

A bottom-up approach to the strong CP problem

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Abstract

The strong CP problem is one of many puzzles in the theoretical description of elementary particle physics that still lacks an explanation. While *top-down* solutions to that problem usually comprise new symmetries or fields or both, we want to present a rather *bottom-up* perspective. The main problem seems to be how to achieve small CP violation in the strong interactions despite large CP violation in weak interactions. Observation of CP violation is exclusively through the Higgs–Yukawa interactions. In this paper, we show that with minimal assumptions on the structure of mass (Yukawa) matrices they do not contribute to the strong CP problem and thus we can provide a pathway to a solution of the strong CP problem within the structures of the Standard Model and no extension at the electroweak scale is needed. However, to address the flavor puzzle, models based on minimal $SU(3)$ flavor groups leading to the proposed flavor matrices are favored. Though we refrain from an explicit a UV completion of the Standard Model, we provide a simple requirement those models should have to intrinsically not show a strong CP problem.

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1 Introduction

The Standard Model (SM) is known to be an incomplete model full of unresolved problems. Among the issues of the SM that still wait to be solved, the strong CP problem appears to be a very central one as it resides in the interplay of non-perturbative effects in Quantum Chromodynamics (QCD) and CP violation (CPV) in weak interactions. Curiously, the majority of present day solutions to many of the problems in high energy physics obey the tendency of going beyond the SM introducing new physics at a higher scale; the strong CP problem seems to follow this tendency. However, here we take a different philosophy and carefully scrutinize the available structures of the SM, offering an alternative approach of the problem. Our point of view might be defined as pragmatic, rather bottom-up, as we only study the mass matrices along with the bi-unitary transformations diagonalizing them. Starting from the SM flavor structures, we are going to present guidelines for model builders to fell the strong CP problem. Before moving to the details of our treatment let us first briefly summarize what the strong CP problem is. For a comprehensive review of this problem, see for instance [1].

The θ_{QCD} parameter of QCD parametrizes the non-equivalence of possible QCD vacua as for non-abelian gauge fields there can be non-vanishing winding numbers defined as

$$n = \frac{g^2}{32\pi^2} \int d^4x F_{\mu\nu}^a \tilde{F}^{a\mu\nu}, \quad (1)$$

leading therefore to an effective action

$$S_{\text{eff}} = \int d^4x \mathcal{L} + in\theta_{\text{QCD}}. \quad (2)$$

The axial anomaly introduces via

$$\partial^\mu j_\mu^5 = \frac{g^2}{16\pi^2} F_{\mu\nu}^a \tilde{F}^{a\mu\nu}, \quad (3)$$

effectively a change in S_{eff} by rotations of the quark fields with $\exp(-i\gamma_5 \frac{\theta_q}{2})$ that shifts the gauge field θ_{QCD} parameter as

$$\theta_{\text{QCD}} \rightarrow \bar{\theta} = \theta_{\text{QCD}} + \theta_q. \quad (4)$$

The same transformation affects quark mass terms as $m\bar{q}_L q_R \rightarrow e^{-i\theta_q} m\bar{q}_L q_R$ and may conversely be used to trace CP violating effects stemming from the masses. Due to this property, we may identify the physical remaining phase after such rephasing with the axial phase trans-

formation and be left with the well-known combination $\bar{\theta} = \theta_{\text{QCD}} + \theta_{\text{QFD}}$, where the quark flavor dynamical contribution is given by

$$\theta_{\text{QFD}} = \arg \det(\mathbf{M}_u) + \arg \det(\mathbf{M}_d) = \arg \det(\mathbf{M}_u \mathbf{M}_d). \quad (5)$$

The parameter $\bar{\theta}$ violates CP and induces an electric dipole moment for the neutron, so bounds are roughly $\bar{\theta} < 10^{-10}$ [2]. Such a huge cancellation between those two contributions in Eq. (4) is to be seen as a fine-tuning problem as they are conceptually independent. The strong CP problem now manifests itself in the question why $\bar{\theta}$ is so small although CPV in weak interactions has been found to be rather large (large, of course, compared to $\bar{\theta}$, not on absolute grounds). Even though both contributions, the pure gauge θ_{QCD} and the quark flavored θ_{QFD} , cannot be treated separately because a chiral phase shift in the quarks always reintroduces a genuine θ_{QFD} -term according to Eq. (3), we want to disaggregate especially the θ_{QFD} -term on a flavor physics groundwork. The viability of the approach is reflected in the fact that one can always find a basis, in which either θ_{QCD} or θ_{QFD} vanishes [3].

Popular solutions to this problem are besides the possibility of having one massless quark (typically the u -quark but the same holds for a massless d -quark), the introduction of at least one new symmetry (like an axial $U(1)$ or Peccei–Quinn [4] symmetry) that gets spontaneously (or softly [5, 6]) broken and comprises a light pseudo Nambu–Goldstone boson, the axion [7, 8].¹ A third way to compass the problem is via mechanisms worked out by Nelson [13] and Barr [14, 15], for a recent review see [16]. The main requirement for this mechanism is a vanishing $\arg \det(\mathbf{M}_q)$ and a way to spontaneously break CP in the context of Grand Unified Theories, alternatively spontaneously [17] or softly broken parity [18] (or a combination of all of them [19, 20]). This proposal is noteworthy in the sense that it provides a solution to the strong CP problem with no low energy consequence; unlike the axion and $m_u = 0$ solutions [16]. It was also shown in a certain kind of toy model that explicit soft CP -violation in the Higgs potential with two complex scalars leads spontaneously to an explicit CP -violating effect in the quarks mass matrices and still keeps $\det(\mathbf{M}_u \mathbf{M}_d)$ real at the tree-level [21]. In this spirit, the study of the SM structure as an effective theory is sufficient to circumvent the original problem as one is left with the open question about the origin of the Yukawa interactions. Another approach with spontaneous CPV is the one involving discrete flavor symmetries [22]. For last, generalized P -invariance in left-right symmetric theories

¹Axions and axion-like particles (ALPs) have a very rich phenomenology, summarized e. g. in [9], with an ongoing experimental effort to detect them (as the ALPS experiment at DESY [10, 11] and future facilities like ALPS-II or SHiP at CERN [12]).

can also provide valuable methods on computing approximately $\bar{\theta}$ through the corresponding right-handed quark mixing matrix [23–25], while supersymmetry helps to protect $\bar{\theta} = 0$ [26, 27].

