Questions from conveners:
How can correlations be used to determine
the size of the interaction and phase transitions?

An approach to QCD phase transition via multiplicity fluctuations and correlations

Kensuke Homma

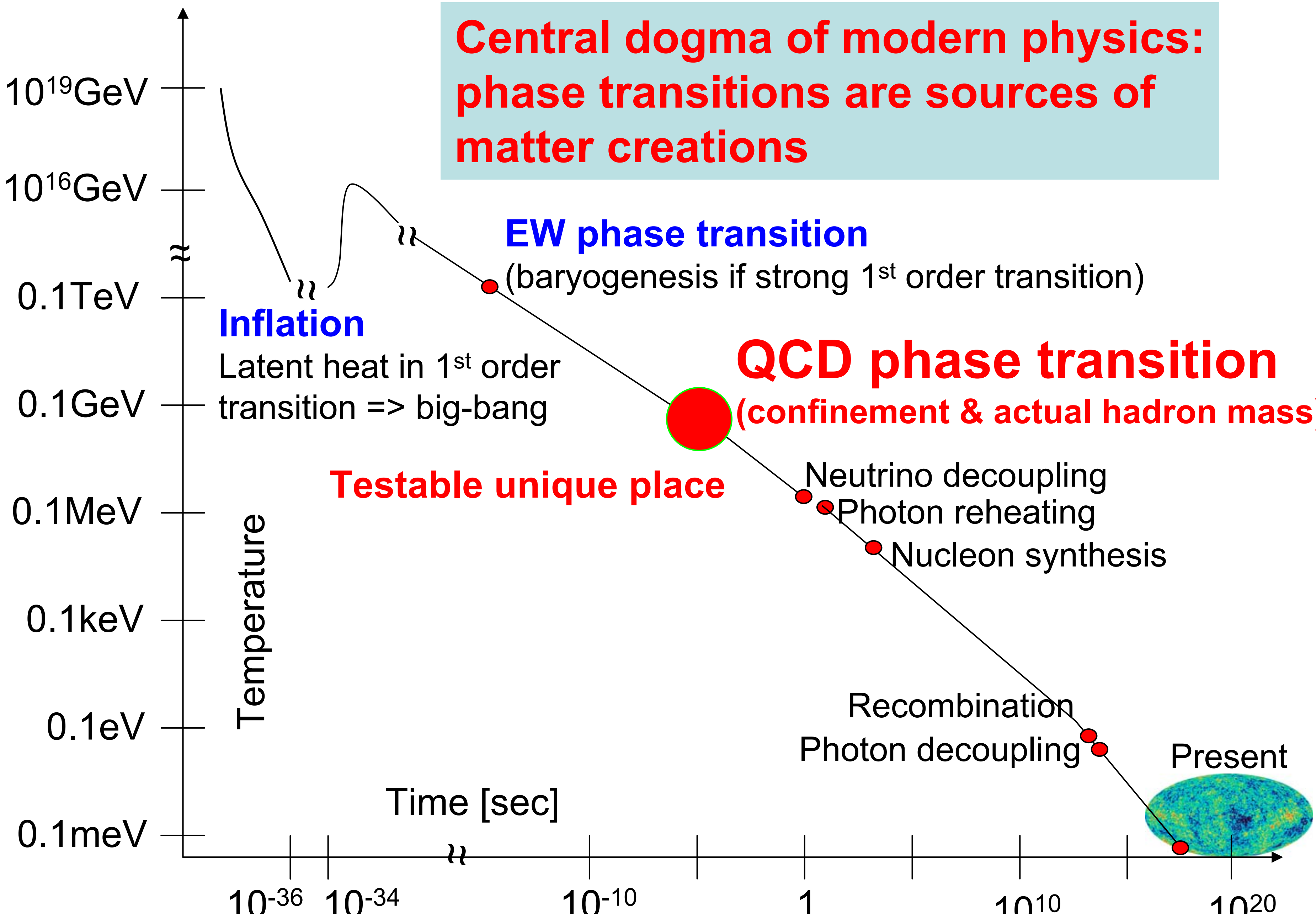
Hiroshima University

