be probed separately. Among the four-body decays that are sensitive to the Z neutrino couplings, it also has the highest rate. The fully leptonic $Z \to ll'\nu_l\nu_{l'}$ decay involves two leptons of distinct flavors. In that case, interferences between diagrams where the Z couples to neutrinos of different flavors render the analysis more involved. On the other hand, in $Z \to ll\nu\nu$, with two leptons of the same flavor, all species of neutrinos can be produced irrespectively of the flavor of the charged leptons (see, e.g., the first diagram of Fig. 3). The presence of two couplings of the Z to neutrinos in the corresponding diagrams also introduces cubic and quartic C_{ν_l} dependences in the decay rate. However, there are no interferences proportional to two different C_{ν_l}'s in this third channel.

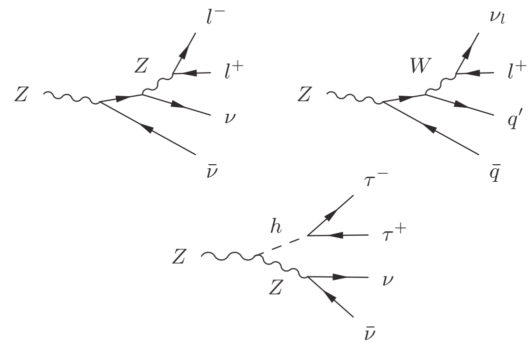

FIG. 3. Some contributions to the four-body $Z \to jjl\nu_l$ and $Z \to ll\nu\bar{\nu}$ decays that do not proceed through a $W l \nu_l$ intermediate state.

Using MadGraph5 [8], we extract the dependence of each decay rate on the C_{ν_l} coefficients. These are given in Eqs. (A6)–(A8) of the Appendix. The τ mass has been kept nonvanishing and marginally affects some of the numerical factors. Because of additional contributions to the four-body decays and of the small phase space available in the three-body decay (which requires an on-shell W), the rate of the four-body processes are much higher than what would have been obtained by using a narrow-width approximation on the three-body decays. Fixing $C_{\nu_l} = 1$, the SM decay widths are

$$\frac{\Gamma^{\mathrm{SM}}(Z \to ll\nu\bar{\nu})}{10^{-8}\ \mathrm{GeV}} \simeq \begin{cases} 2.4 & \text{for } l = e, \mu \\ 2.3 & \text{for } l = \tau \end{cases}$$

$$\frac{\Gamma^{\mathrm{SM}}(Z \to l\nu_l jj)}{10^{-8}\ \mathrm{GeV}} \simeq \begin{cases} 6.5 & \text{for } l = e, \mu \\ 6.3 & \text{for } l = \tau \end{cases}$$

$$\frac{\Gamma^{\mathrm{SM}}(Z \to ll'\nu_l\nu_{l'})}{10^{-8}\ \mathrm{GeV}} \simeq \begin{cases} 1.5 & \text{for } l = e, \quad l' = \mu \\ 1.4 & \text{for } l = e, \mu, \quad l' = \tau \end{cases}$$

for each lepton charge assignment.

The $Z \to jjl\nu_l$ process has the highest rate and the simplest dependence in the C_{ν_l} couplings. When $C_{\nu_l} = -1$, the destructive interferences discussed in Sec. III cause a dramatic increase of the width by a factor of 4.1 (to 27×10^{-8} GeV for $l = e$, μ, and 26×10^{-8} GeV for $l = \tau$). The changes induced by positive C_{ν_l}'s in differential distributions of $Z \to jjl\nu_l$ are moderate (see Fig. 4). An increased sensitivity could be obtained by selecting dijet invariant masses in the [15, 75] GeV interval (see Fig. 5).