Interpreting θ_{QCD} as a Lagrangian parameter, it is the only parity violating term in the QCD Lagrangian (and because charge conjugation is conserved, the θ_{QCD} -term explicitly violates CP). In the bottom-up approach, we take a vanishing θ_{QCD} for granted and unlike the Nelson–Barr approach stay at first ignorant about possible symmetries and physics at higher energy scales. Whether or not some variant of P or CP has to be employed as symmetry of nature depends on the specific realization. Instead, we pursue the option that CPV shall only arise from the Yukawa interactions alone, not even from the Higgs vacuum expectation values that multiply the Yukawa couplings. We do not require necessarily spontaneous CPV but allow in principle for explicit breaking in the (effective) Yukawa couplings. Imposing global CP -invariance of the QCD gauge interactions (though parity is enough) suffices for our main argument. We treat the Yukawa Lagrangian as an effective Lagrangian hiding the UV completion in the dimensionless Yukawa matrices. In that way, we obtain $\theta_{\text{QCD}} = 0$ and hence reduce the problem to understand why $\arg \det(\mathbf{M}_u \mathbf{M}_d)$ is such a small number (or why it should exactly vanish).

In the course of this paper, we accordingly suppose $\theta_{\text{QCD}} = 0$ and show how $\arg \det(\mathbf{M}_q)$ vanishes by imposing a minimal constraint on certain flavor phases and still providing sufficiently large weak CPV.² We shall argue that the physical CPV in the weak interactions is unrelated to any other phases appearing in $\arg \det(\mathbf{M}_q)$ and may only give a small finite contribution at higher orders as $\theta_{\text{QCD}} = 0$ at tree-level [28]. In the following, we give an explicit example of symmetry structures of mass matrices based on special unitary transformations that have the desired property and start by finding the minimal assumptions for the mass matrices to fulfill that. These assumptions easily find their way in any extension of the SM that generate Yukawa couplings dynamically, either by spontaneous breaking of the underlying flavor symmetry or by the moduli fields of string theory [29]. The generalization to an arbitrary number of quark families illustrates the universal validity of our idea.

Wrapping up our philosophy in order to clarify the new approach, we want to point out that a deliberate solution to the strong CP problem does not require a distinct statement about fundamental CPV. First of all, CP -invariance of the gauge interactions is sufficient,

²We call *flavor phases* the ones that appear both in the Yukawa matrices in a certain weak basis and remain, after moving to the mass basis, in the Jarlskog invariant. Of course, within the minimal SM only one physical will remain (that would be a linear combination of all the initial ones). But more physical phases could remain in extensions of the SM such as those including a fourth family and N extra Higgs doublets.

as we will show, to circumvent one aspect of the strong CP problem. Second, CP may be violated explicitly (or spontaneously) in the Yukawa interactions of SM fermions to the Higgs scalar as these interactions are the less understood in the context of the SM and do not necessarily have to respect CP . Third, we shall identify types of fermion mass matrices that automatically cancel out the undesired contribution to the $\bar{\theta}$ -parameter and thus solve the strong CP problem without the need of additional degrees of freedom in the theory as new fields assisting in this process. Finally, we stay open towards the remaining solution of the flavor puzzle in the SM, namely the question why there are three families of fermions and why they behave and mix as they do. However, we give an explicit realization of a parametrization (not yet a model) that helps to explain the mixing angles fully in terms of mass ratios and additionally provides the weak CP -phase of the SM in the same form.

This paper is organized as follows. In Section 2, we disentangle the origins of strong and weak CPV despite the fact that both of them can be expressed in terms of the same quark mass matrices. In Section 3, we write the Kobayashi–Maskawa phase in terms of quark mass ratios as it follows from [30], which is seen to be independent from the previous considerations. In Section 4, we study general consequences of the conditions provided in this work and how they are related to concrete models. Finally, we conclude in Section 5.

2 Disentangling weak and strong CPV

A complete knowledge of the quark mass matrices, $\mathbf{M}_u$ and $\mathbf{M}_d$, tackles down the flavor puzzle in the SM and finally evades the strong CP problem (since the quark flavor contribution to $\bar{\theta}$ can be absorbed in the masses and $\theta_{\text{QCD}} = 0$). Strong and weak CPV, θ_{QFD} and the rephasing invariant J_q , respectively, are of different origins on one hand. And on the other hand sufficiently large CPV in the weak sector³ therefore does not necessarily imply strong CPV despite the fact that both reside in the same mass matrices

- weak CPV: the Jarlskog invariant

$$J_q \sim \text{Im} \left[\det([\mathbf{M}_u \mathbf{M}_u^\dagger, \mathbf{M}_d \mathbf{M}_d^\dagger]) \right], \text{ see [33],}$$

- strong CPV: the θ_{QFD} -term from above

$$\theta_{\text{QFD}} \equiv \arg \det(\mathbf{M}_u \mathbf{M}_d).$$

³The expression “large” does not have to be correlated to a numerically large value of the only CP -violating phase in weak interactions, since, depending on the chosen parametrization, this phase can be rather small though the overall CP -violation appears to be actually large (see [31, 32]). The rephasing invariant J_q does not depend on the particular parametrization and serves as a better measure of comparison.

