Theoretical concepts of parton saturation - from HERA to LHC

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Abstract

We present a short summary of parton saturation concepts as seen in deep inelastic scattering.

1 Introduction

The deep inelastic scattering (DIS) experiments, in which leptons probe nucleons with the help of electroweak bosons, reveal that nucleons consist of partons. These are colored quarks of Quantum Chromodynamics (QCD) which carry approximately half of the nucleon's momentum. The missing half is provided by gluons to which the electroweak bosons do not couple. Thus, although not directly probed, gluons are extremally important for the description of the nucleon structure. Quantitatively, this is summarized by the DGLAP evolution equations of QCD which govern the dependence of the quark and gluon distributions in a nucleon on a scale Q^2 (identified in DIS with photon's virtuality $q^2 = -Q^2$). The sign of the logarithmic derivative, $\partial F_2 / \partial \log Q^2$, at different values of the Bjorken variable x is determined by the relative contribution of quarks to gluons. In the limit $x \to 0$, studied intensively by the experiments at HERA, the deep inelastic processes are dominated by a strongly rising gluon distribution. Therefore, in the small-x limit, gluonic systems inside the nucleon are predominantly studied. The description of processes in such systems, using perturbative QCD (pQCD), is the aim of this presentation.

2 Collinear factorization versus k_T-factorization

In the electron–proton DIS, the measured proton structure functions, F_T and F_L, are related to the parton distributions through the collinear factorization formula resulting from pQCD:

$$F_{T,L}(x, Q^2) = \sum_{i=q,\bar{q},g} \{C_{T,L}^{(i)} \otimes f_i\}(x, Q^2) + \sum_{n=1} \frac{\Lambda_{T,L}^{(n)}(x, \alpha_s)}{Q^{2n}} \tag{1}$$

where $\otimes$ indicates integral convolution in parton longitudinal momentum fractions, $\alpha_s = \alpha_s(Q^2)$ is the running strong coupling constant, $C_{T,L}^{(i)}(z, \alpha_s)$ are perturbatively computed coefficient functions and $f_i(x, Q^2)$ are quark, antiquark and gluon distributions (multiplied by x). The Q^2-dependence of the parton distributions is determind by the DGLAP evolution equations [1] with initial conditions which are fitted to data. The first term on the r.h.s. of eq. (1) provides the leading twist-2 description with logarithmic dependence on Q^2 while the remaining terms, called higher twists, seem to be suppressed for large Q^2. In the standard analysis, a global fit of the leading twist formula to the HERA data on $F_2 = F_T + F_L$, together with cross sections of other hard processes, leads to the determination of the parton distributions shown in Fig. 1. A
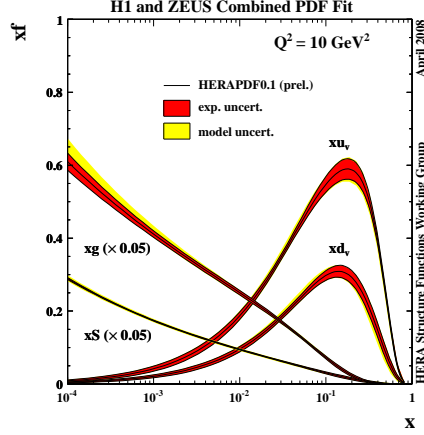
Fig. 1: *Parton distributions from a global fit to the HERA data as functions of x for fixed $Q^2 = 10 \text{ GeV}^2$.*

distinct feature of this determination is a strong rise of the gluon and sea quark distributions for $x \to 0$.

A closer theoretical examination of the small-x scattering reveals that for not too high Q^2, the higher twist terms cannot be neglected since they are enhanced by powers of $\alpha_s \log(1/x)$, when the smallness of α_s is compensated by a large logarithm of x. The relevant resummation of such terms in the leading (LO) and next-to-leading (NLO) logarithmic approximation leads to the BFKL approach to the structure functions with the following k_T-factorized form [2]:

$$F_2(x, Q^2) = Q^2 \int \frac{d^2 k_T}{k_T^4} \, \Phi(k_T^2/Q^2, \alpha_s(k_T)) \, f(x, k_T) \tag{2}$$

where the impact factor $\Phi(k_T^2/Q^2, \alpha_s(k_T))$ describes the interaction of the virtual photon with a gluon with nonzero transverse momentum k_T. In the LO this is the process: $\gamma^*(Q^2)g(k_T) \to q\bar{q}$. The function $f(x, k_T)$ is called unintegrated gluon distribution which obeys the BFKL equation [3] and is related to the gluon distribution $g(x, Q^2)$ through the formula

$$xg(x, Q^2) = \int \frac{d^2 k_T}{k_T^2} \, f(x, k_T) \, \theta(|k_T| < Q^2) \,. \tag{3}$$