1. Why we search for QCD phase transition
2. An experimental approach to critical phenomena at RHIC-PHENIX

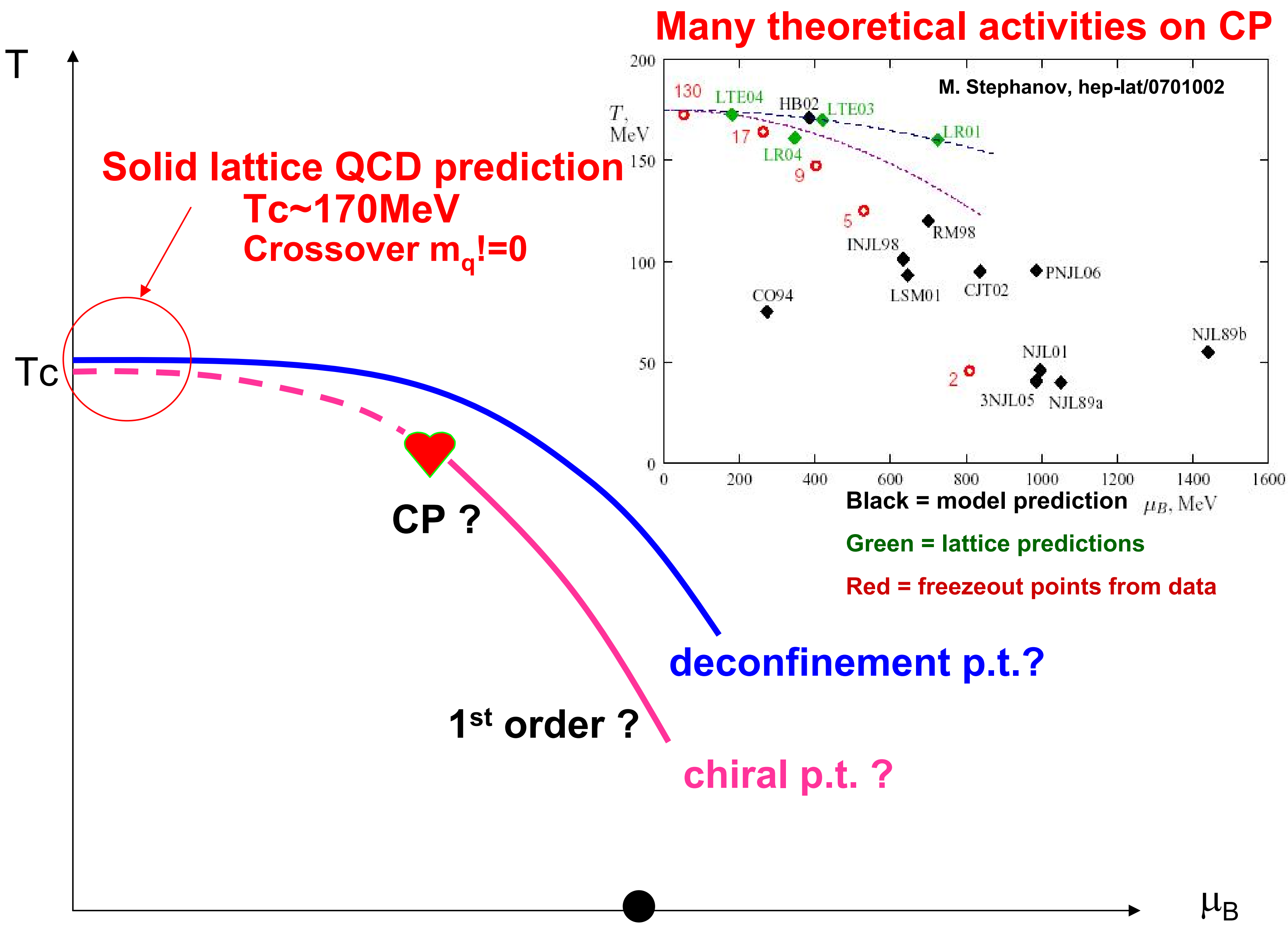
ISMD 2008

19 Sep, 2008 in DESY, Hamburg, Germany

Phase transitions in the early universe



Conjectured QCD phase diagram



What can we refute?