It is not only the functional dependence on the mass matrices (and thus relevant phases) that is apparently different for weak and strong CPV but rather their individual transformation properties of parity and charge conjugation [31]. While both objects are CP -odd, θ_{QCD} transforms even under C and odd under P whereas J_q behaves the other way round, see [34]. For “complete knowledge” of mass matrices, it is adequate to set up each matrix in terms of known parameters, even though a full theory of flavor is still lacking. By the freedom of relying on an effective description of the mass matrices, we encompass the strong CP problem by an ansatz to understand the flavor puzzle.

In the following, we will assume that all quark masses are different from zero, as suggested by lattice calculations [35]. Our minimal requirement for the mass matrices follows very obviously from the usual Singular Value Decomposition⁴

$$\mathbf{M}_q = \mathbf{L}_q^\dagger \boldsymbol{\Sigma}_q \mathbf{R}_q, \quad (6)$$

with $\mathbf{L}_q$ and $\mathbf{R}_q$ unitary transformations and $\boldsymbol{\Sigma}_q = \text{diag}(m_{q_1}, m_{q_2}, m_{q_3})$ a positive diagonal matrix, $m_{q_i} > 0$. Consequently, we find

$$\begin{aligned} \arg \det(\mathbf{M}_u \mathbf{M}_d) &= \arg \det(\mathbf{L}_u^\dagger \boldsymbol{\Sigma}_u \mathbf{R}_u \mathbf{L}_d^\dagger \boldsymbol{\Sigma}_d \mathbf{R}_d) \\ &= \arg(\det \mathbf{L}_u^\dagger \det \mathbf{R}_u \det \mathbf{L}_d^\dagger \det \mathbf{R}_d), \end{aligned} \quad (7)$$

after using the well-known property of the determinant, $\det(\mathbf{AB}) = \det \mathbf{A} \det \mathbf{B}$, with the args of the diagonal matrices vanishing (as they are real and positive). The decomposition of Eq. (6) is completely arbitrary in the sense that different choices of $\mathbf{L}_q$ and $\mathbf{R}_q$ lead to different flavor representations of the mass matrices and additionally the particular choice of $\mathbf{L}_u = \mathbf{L}_d$ leaves the weak interaction basis invariant. In case, one desires to build mass matrices $\mathbf{M}_q$ on a certain family of flavor symmetries, this fixes the allowed classes of transformations $\mathbf{L}_q$ and $\mathbf{R}_q$. We give constraints on these transformations that can be easily verified in any proposal of a flavor model to be inherently free of a strong CP problem.

In the limit of vanishing mass matrices, the SM Lagrangian (the kinetic terms) obeys for n generations a $U(n)$ symmetry for each gauge multiplet (i. e. left-handed quark doublets and the up- and down-type right-handed singlets; in total $U(n)^3$). The maximal freedom to rotate

⁴A similar conclusion can be drawn by virtue of the Polar Decomposition, $\mathbf{M}_q = \mathbf{H}_q \mathbf{U}_q$ [31, 34], with $\mathbf{H}_q$ a Hermitian and $\mathbf{U}_q$ a unitary matrix (note that $\mathbf{U}_q \neq \mathbf{R}_q$ in the Singular Value Decomposition).

Table 1: Complex phases contributing to either the strong or weak CPV phase with an arbitrary number of fermion families. Notice how always the same linear combination of phases appears in the strong CP case. Through this table it becomes very apparent that the origin of strong CPV is completely independent from the one generating quark mixing or weak CPV.

Number of fermion families	θ_{QFD}	$\delta_{CP}^{\text{weak}}$
1	$\alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)}$	0
2	$\alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)}$	0
3	$\alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)}$	δ_{CP}^{CKM}
4	$\alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)}$	$\delta_{CP}^{\text{CKM}}, \omega_{CP}, \omega'_{CP}$
n	$\alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)}$	$\delta_{CP}^{\text{CKM}}, \omega_{CP}^k \ (k = 1, 2, \dots, \frac{n(n-3)}{2})$

these fields is parametrized hence by $U(n)$ transformations U with $\det U = e^{i\phi}$, such that

$$\arg \det(\mathbf{M}_u \mathbf{M}_d) = \alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)}, \quad (8)$$

where $\arg \det(\mathbf{K}_q) = \alpha_K^{(q)}$ and $\mathbf{K}$ either L or R . Let us remark that this result is independent of n and applies in the same way for any number of fermion families, see Table 1.

We emphasize that strong and weak CPV, θ_{QFD} and J_q , are of different origin and thus can be treated independently. If the left and right unitary rotations L_q and R_q , respectively, were either *special* unitary rotations or the same (as for Hermitian mass matrices), Eq. (8) would vanish trivially and still allow for a non-vanishing Jarlskog invariant. [Recall that unitary transformations are equal to special unitary transformations times a global phase, $U(n) = \text{SU}(n) \otimes U(1)/Z_n$ [36].] Hence, the global phases give rise to strong CPV and consequently θ_{QFD} is sensitive to these global phases only and insensitive to the complex structure of the underlying $SU(3)$ transformations responsible for flavor mixing phenomena. Weak CPV on the other hand has exactly the opposite relation: it is sensitive to the complex nature of the special bi-unitary transformations (which give rise to flavor mixing in weak interactions) and insensitive to global phases (a property exactly expressed by Jarlskog invariant which is known to be rephasing invariant). Even in the absence of a global phase that violates CP strongly, we can have sufficiently large weak CPV: the existence or non-existence of strong CPV is completely unrelated to the existence or non-existence of weak CPV as can be seen from the two family case which has no weak CPV while showing, in principle, strong CPV. Conversely, comparison of the expressions of J_q and θ_{QFD} as listed above already shows that the phase difference of $\mathbf{M}_u$ and $\mathbf{M}_d$ responsible for $\theta_{\text{QFD}} \neq 0$ drops out in the expression of J_q . Note that the existence of these global $U(1)$ -phases and the invariance of the SM field

Table 2: Different cases for which there is no contribution in the strong CP phase, $\bar{\theta}$, stemming from the global phases, θ_{QFD} .