From the solution of the BFKL equation, the small-x limit is dominated by the gluon distribution with the power-like rise, $f(x, k_T) \sim x^{-\lambda}$ and $\lambda \approx 0.3$. There is a general agreement, based on the experience with the Froissart-Martin bound, that such a rise of the gluon distribution, and in consequence F_2, violates unitarity and eventually must be tamed. The BFKL solution is also plagued by diffusion to infrared, namely, the k_T-integration in the pQCD formula (2) is quickly dominated by the contribution from the soft momenta region, $k_T \approx \Lambda_{QCD}$, where the Landau pole of $\alpha_s(k_T)$ is encountered. A cure for these problems is absolutely necessary.

3 Parton saturation

The taming of the power-like rise of the gluon distribution $xg(x, Q^2)$ was addressed for the first time by Gribov, Levin and Ryskin in [4] in the double logarithmic approximation. Summing fan
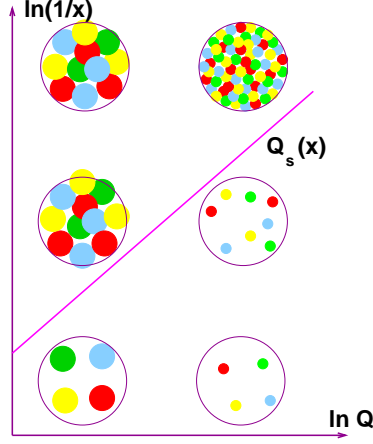
Fig. 2: *Saturation line in the (x, Q^2)-plane.*

diagrams, which take into account the fusion of t-channel gluons, the linear DGLAP equation for the gluon distribution receives a negative, nonlinear term,

$$\frac{\partial^2 xg(x,Q^2)}{\partial \ln(1/x)\partial \ln Q^2} = \overline{\alpha}_s xg(x,Q^2) - \frac{\alpha_s^2}{\pi^2 R^2}\frac{[xg(x,Q^2)]^2}{Q^2}\,, \tag{4}$$

where $\overline{\alpha}_s = N_c\alpha_s/\pi$ and the parameter R controls the strength of the nonlinearity. With such a modification, the gluon distribution saturates for $x \to 0$, and so does the structure function. This result was extended in [5] by including nonlinear modifications for the sea quark distributions. A crucial feature introduced by the nonlinearity is an x-dependent saturation scale $Q_s^2(x)$, defined as a value of Q^2 for which the nonlinear term in eq. (4) is comparable with the linear one:

$$xg(x,Q_s^2)\frac{\alpha_s(Q_s^2)}{Q_s^2} \sim \pi R^2\,. \tag{5}$$

Therefore, saturation effects are important when the number of gluons per unit of rapidity, xg, times the gluon-gluon interaction cross section, α_s/Q^2, approaches the geometric size of the nucleon or a gluonic system inside the nucleon ("hot spot"). In such a case, a simple additive treatment of parton emission breaks down and gluons start to annihilate. Since from (5) $Q_s^2 \sim xg$ and $xg \sim x^{-\lambda}$ before the saturation limit is reached, we find that $Q_s^2 \gg \Lambda_{QCD}^2$ for sufficiently small x, and the presented approach based on perturbative QCD is justified . This is schematically illustrated in Fig. 2 where two regions separated by the saturation line, $Q^2 = Q_s^2(x)$, are shown. Below this line, in the dilute region, the linear evolution equations are valid, while approaching the line, the saturation region is entered with nonlinear equations describing parton saturation.