Good scientific subjects in a strict sense:

- If we find an octopus on Mars→we can refute a hypothesis that there is no creature on Mars.
- If we can not find Higgs below 1TeV→we can refute the Higgs sector of the standard model.
- If we find that transition at finite T and μ_B/T_c<<1 is NOT crossover→we can refute QCD in non perturbative region.

However,

even if we can not find a critical point, we can not refute QCD at finite T and finite μ_B (at this moment).

Then, what can we refute?

If we find a critical point→we can refute the empty diagram at the QCD scale→it supports the central dogma that vacuum phase transitions can be sources of matter creations.

This subject has an impact even beyond the QCD scale !

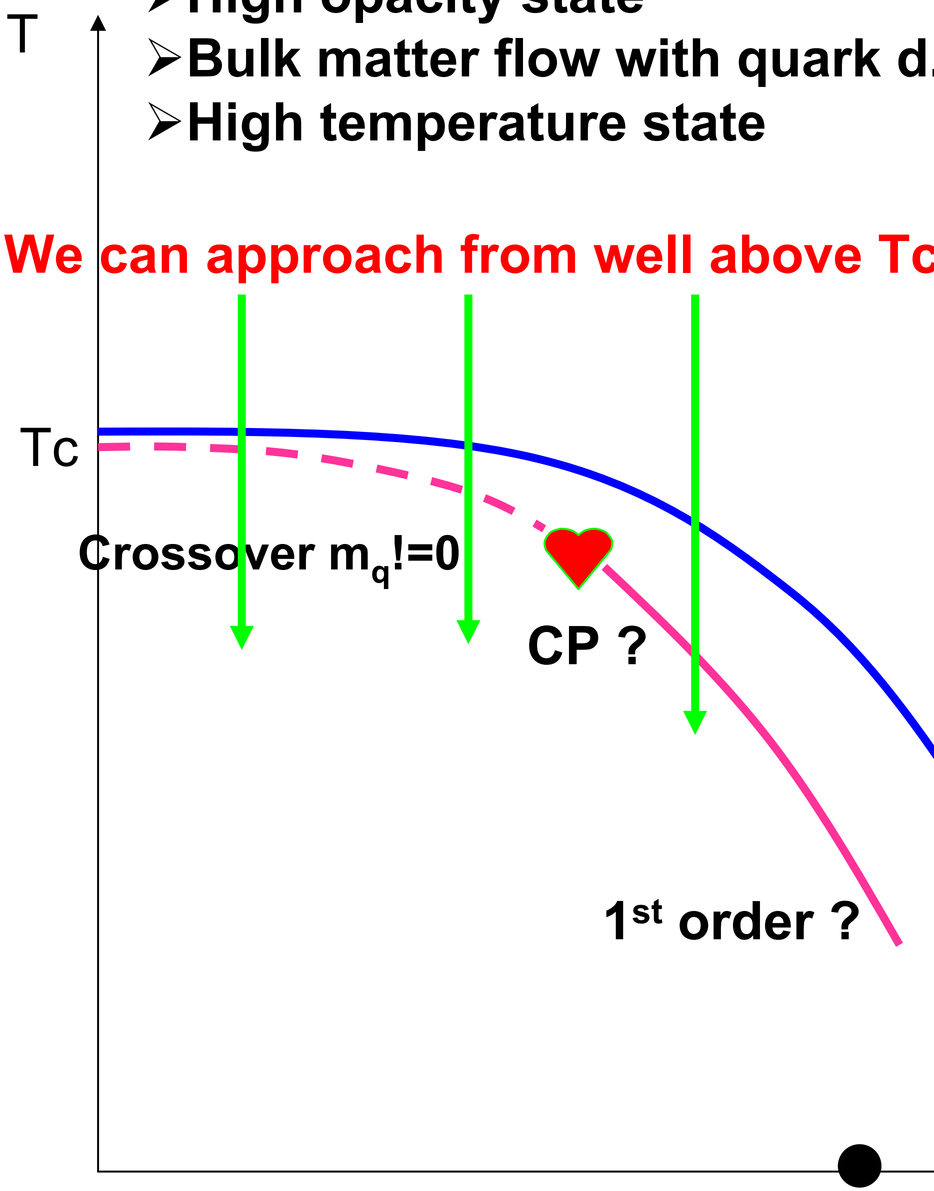
RHIC achievements

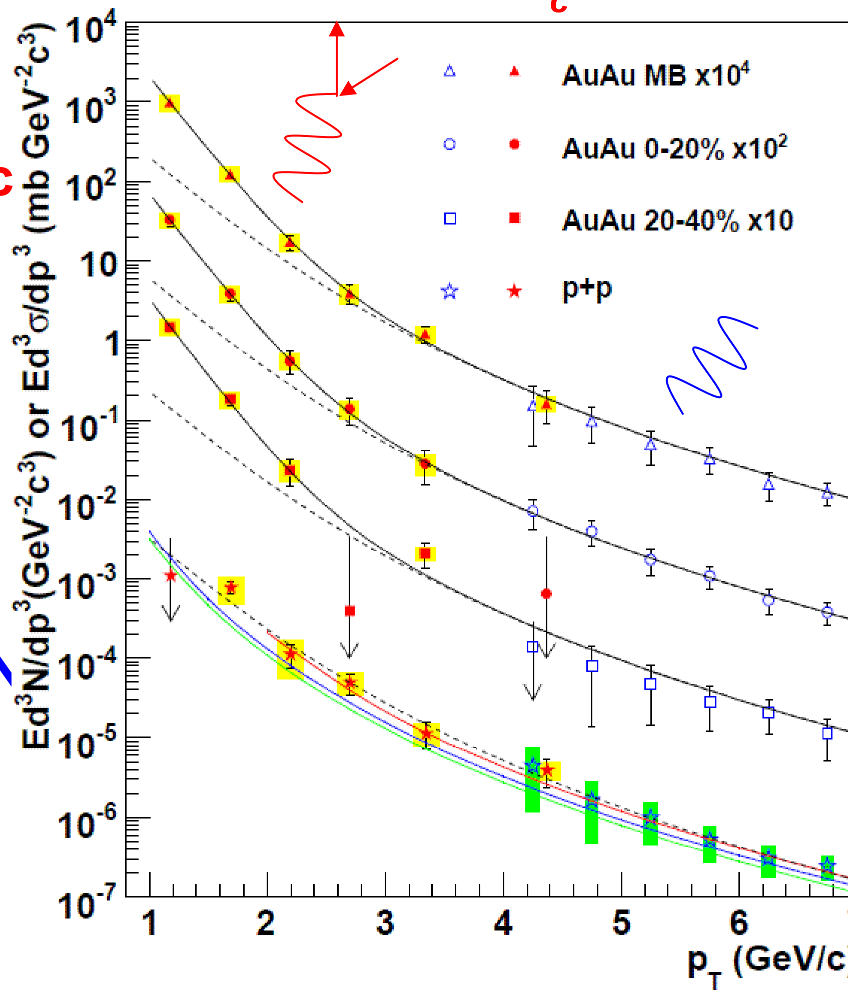
- High opacity state
- Bulk matter flow with quark d.o.f
- High temperature state

T=221±23(stat)±18(sys)