Case	$\alpha_L^{(u)}$	$\alpha_L^{(d)}$	$\alpha_R^{(u)}$	$\alpha_R^{(d)}$	Condition on the Yukawa couplings	Weak CPV
Ia	0	0	0	0	CP -invariance	No
Ib	0	0	0	0	$\mathcal{G} \subseteq \text{SU}(3)$ -invariance	Yes
II	α_u	α_d	α_u	α_d	P -invariance	Yes
III	α_L	$-\alpha_L$	α_R	$-\alpha_R$	Unknown	Yes
IV	α	β	β	α	Unknown	Yes
V	α_L	α_L	x	$2\alpha_L - x$	$\text{SU}(2)_L$ gauge	?

content under a particular combination of such rephasings is just the conservation of baryon number, which, however, is accidental.

We have shown that the strong CPV parameter (θ_{QFD}) can be treated independently from the weak one, which already on its own is quite distinct from other approaches. However, to identify how the above conditions feed into a viable UV complete model one needs to go farther. In this regard, it helps to recognize certain benchmark scenarios that have $\theta_{\text{QFD}} = \alpha_R^{(u)} - \alpha_L^{(u)} + \alpha_R^{(d)} - \alpha_L^{(d)} = 0$. The different ways to achieve this goal are sketched in Table 2, note that in principle one should be able to smoothly interpolate between those scenarios. These conditions should be seen as applied to a more fundamental theory where $\theta_{\text{QCD}} = 0$ and which when taken to lower energies one delivers the SM. In this sense, the Yukawa part of the SM Lagrangian could be treated as an effective Lagrangian. Case Ia considers a CP-invariant Lagrangian where there are no phases; in order to get weak CPV one must either spontaneously or explicitly break it. Case Ib refers to a flavor theory employing an $\text{SU}(3)$ symmetry group or subgroup. Case II embraces those Left-Right (LR) symmetric theories where parity is conserved. Cases III and IV, are other examples with no explicit model present in which the sum of global phases could be canceled.

Therefore, within this context, the vanishing $\arg \det(\mathbf{M}_u \mathbf{M}_d)$ is automatic, contrary to the common folklore. Instead, certain conditions, that are summarized in Table 2, could be taken as forced by symmetry reasons. Our approach to solve the strong CP problem reduces essentially to explain why $\alpha_L^{(q)} = \alpha_R^{(q)}$ (similar to Case II in Table 2 without explicit P -invariance) while, simultaneously, explain the observed amount of weak CPV, $\delta_{CP}^{\text{CKM}} = (1.19 \pm 0.15)$ rad, see [37].

Before we move to an explicit realization of our findings, let us briefly summarize what we have so far: even if all quarks are massive, we have no strong CP problem without imposing any new symmetries. The requirement $\arg \det(\mathbf{M}_u \mathbf{M}_d) = 0$ can be achieved by all of the

benchmark cases of Table 2 and any interpolation between them. For example, Case I or II, ensures $\alpha_L^{(q)}$ and $\alpha_R^{(q)}$ for $q = u, d$ to be zero or equal, respectively; the minimal way for the former scenario would be to propose either CP -invariance or $SU(3)$ transformations for the diagonalization of the mass matrices which conversely means that any flavor model based on $SU(3)$ transformations gives a solution to the strong CP problem [38,39]. Finally, we can still have (arbitrarily large) CPV in weak interactions as this is unrelated to strong CPV. The main task is somewhat to reduce the arbitrariness in complex phases that are generally allowed for the mass matrices and give a restrictive prescription for weak CPV. It is comparably simple to define generic CP -violating textures of mass matrices that have no strong CP -phase and still allow for a CKM-phase according to a mismatch between phases in up and down type mass matrices [40]. Our approach, however, is still even more generic as we do not rely on a certain flavor basis in which the phases are apparent and stay rather basis and model independent.

3 A suggestive way of calculating weak CPV

In the previous section, we have stated that the $SU(3)$ symmetry transformations in family space, acting in the left and right handed fields of up-quark and down-quark types, are enough to deliver any amount of weak CPV independently of having previously eliminated the combination of $U(1)$ global phases which give rise to the strong CP phase, θ_{QFD} . Now, we want to outline in a recently proposed mixing parametrization [30], how to represent the weak CP phase. This parametrization relates the entries of the Cabibbo–Kobayashi–Maskawa (CKM) matrix to functions of the quark mass ratios and thus the remaining CP phase can be written as function of those.