Eq. (4) is a rather crude approximation since it is valid in the extreme case, $x \to 0$ and $Q^2 \to \infty$. In the k_T-factorization approach the latter limit is relaxed and only large logarithms $\log(1/x)$ are relevant. Summing BFKL pomeron fan diagrams with triple pomeron vertices in the leading logarithmic approximation and in the limit of large number of colors N_c, the Balitsky-

Kovchegov (BK) equation for the unintegrated gluon density $\phi(x, k_T)$ is found[1] [6, 7]:

$$\partial_Y \phi(x, k_T) = \overline{\alpha}_s \chi(-\partial_L) \phi - \overline{\alpha}_s \phi^2 \qquad (6)$$

where $Y = \log(1/x)$ is rapidity, $L = \log k_T^2$ and χ is the BFKL characteristic function. This nonlinear equation generalizes the linear BFKL equation. The properties of its solutions were intensively studied both analytically [8] and numerically [9]. The most fruitful approach is based on the relation to the known from statistical physics Fisher-Kolmogorov equation, which admits travelling wave solutions. In our language, it means that the BK solution develops a saturation scale, $Q_s(x) \sim x^{-\lambda}$ with known value of λ [8], such that for small x we have

$$\phi(x, k_T) = \phi(k_T/Q_s(x)). \qquad (7)$$

This property, called geometric scaling, was observed in the data from HERA [10]. Looking more carefully, for $k_T \gg Q_s(x)$ the gluon distribution $\phi \sim 1/k_T^2$, while for small transverse momenta, $\Lambda_{QCD} \ll k_T < Q_s(x)$, the behaviour changes to logarithmic, $\phi \sim \ln(Q_s(x)/k_T)$. This is the illustration of the transition to saturation, when both the power-like growth in x and infrared diffusion in k_T of the gluon distribution are tamed, see again Fig. 2 with $Q^2 \equiv k_T^2$.

4   Color dipole approach and beyond

A more intuitive approach to parton saturation is provided by the color dipole approach [11, 12]. In the target rest frame, the DIS at small x can be formulated as the eikonal scattering of a color quark-antiquark dipole, formed by the splitting $\gamma^* \to q\bar{q}$, on the target color field. The dipole scattering amplitude $N(\mathbf{x}, \mathbf{y})$ is given by two Wilson lines collinear to quarks' velocity u

$$N(\mathbf{x}, \mathbf{y}) = 1 - \frac{1}{N_c} \mathrm{Tr}\, U(\mathbf{x}) U^\dagger(\mathbf{y}), \qquad U(\mathbf{x}) = \mathrm{P} \exp\left\{ ig \int_{-\infty}^{\infty} d\lambda\, u \cdot A(\lambda u + \mathbf{x}) \right\} \qquad (8)$$

where $\mathbf{x}$ and $\mathbf{y}$ are two dimensional vectors of the quark transverse positions, conserved during the collision, and A is a target color field. The deviation of the classcal quark trajectory from the light-like line defines the change of N with rapidity Y, which leads to the new BK equation for the dipole scattering amplitude [6]. Its solutions fulfil the unitarity bound, $N \leq 1$. When the dependence on the impact parameter, $\mathbf{b} = (\mathbf{x} + \mathbf{y})/2$ is neglected, the new equation is equivalent to eq. (6) after Fourier transforming of $N/\mathbf{r}^2$ with respect to $\mathbf{r} = \mathbf{x} - \mathbf{y}$. The BK equation in the transverse space was also obtained in the Mueller's dipole approach [12] in which the $q\bar{q}$ dipole develops a system of dipoles (by radiating soft gluons in the large N_c approximation) which subsequently multiply interact with a large nucleus target [7].

The dipole scattering amplitude is the basic ingredient in the computation of the nucleon structure functions at small x. In the last ten years, this amplitude was also modelled using the properties of the BK solutions such as color transparency, $N \sim r^2$ for a small dipole size $r = |\mathbf{r}|$; geometric scaling, $N = N(rQ_s(x))$; and the unitarity bound, $N \leq 1$. A recent comprehensive review on the dipole models of DIS processes is presented in [13].

The BK equation describes unitarity corrections in the asymmetric configuration when the target is extended and dense and the projectile is small and dilute. In a more symmetric

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[1]ϕ is related to the unintegrated gluon density f from Section 2 by $f(x, k) \sim k^2 \nabla_k^2 \phi(x, k)$.

configuration, e.g. in the pp scattering at LHC, the BK equation is no longer sufficient, which means that in the diagrammatic approach closed pomeron loops have to be taken into account besides fan diagrams. An interesting attempt in this direction was made in [14] where pomeron loops were modelled as color reconnections in the dipole cascades. The resulting scattering amplitudes respect the target-projectile symmetry and describe reasonable well the existing total and diffractive cross sections in the $p\overline{p}$ scattering. The pomeron loops were also studied in a statistical approach, based on the stochastic Fisher-Kolmogorov equation, finding a new kind of scaling called diffusive scaling [15]. Recently, high energy factorization theorems for the gluon production in heavy nucleus collisions were proven in the color glass condensate approach [16].