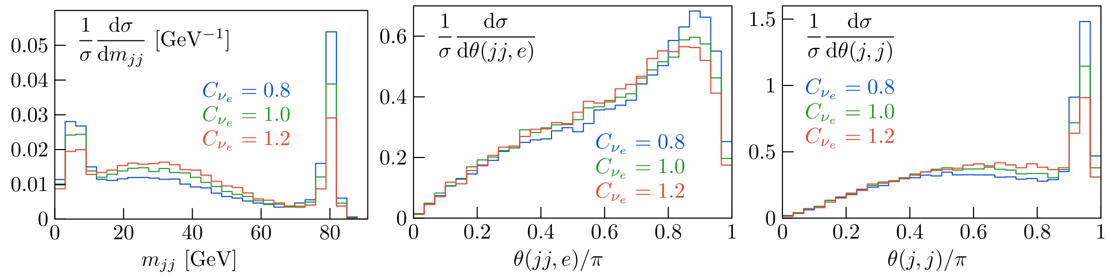

FIG. 4. The effect of positive C_{ν_l} variations on some differential distributions in $Z \to jjl\nu_l$. The dijet invariant mass, angle between the dijet system and charged lepton, and angle between the two jets are displayed.

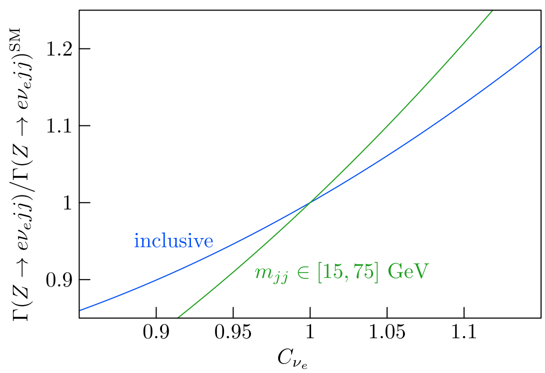

FIG. 5. Improvement obtained in the sensitivity to positive C_{ν_e} when a $m_{jj} \in [15, 75]$ GeV cut is imposed on the dijet invariant mass of the $Z \to jje\nu_e$ decay. Similar results are expected for $Z \to \mu\nu_\mu jj$ and $Z \to \tau\nu_\tau jj$.

Regions of the phase space with enhanced sensitivities could also be studied and exploited in the channels involving two final-state charged leptons.

In the $C_{\nu_e} - C_{\nu_\tau}$ plane of Fig. 6, we display the lines along which several relevant partial decay widths take their SM values. The muon-neutrino coupling to the Z has been

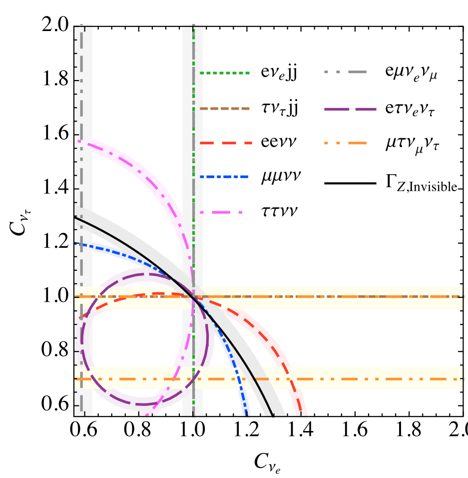

FIG. 6. Bounds on C_{ν_e} and C_{ν_τ} from all the relevant decay channels as listed in Eqs. (A6)–(A8), with $C_{\nu_\mu} = C_{\nu_l}^{\mathrm{SM}} = 1$, assuming 2% uncertainty in the rate measurement.

fixed to its standard-model value $C_{\nu_\mu}^{\mathrm{SM}} = 1$. The bands around each line assume 2% uncertainties on the rate measurements, a precision that should be achieved at future colliders. The current bound on $\sum_l C_{\nu_l}^2$ from the total Z-boson width is also displayed. It should be noted that $Z \to \mu\mu\nu\nu$ has a strong dependence on C_{ν_μ} and the band is expected to get broadened when allowing C_{ν_μ} to vary within its allowed range. A combination of several channels would bring the constrains on the C_{ν_e} and C_{ν_τ} couplings down to the percent level at which the magnitude of C_{ν_μ} is currently known. A negative value for the latter could be unambiguously excluded. Only a tiny volume of the full $C_{\nu_e} - C_{\nu_\tau} - C_{\nu_\mu}$ parameter space would remain allowed if no deviation is observed.