A very famous expression of a mixing angle as a function of a mass ratio was provided by the well-known Gatto–Sartori–Tonin (GST) relation, $\tan \theta_c \approx \sqrt{m_d/m_s}$, for the Cabibbo angle θ_c [41]. Based on this finding, a parametrization of the fermion mixing matrices was proposed that only uses the mass ratios as input [30]. Besides the phenomenological observation $m_i \ll m_j$ for $i < j$ with masses of the i -th and j -th generation, a crucial assumption behind this parametrization is that the Euler rotations can be individually expressed by $\tan \theta_{ij} = \sqrt{m_i/m_j}$. Likewise, symmetrical structures in the mass matrices have been detected that lead to exactly this kind of mixing matrices [42]. In that view, the final quark mixing matrix can be decomposed into a chain of successive two-family rotations where each planar

SU(2) rotation can then be written as

$$U'_{ij}(\mu_{ij}, \delta_{ij}) = \begin{pmatrix} \frac{1}{\sqrt{1+\mu_{ij}}} & \sqrt{\frac{\mu_{ij}}{1+\mu_{ij}}} e^{-i\delta_{ij}} \\ -\sqrt{\frac{\mu_{ij}}{1+\mu_{ij}}} e^{i\delta_{ij}} & \frac{1}{\sqrt{1+\mu_{ij}}} \end{pmatrix}, \quad (9)$$

with $\mu_{ij} = m_i/m_j$ and an *a priori* arbitrary complex phase $\delta_{ij} \in [0, 2\pi)$. We identify $\sin \theta_{ij} = \sqrt{\frac{\mu_{ij}}{1+\mu_{ij}}}$ and $\cos \theta_{ij} = \frac{1}{\sqrt{1+\mu_{ij}}}$. For example, the full SU(3) transformation for the 2-3 sub-sector is then given by

$$U'_{23}\left(\frac{m_2}{m_3}, \delta_{23}\right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{\sqrt{1+m_2/m_3}} & \sqrt{\frac{m_2/m_3}{1+m_2/m_3}} e^{-i\delta_{23}} \\ 0 & -\sqrt{\frac{m_2/m_3}{1+m_2/m_3}} e^{i\delta_{23}} & \frac{1}{\sqrt{1+m_2/m_3}} \end{pmatrix}. \quad (10)$$

Now, defining the CKM-matrix as

$$V_{\text{CKM}} \equiv L_u L_d^\dagger, \quad (11)$$

with the U(3) transformations $L_{u,d}$ defined via Eq. (6), we have in the formulation of [30] four mass ratios entering the game and six phases from which three can be removed by choosing the up-type mass matrix real.⁵ From the remaining three, only *one maximally CP-violating phase* sitting in the 1-2 rotation is needed to fully reproduce the CKM-phase, details may be found in [30]. It was also pointed out in Ref. [43] that the same follows for certain 1-3 texture zero mass matrices. The approach of [30] is however more general as it does not rely on specific texture zeros but merely on symmetrical structures à la Ref. [42].

Using this mass ratios parametrization, we similarly compute the Kobayashi–Maskawa CP-phase in terms of mass ratios. In the standard parametrization, the most recent global fit obtains for it $\delta_{\text{CP}}^{\text{CKM}} = (1.19 \pm 0.15) \text{ rad}$ [37]. In the following, we want to estimate the corresponding theoretical value. After imposing individual rotations of the type (9), we can finally build up a quark mixing matrix that has non-vanishing CPV.

The procedure introduced in Ref. [30] gives a mixing matrix which cannot be directly compared to the conventional parametrization. In order to do that, we first need to rephase

⁵This rephasing should not introduce a new strong CP-phase as we only shuffle complex entries from M_u to M_d . Moreover, any global phase does not play a role for weak CP-violation as the relevant objects are the left-Hermitian products $M_{u,d} M_{u,d}^\dagger$.

both the up and down type quark fields

$$\tilde{V}_{\text{CKM}} = \boldsymbol{\chi}_u V_{\text{CKM}} \boldsymbol{\chi}_d^\dagger, \quad (12)$$

in such a way that we are able to produce the following structure

$$\tilde{V}_{\text{CKM}} \sim \begin{pmatrix} \text{Re} & \text{Re} & \mathcal{C} \\ \mathcal{C} & \mathcal{C} & \text{Re} \\ \mathcal{C} & \mathcal{C} & \text{Re} \end{pmatrix}, \quad (13)$$

where $\boldsymbol{\chi}_q = \text{diag}(e^{i\phi_q}, 1, 1)$ and Re and $\mathcal{C}$ mean real and complex entries. After rephasing, we get the following expression for the Kobayashi–Maskawa CP -phase

$$\delta_{\text{CP}}^q \approx \arctan \left[\sqrt{\frac{\mu_{\text{ds}}(1 + \mu_{\text{ds}})}{\mu_{\text{uc}}(1 + \mu_{\text{uc}})}} \right] \approx (1.38 \pm 0.10) \text{ rad}, \quad (14)$$

which after insertion of the values of the quark mass ratios, $\mu_{\text{ds}} = m_{\text{d}}/m_{\text{s}} = 0.051 \pm 0.001$ and $\mu_{\text{uc}} = m_{\text{u}}/m_{\text{c}} = 0.0021 \pm 0.0001$, we find it to be in agreement to the experimental value, $\delta_{\text{CP}}^{\text{CKM}} = (1.19 \pm 0.15)$.⁶ Notice how when the decoupling limit, $m_{b,t} \rightarrow \infty$, is considered the CP -phase does not go to zero as one would expect it in other parametrizations [32]. Nevertheless, there is no inconsistency in this result as simultaneously the magnitudes of the mixing matrix elements vanish, $|\mathbf{V}_{13}^{\text{CKM}}| \rightarrow 0$ and $|\mathbf{V}_{23}^{\text{CKM}}| \rightarrow 0$ as $m_{t,b} \rightarrow \infty$. Conversely, the CP phase gets closer to its maximum value $\frac{\pi}{2}$ when the ratio between the up and the charm quark gets more suppressed, $m_{\text{u}}/m_{\text{c}} \rightarrow 0$.