References

[1] V. N. Gribov and L. N. Lipatov, Sov. J. Nucl. Phys. **15**, 438 (1972);
G. Altarelli and G. Parisi, Nucl. Phys. **B126**, 298 (1977);
Y. L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977).

[2] S. Catani, M. Ciafaloni, and F. Hautmann, Phys. Lett. **B242**, 97 (1990);
S. Catani, M. Ciafaloni, and F. Hautmann, Nucl. Phys. **B366**, 135 (1991);
J. C. Collins and R. K. Ellis, Nucl. Phys. **B360**, 3 (1991).

[3] V. S. Fadin, E. A. Kuraev, and L. N. Lipatov, Phys. Lett. **B60**, 50 (1975);
L. N. Lipatov, Sov. J. Nucl. Phys. **23**, 338 (1976);
E. A. Kuraev, L. N. Lipatov, and V. S. Fadin, Sov. Phys. JETP **45**, 199 (1977);
I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28**, 822 (1978).

[4] L. V. Gribov, E. M. Levin, and M. G. Ryskin, Phys. Rept. **100**, 1 (1983).

[5] A. H. Mueller and J.-w. Qiu, Nucl. Phys. **B268**, 427 (1986).

[6] I. Balitsky, Nucl. Phys. **B463**, 99 (1996). `hep-ph/9509348`.

[7] Y. V. Kovchegov, Phys. Rev. **D60**, 034008 (1999). `hep-ph/9901281`;
Y. V. Kovchegov, Phys. Rev. **D61**, 074018 (2000). `hep-ph/9905214`.

[8] S. Munier and R. B. Peschanski, Phys. Rev. Lett. **91**, 232001 (2003). `hep-ph/0309177`;
S. Munier and R. B. Peschanski, Phys. Rev. **D69**, 034008 (2004). `hep-ph/0310357`;
S. Munier and R. B. Peschanski, Phys. Rev. **D70**, 077503 (2004). `hep-ph/0401215`.

[9] N. Armesto and M. A. Braun, Eur. Phys. J. **C20**, 517 (2001). `hep-ph/0104038`;
K. J. Golec-Biernat, L. Motyka, and A. M. Stasto, Phys. Rev. **D65**, 074037 (2002). `hep-ph/0110325`.

[10] A. M. Stasto, K. Golec-Biernat, and J. Kwiecinski, Phys. Rev. Lett. **86**, 596 (2001). `hep-ph/0007192`.

[11] N. N. Nikolaev and B. G. Zakharov, Z. Phys. **C49**, 607 (1991).

[12] A. H. Mueller, Nucl. Phys. **B415**, 373 (1994);
A. H. Mueller and B. Patel, Nucl. Phys. **B425**, 471 (1994). `hep-ph/9403256`;
A. H. Mueller, Nucl. Phys. **B437**, 107 (1995). `hep-ph/9408245`.

[13] L. Motyka, K. Golec-Biernat, and G. Watt (2008). `0809.4191`.

[14] E. Avsar, G. Gustafson, and L. Lonnblad, JHEP **07**, 062 (2005). `hep-ph/0503181`;
E. Avsar, G. Gustafson, and L. Lonnblad, JHEP **01**, 012 (2007). `hep-ph/0610157`;
E. Avsar, G. Gustafson, and L. Lonnblad, JHEP **12**, 012 (2007). `0709.1368`.

[15] A. H. Mueller and A. I. Shoshi, Nucl. Phys. **B692**, 175 (2004). `hep-ph/0402193`;
S. Munier, Nucl. Phys. **A755**, 622 (2005). `hep-ph/0501149`;
E. Iancu, A. H. Mueller, and S. Munier, Phys. Lett. **B606**, 342 (2005). `hep-ph/0410018`.

[16] F. Gelis, T. Lappi, and R. Venugopalan (2008). `0804.2630`;
F. Gelis, T. Lappi, and R. Venugopalan (2008). `0807.1306`.