V. CONCLUSIONS

We have demonstrated the high and differentiated sensitivities of certain four-body decays of the Z to its couplings to each flavor of neutrino. They are sourced by large destructive interferences. While, in our study, we concentrated on the coupling of the Z to neutrinos, deviations from the SM could also occur in several other couplings that enter the decays discussed. So, we emphasize that nonstandard interactions should be probed in several independent ways. We expect that future circular colliders running at the Z peak will measure the suggested decays at the one-percent level. Future neutrino scattering and oscillation experiments will also further probe the low-energy limit of interactions that depend on the same couplings of the Z to neutrinos. It is the combination of these experiments that will give the strongest probing power and ensure the robustness of the obtained limits.

There are several possible outcomes to such a program. These experiments may agree with SM predictions, and set stronger bounds on the deviations of the couplings from their SM values. Alternatively, some deviations might be established. In that case, a combination of experiments should be used to identify unambiguously their origin. We can imagine a situation in which neutrino oscillation observations deviate from the SM expectations while Z decays rates agree with them. That could be an indication of a new heavy mediator of neutrino interactions. Another possible scenario could be that of a deviation only found in the four-body decay of the Z involving two charged leptons of identical flavor, but neither in that featuring jets, nor in that involving charged leptons of different flavors. Such an outcome could be explained by a new source of a triple-Z vertex.

ACKNOWLEDGMENTS

We thank Jakub Scholtz for useful discussions. E. K. thanks Josh Ruderman for initial discussion on this project. The work of G. D. was supported in part by the FNRS, Belgium and by the Belgian American Education Foundation, USA. The work of Y. G. and E. K. is supported in part by the U.S. National Science Foundation through Grant No. PHY-1316222 and by a grant from the BSF. M. K. is supported by the DFG Graduate School Symmetry Breaking in Fundamental Interactions (GRK 1581) and is particularly grateful for having had the opportunity to engage in this collaboration, enabled by the European Research Council (ERC) under the Advanced Grant EFT4LHC. S. R. would like to thank the Department of Science and Technology (DST), Government of India for supporting the visit through the INSA-INSPIRE Faculty Fellowship.

APPENDIX: ANALYTICAL AND NUMERICAL RESULTS

1. Analytical results for $Z \to Wl\nu$

The amplitude for the decay $Z \to Wl\nu$ is obtained from evaluating the diagrams shown in Fig. 2. We assign the momenta k^μ, p^μ and q^μ to the final state W, lepton and neutrino respectively. Allowing for modifications of the SM couplings, the amplitude can be written as

$$i\mathcal{A}_{Z\to Wl\nu} = -i\bar{u}(p)[\Delta_\nu^Z A_1^{\mu\nu} + \Delta_l^Z A_2^{\mu\nu} + \Delta_W^Z A_3^{\mu\nu}]P_L v(q)\varepsilon_\mu^Z \varepsilon_\nu^{W*},$$

with

$$\begin{aligned} A_1^{\mu\nu} &= \frac{g^2}{\sqrt{2}c_W}(g_V^\nu + g_A^\nu)\gamma^\nu \frac{\not{p}+\not{k}}{(p+k)^2}\gamma^\mu, \\ A_2^{\mu\nu} &= -\frac{g^2}{\sqrt{2}c_W}(g_V^l + g_A^l)\gamma^\mu \frac{\not{k}+\not{q}}{(k+q)^2}\gamma^\nu, \\ A_3^{\mu\nu} &= \frac{g^2 c_W}{\sqrt{2}} \frac{V^{\mu\nu\rho}(p+k+q, k, p+q)}{(p+q)^2 - m_W^2 - i\Gamma_W m_W}\gamma_\rho, \end{aligned} \tag{A1}$$

where

$$\begin{aligned} &V^{\mu\nu\rho}(P, p_-, p_+) \\ &\quad = g^{\mu\nu}(P+p_-)^\rho - g^{\mu\rho}(p_+ + P)^\nu + g^{\nu\rho}(p_+ - p_-)^\mu. \end{aligned} \tag{A2}$$