Hence, we have shown that within the SM without adding new degrees of freedom the amount of weak CPV can be calculated by means of the quark mass ratios if one allows for a relation among CKM angles and mass ratios.

What about higher order corrections? The impact of the weak CP phase in the CKM model on the strong CP phase was first and extensively studied in Ref. [28] where the generic contribution was shown to be small. However, at very high (i. e. 14th) order in perturbation theory there is an “infinite” contribution which actually turns out to be rather tiny when the original θ_{QCD} parameter is renormalized to zero at around the Planck scale. Even with some “New Physics” contribution (of heavy quarks above the electroweak scale—remembering that

⁶The quark masses have been treated as running $\overline{\text{MS}}$ masses evaluated at the weak scale ($Q^2 = M_Z^2$), numbers are taken from App. A in Ref. [30].

at the time this reference originates the mass of the top quark was expected to be well below its today's value) there is no huge effect. Proper New Physics contributions, however, strongly depend on the implementation of New Physics and shall be rather tuned to avoid a strong effect on θ_{QCD} anyway. Spurious contributions at low energies are not to be expected once the Standard CKM model has been generated in a top-down approach.

4 Towards a UV completion

After having paved the path towards a UV complete theory of flavor without strong CP problem, we explore the possible consequences of the conditions above to see where the path may end. Through this reasoning, the functional dependence of the Kobayashi–Maskawa phase on the quark mass ratios in a parametrization suggested by some of the authors [30] helps to describe the phenomenological properties of the true flavor dynamics realized in nature. In this regard, we start with certain flavor matrix structures [42], which have successfully described fermion mixing [30], and study their connection to the strong CP problem.

We explicitly refrain from providing a UV complete model extending the field content of the SM in order not to spoil the generality of our results. What follows shall be rather seen as matching conditions of any theory of flavor generating mass or Yukawa matrices for the low energy effective theory where all relevant heavy degrees of freedom are integrated out. We leave it to either the interested reader to construct such a model which give such conditions or to future works of the authors. As the framework itself leading the results in [30, 42], is directly related to the decomposition of 3×3 matrices into 2×2 submatrices, we show some relations in the two-family case only, *not losing by this any generality in our treatment*.

It is outside the scope of this work to provide the full details of the two mentioned publications [30, 42]. Nonetheless, here we briefly comment the essential ideas behind them which might be incorporated in any UV complete model wishing to serve as a theory of flavor. The first work delivers a new mixing parametrization which applies to both quarks and leptons [30]. A systematical procedure was built through the phenomenological observation of hierarchical fermion masses, $m_3^2 \gg m_2^2 \gg m_1^2$, along with a lower rank approximation theorem known as Schimdt–Mirsky. In a similar fashion as the Wolfenstein parametrization, where the four mixing parameters are real but still the parametrization is complex, the four independent mass ratios of either the quark or lepton sector are used as mixing parameters,

$$V_f = V_f \left(\frac{m_1^a}{m_2^a}, \frac{m_2^a}{m_3^a}, \frac{m_1^b}{m_2^b}, \frac{m_2^b}{m_3^b} \right), \quad (15)$$

where $f = q, \ell$ is the CKM or PMNS mixing matrix, respectively, and $a = u, \nu$ and $b = d, e$. This procedure exploits the mathematical properties of matrices under the fact of hierarchical singular values. There is no loss of generality in any of the involved approximations as cautious steps are made. Two main issues, however, are left: what symmetry or principle dictates that Yukawa matrices should arise with the following structure,

$$\mathbf{M} \sim \begin{pmatrix} & & \\ & & \times \\ \times & & \end{pmatrix} + \begin{pmatrix} & & \\ & \times & \\ \times & & \end{pmatrix} + \begin{pmatrix} & \times & \times \\ \times & & \\ \times & & \end{pmatrix} ? \quad (16)$$

and what is producing among them a hierarchy?

The second work [42] precisely offers an answer to the first question. The mass matrix which gets diagonalized by a transformation of the type (9) can be found very easily in the two family case and can be generalized to $n > 2$ generations according to [42]. In Ref. [42], mass matrices are constructed in such a way to allow the sequential diagonalization of [30] without preference to any of the families. The basic assumption behind this approach is that Higgs–Yukawa interactions (or conversely mass matrices if one does not specify the mass generating mechanism directly) are symmetric under permutations of the fermion fields. This permutation symmetry is then supposed to be broken stepwise as $S_{3L} \otimes S_{3R} \rightarrow S_{2L} \otimes S_{2R} \rightarrow S_{2A} \oplus S_{2S}$, where the last step proceeds to a sum of anti-symmetric and symmetric permutation matrices of two objects. This proposal can be fulfilled employing textures like the one appearing in Eq. (16) which allow to study the corresponding mixing by three different rotations in a two-family space each.