Lattice result T_c~170MeV

We can approach from well above Tc

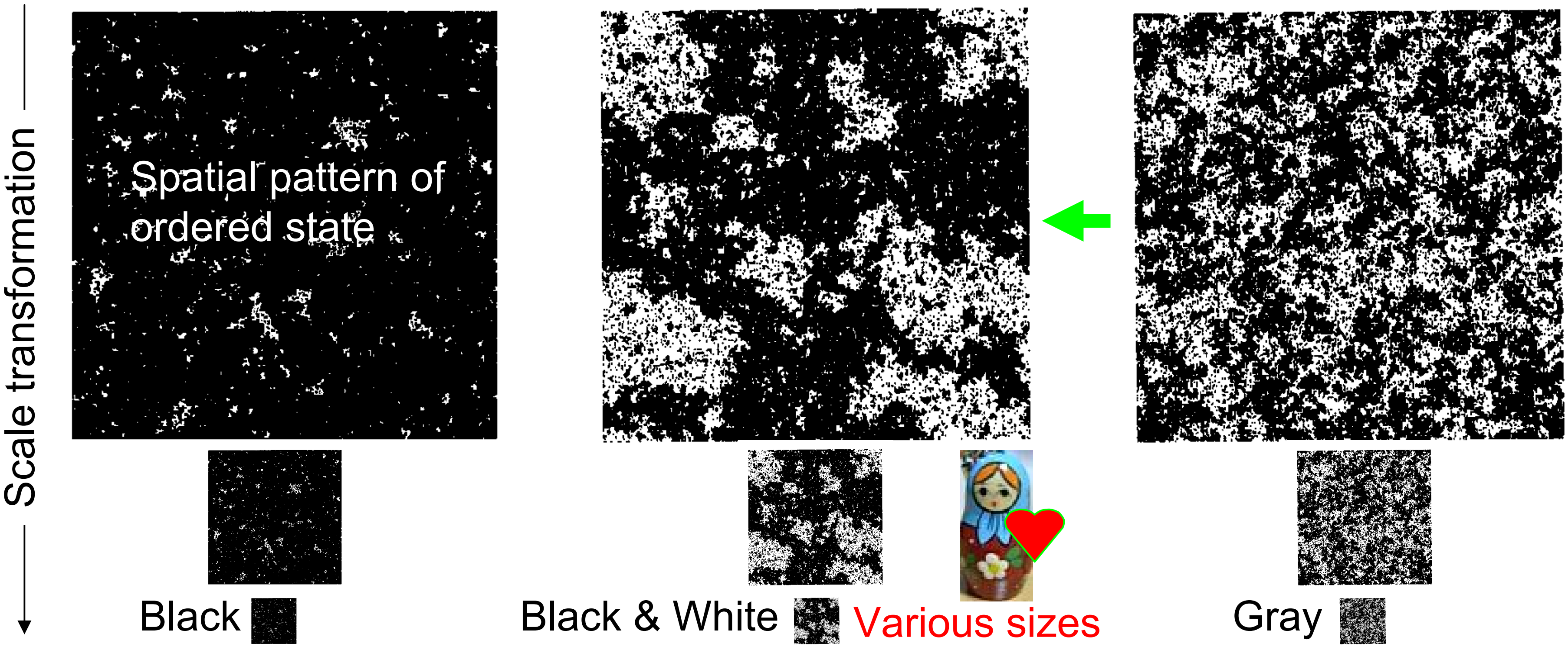




arXiv:0804.4168v1 [nucl-ex] 25 Apr 2008

What is the critical behavior ?