Here $m_W = 80.385$ GeV is the W-boson mass, $\Gamma_W = 2.085$ GeV the W-boson full width and $c_W = 0.8768$ is the cosine of the electroweak mixing angle. We extract the electroweak coupling g from $g^2 = 8G_F m_W^2/\sqrt{2}$, where $G_F = 1.1663787 \times 10^{-5}$ GeV^{-2} is Fermi's constant [7]. We neglect the charged lepton mass throughout this discussion. When the functions $A_k^{\mu\nu}$ are written in terms of the invariant masses

$$m_{Wl}^2 = (p+k)^2, \qquad m_{l\nu}^2 = (p+q)^2, \tag{A3}$$

the matrix appearing in Eq. (14) is defined by

$$\begin{aligned} M_{ij} = &\frac{1}{64(2\pi)^3 m_Z^3}\int_{m_W^2}^{m_Z^2} dm_{Wl}^2 \int_0^{\hat{m}_{l\nu}^2} dm_{l\nu}^2 \\ &\times \left\{ \mathrm{tr}[(A_i^{\mu\nu})^\dagger \not{p} A_j^{\rho\sigma} \not{q}] \left(\frac{1}{3}\mathcal{P}_{\mu\rho}^Z(p+k+q)\mathcal{P}_{\nu\sigma}^W(k)\right)\right\}, \end{aligned} \tag{A4}$$

where $\mathcal{P}_{\mu\nu}^X(k) = -g_{\mu\nu} + k_\mu k_\nu/m_X^2$ is the transverse projector for a gauge boson X with momentum k and mass m_X, $m_Z = 91.19876$ GeV is the Z-boson mass and the upper integration boundary of the phase space integral is given by

$$\hat{m}_{l\nu}^2 = \frac{(m_{Wl}^2 - m_W^2)(m_Z^2 - m_{Wl}^2)}{m_{Wl}^2}. \tag{A5}$$

Performing this integration numerically, and setting the width of the W to zero, leads us to the numbers shown in Eq. (14).

2. Numerical results for four-body Z-decays

Here we present the numerical results of the four-body decays discussed in the main text:

$$\frac{\Gamma(Z\to ll\nu\bar{\nu})}{10^{-8}\ \mathrm{GeV}} \simeq \begin{cases} 2.8 - 4.3C_{\nu_l} + 3.2C_{\nu_l}^2 - 1.3C_{\nu_l}^3 + \sum_{\alpha=e,\mu,\tau}(0.077C_{\nu_\alpha}^2 + 0.27C_{\nu_\alpha}^3 + 0.33C_{\nu_\alpha}^4) & \text{for } l = e, \mu \\ 2.7 - 4.0C_{\nu_l} + 3.0C_{\nu_l}^2 - 1.4C_{\nu_l}^3 + \sum_{\alpha=e,\mu,\tau}(0.076C_{\nu_\alpha}^2 + 0.26C_{\nu_\alpha}^3 + 0.31C_{\nu_\alpha}^4) & \text{for } l = \tau \end{cases} \tag{A6}$$

$$\frac{\Gamma(Z \to jjl\nu_l)}{10^{-8}\ \mathrm{GeV}} \simeq \begin{cases} 8.2 - 10C_{\nu_l} + 8.7C_{\nu_l}^2 & \text{for } l = e, \mu \\ 8.1 - 9.9C_{\nu_l} + 8.0C_{\nu_l}^2 & \text{for } l = \tau \end{cases} \tag{A7}$$

$$\frac{\Gamma(Z \to ll'\nu_l\nu_{l'})}{10^{-8}\ \mathrm{GeV}} \simeq \begin{cases} 2.8 - 2.3(C_{\nu_l} + C_{\nu_{l'}}) - 0.085C_{\nu_l}C_{\nu_{l'}} + 1.5(C_{\nu_l}^2 + C_{\nu_{l'}}^2) & \text{for } l = e, \quad l' = \mu \\ 2.7 - 2.4C_{\nu_l} - 2.3C_{\nu_{l'}} - 0.080C_{\nu_l}C_{\nu_{l'}} + 1.5C_{\nu_l}^2 + 1.4C_{\nu_{l'}}^2 & \text{for } l = e, \mu, \quad l' = \tau \end{cases} \tag{A8}$$

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