We exemplarily study the two-family case where the mass matrix originated in the sequential breakdown of permutation symmetries [42] is given in a *preferred* basis⁷ as

$$\mathbf{M} = \begin{pmatrix} 0 & \sqrt{m_1 m_2} e^{-i\delta_m} \\ -\sqrt{m_1 m_2} e^{i\delta_m} & m_2 - m_1 \end{pmatrix}. \quad (17)$$

Now, we want to map this structure resulting in a GST relation to the most general case of a 2×2 mass matrix. The GST relation gives Eq. (9) as the corresponding unitary transformation. A general $U(2)$ matrix has two more parameters that can be expressed as an additional

⁷The preferred basis corresponds to the mass basis for $m_1 \rightarrow 0$.

phase on the diagonal and an overall phase factor,

$$U = e^{i\phi/2} \begin{pmatrix} \cos \theta e^{i\eta} & \sin \theta e^{-i\delta} \\ -\sin \theta e^{i\delta} & \cos \theta e^{-i\eta} \end{pmatrix}, \quad (18)$$

such that $\det U = e^{i\phi}$. According to Eq. (18), the relevant left rotation of a generic mass matrix should also have the form

$$L = e^{i\alpha_L/2} \begin{pmatrix} \cos \theta_L e^{i\beta_L} & \sin \theta_L e^{-i\delta_L} \\ -\sin \theta_L e^{i\delta_L} & \cos \theta_L e^{-i\beta_L} \end{pmatrix}. \quad (19)$$

The same expression follows for the right transformation R with $L \leftrightarrow R$ in Eq. (19) and the individual entries of the mass matrices can be expressed via

$$M = e^{i\frac{\alpha_R - \alpha_L}{2}} \begin{pmatrix} \cos \theta_L e^{-i\beta_L} & -\sin \theta_L e^{-i\delta_L} \\ \sin \theta_L e^{i\delta_L} & \cos \theta_L e^{i\beta_L} \end{pmatrix} \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} \begin{pmatrix} \cos \theta_R e^{i\beta_R} & \sin \theta_R e^{-i\delta_R} \\ -\sin \theta_R e^{i\delta_R} & \cos \theta_R e^{-i\beta_R} \end{pmatrix}, \quad (20)$$

and thus

$$\begin{aligned} M_{11} &= e^{i\frac{\alpha_R - \alpha_L}{2}} [e^{-i(\beta_L - \beta_R)} m_1 \cos \theta_L \cos \theta_R + e^{-i(\delta_L - \delta_R)} m_2 \sin \theta_L \sin \theta_R], \\ M_{12} &= e^{i\frac{\alpha_R - \alpha_L}{2}} [e^{-i(\beta_L + \delta_R)} m_1 \cos \theta_L \sin \theta_R - e^{-i(\beta_R + \delta_L)} m_2 \cos \theta_R \sin \theta_L], \\ M_{21} &= e^{i\frac{\alpha_R - \alpha_L}{2}} [e^{i(\beta_R + \delta_L)} m_1 \cos \theta_R \sin \theta_L - e^{i(\beta_L + \delta_R)} m_2 \cos \theta_L \sin \theta_R], \\ M_{22} &= e^{i\frac{\alpha_R - \alpha_L}{2}} [e^{i(\beta_L - \beta_R)} m_2 \cos \theta_L \cos \theta_R + e^{i(\delta_L - \delta_R)} m_1 \sin \theta_L \sin \theta_R]. \end{aligned} \quad (21)$$

Matching this set of equations to the matrix form of Eq. (17) reduces the freedom of the $U(2)$ rotations as the structure is dictated by the simple symmetry patterns. We find the conditions

$$\alpha_L = \alpha_R, \quad \beta_L - \beta_R = \delta_L - \delta_R = 0, \quad \theta_R = -\theta_L. \quad (22)$$

Identifying the impact of those relations is rather trivial in comparison with Eq. (17), as the mass matrix there clearly exhibits no global phase (and thus $\alpha_L = \alpha_R$), and the off-diagonal phase is given by $\delta_m = \delta_{L(R)} + \beta_{L(R)}$, as combination of the two relevant phases in the $SU(2)$ rotation. Notice that, although Eq. (17) is not left-right symmetric (the mass matrix is anti-Hermitian), one gets roughly the same conditions on the left and right rotation matrices. The last relation, $\theta_L = -\theta_R$ follows after commuting the phase matrices through and absorbing phases in a redefinition of the fermion fields. This redefinition does not fully apply to the three-family case and thus there is a remaining CP -violating phase in the mixing.

The constraints of Eqs. (22) provide very valuable information especially on the *right-handed* rotations coded in $\mathbf{R}$ resulting in the clear prediction that the individual mixing angles of the right-handed sector have exactly the same magnitude as the known left-handed (i. e. CKM) ones. The future detection of right-handed currents may be a razor to finally rule out the proposed description. Note, that we do not predict right-handed currents at all. If, however, a right-handed counterpart to the electroweak gauge group exists, the corresponding CKM matrix cannot be arbitrary in that description.

Fulfillment of conditions similar to (22) is very natural in left-right symmetry models. It is well-known that such parity-invariant models offer a solution to the strong CP problem on their own without the need of an axion solution [18]. A minimal model is based on the gauge group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, where the left- and right-handed fermions transform as doublets of $SU(2)_L$ and $SU(2)_R$, respectively. Parity invariance requires the Yukawa matrices to be Hermitian and the mass matrices are of the form

$$\mathbf{M}_f = \sum_i \mathbf{Y}_i^{(f)} \langle \chi_i^0 \rangle, \quad (23)$$

with the vacuum expectation value (vev) $\langle \chi_i^0 \rangle$ of the relevant set of Higgs multiplets taking part in electroweak symmetry breaking. Consequently, for the necessary condition on the *mass matrices*, all the vevs have to be real in order not to spoil the Hermiticity of the Yukawa matrices. Generically, however, such multi Higgs models easily have spontaneous CP violation with at least one complex vev. Supersymmetry helps to cure this problem, introduces on the other hand a new strong CP problem connected to the potentially complex gluino mass [44, 45]. Another avenue involves the complete doubling of fermions and gauge group [17, 46], which includes additional mirror fermions as singlets under the SM gauge group but charged under a mirror gauge group $SU(2)_R \times U(1)_X$. The concept of a hidden sector together with LR symmetry applies also to radiative solutions of the flavor hierarchy problem—and automatically complies with the conditions presented here [47]. Conversely, LR-inspired models of flavor model building have no need for a flavored axion as recently proposed on basis of a Froggatt–Nielsen mechanism [48, 49].

We see several viable approaches to build reasonable flavor models that are intrinsically free of the strong CP problem:

- Multi-Higgs models with spontaneous CPV where the mass matrices can be constructed as linear combinations of Yukawa matrices and vacuum expectation values like Eq. (23) that carry complex phases. This approach potentially suffers from unacceptably large

corrections as also discussed in [28].

- LR-inspired models that have Eq. (8) automatically implemented.
- Radiative constructions similar to [47] where LR symmetry may not be necessary to fulfill condition (8). Here we leave the field open to play with the ingredients.
- Non-Abelian flavor models with Yukawa spurion fields as remaining vevs of heavy scalars and in such a way Eq. (8) is achieved dynamically by the flavon dynamics as proposed in Ref. [50].

As a side remark, let us note that recent investigations on minimal left-right symmetric models hint toward the conclusion of $V_{\text{CKM}}^R = V_{\text{CKM}}^L$ [24, 25]. Surprisingly, we do not get an exact equality but rather find for the right-handed sector the angles $\theta_{12}^{\text{CKM},R} = \theta_{12}^{\text{CKM},L}$, $\theta_{23}^{\text{CKM},R} = \theta_{23}^{\text{CKM},L}$, and $\theta_{13}^{\text{CKM},R} \approx \theta_{13}^{\text{CKM},L}/10$, which results from the intricate structure of V_{CKM} in Ref. [30].

We do not have to rely on strict parity invariance of the fermion Yukawa sector in order to reply the findings presented here. Parity symmetry is broken in SM at low energies anyway and what we observe applying the rules from above is rather a fake Parity built in the Yukawa matrices which may be of a different origin than a GUT-inspired remnant Parity invariant structure.

5 Conclusions

We have addressed the strong CP problem by following a bottom-up approach. We have determined the necessary conditions a more fundamental theory should have in order to intrinsically not show a strong CP phase, $\bar{\theta} = 0$, not only at higher energies but also at lower ones. As this phase is made out of two conceptually independent contributions, $\bar{\theta} = \theta_{\text{QCD}} + \theta_{\text{QFD}}$, we have studied the conditions for each of them to be zero, $\theta_{\text{QCD}} = \theta_{\text{QFD}} = 0$, while simultaneously allowing weak CPV. The first condition demands that within a UV complete model one should have either P or CP invariance. This is not a new statement as it is well known, that this automatically sets both contributions equal to zero. However, as one wishes to explain the observed amount of weak CPV stemming from the quark masses, this initial symmetry must be broken. However, in general, this induces at tree level a new strong CP phase, here denoted as $\theta_{\text{QFD}} = \arg \det(\mathbf{M}_{\text{u}} \mathbf{M}_{\text{d}}) \neq 0$. The main challenge, which is naturally present within the Nelson–Barr type of models, then basically consists in explaining why the

amount of strong CPV stemming from the quark masses should be zero, while simultaneously a sufficiently large value (compared to θ_{QFD}) of weak CPV appears, which is coded in the Jarlskog invariant J_q of the experimentally measured (fitted) CKM-matrix. We have realized that there is no difficult challenge in solving the previous problem. Splitting the generational freedom of the gauge kinetic terms as $U(3) = SU(3) \otimes U(1)/Z_n$, it can be clearly seen that arbitrary $U(1)$ factors lead to $\theta_{\text{QFD}} \neq 0$ while the $SU(3)$ nature is responsible for $J_q \neq 0$. Hence, the complex phases implied by θ_{QFD} are entirely unrelated to the phases of weak CPV, as shown in Table 1. The absence of the strong CPV is guaranteed by imposing one of the four possible conditions appearing in Table 2. In particular, Case II has a very minimal condition on the mass matrices such that $\theta_{\text{QFD}} = \sum_{q=u,d} \alpha_R^{(q)} - \alpha_L^{(q)} = 0$, if $\alpha_L^{(q)} = \alpha_R^{(q)}$, though the basic constraint is much weaker. (This gets important in the context of some Grand Unification when up- and down-quark mass matrices are related to each other.) It has been shown that minimal symmetrical requirements on the Higgs–Yukawa interactions according to [42] lead to the given constraint and non-trivial CKM-mixing. As a consequence of this, the mixing of the right-handed sector is fixed and predicts for the right-handed CKM-matrix $\theta_{12}^{\text{CKM},R} = \theta_{12}^{\text{CKM},L}$, $\theta_{23}^{\text{CKM},R} = \theta_{23}^{\text{CKM},L}$, and $\theta_{13}^{\text{CKM},R} \approx \theta_{13}^{\text{CKM},L}/10$. This fingerprint can be tested in future experiments within a variety of extensions of the Standard Model.

Moreover, for the weak CPV we have showed that in the recently proposed fermion mass ratios parametrization [30] the leading contribution to the CKM-phase, after insertion of the value for the mass ratios m_u/m_c and m_d/m_s , implies the value $\delta_{\text{CP}}^q \approx (1.38 \pm 0.10)$ rad which is in agreement to the observed one, $\delta_{\text{CP}}^{\text{CKM}} = (1.19 \pm 0.15)$ rad.

We have not provided a *solution* to the strong CP problem but rather argued that it can be addressed without the need of an axial $U(1)$ symmetry from a flavor physics point of view by modeling the quark mass matrices and thus does not come along with a flavored axion (Flaxion [48] or Axiflavor [49]). Instead there may be several paths to implement the condition to pass by the strong CP problem via flavor model building especially based on spontaneous breaking of the maximal flavor group.

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