Ordered $T=0.995T_c$ Critical $T=T_c$ Disordered $T=1.05T_c$

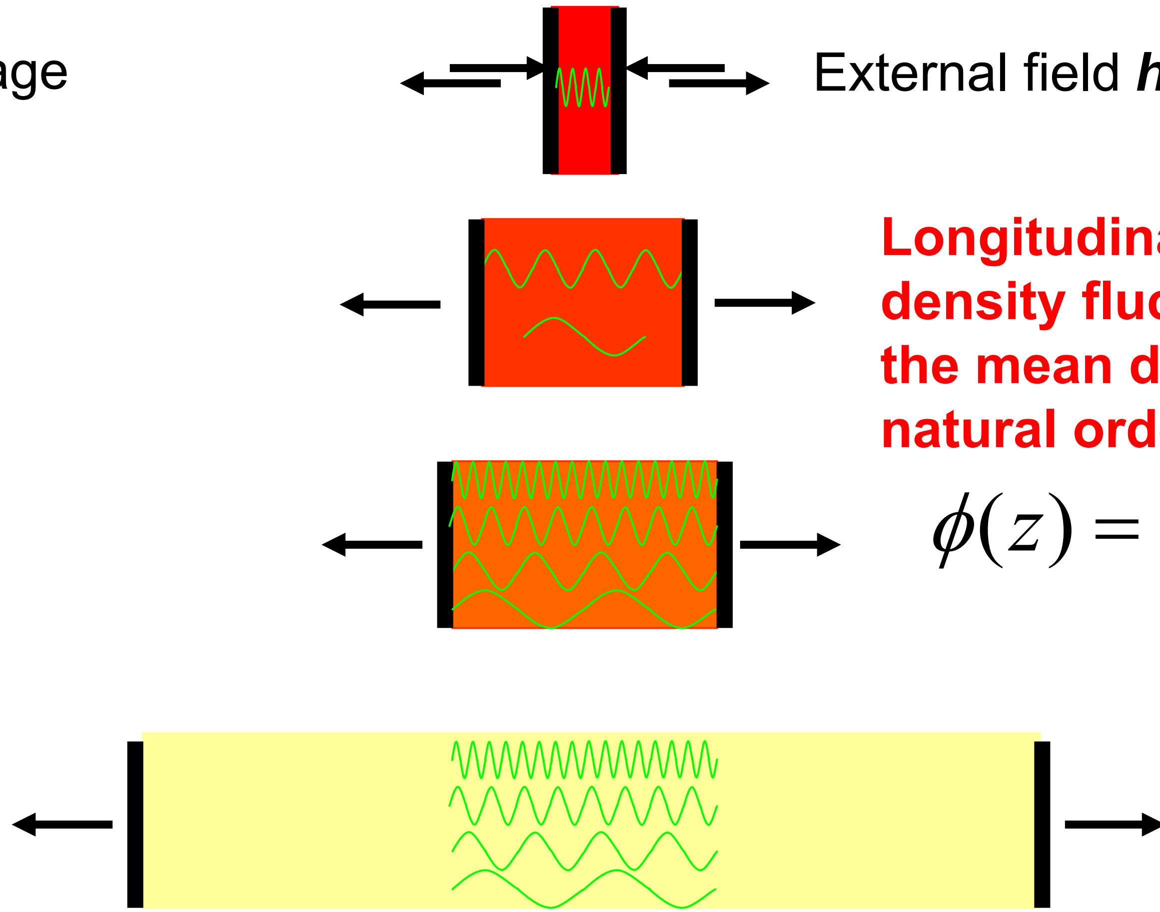


Various sizes
from small to large

Focus of this talk is search for a transition of the correlation size from T>Tc to T=Tc at RHIC

A simulation based on two dimensional Ising model
from ISBN4-563-02435-X C3342I

A picture of expanding medium in early stage



Longitudinal field density fluctuations from the mean density is a natural order parameter

$$\phi(z) = \rho(z) - \langle \rho \rangle$$

We may expect freeze of initially embedded fluctuation due to rapid dilution of medium in the longitudinal direction

Density-density correlation in longitudinal space

Longitudinal space coordinate z can be transformed into rapidity coordinate in each proper frame of sub element characterized by a formation time τ at which dominant density fluctuations are embedded.

$$z = \tau \sinh(y)$$

$$t = \tau \cosh(y)$$

$$dz = \tau \cosh(y) dy$$

Due to relatively rapid expansion in y, analysis in y would have an advantage to extract initial fluctuations compared to analysis in transverse plane in high energy collision.

$$g(T,\phi,h) - g_0 = \int_{\delta y} dy \cancel{\int_{S\perp} d^2 x_\perp}$$

$$\left[\frac{1}{2\tau^2 \cosh(y)} \left(\frac{\partial \phi}{\partial y} \right)^2 + \cosh(y) \left(\cancel{\frac{1}{2} (\nabla_\perp \phi)^2} + U(\phi) \right) \right]$$

In narrow midrapidity region like PHENIX, cosh(y)~1 and y~η.

Direct observable for Tc determination

GL free energy density g with $\phi \sim 0$ from high temperature side is insensitive to transition order, but it can be sensitive to Tc

$$g(T,\phi,h) = g_0 - \underbrace{\frac{1}{2}A(T)(\nabla\phi)^2}_{\text{spatial correlation}} + \underbrace{\frac{1}{2}a(T)\phi^2}_{\phi \text{ disappears at Tc}} + \cancel{\frac{1}{4}b\phi^4 + \frac{1}{6}c\phi^6} \cdots - h\phi$$

spatial correlation

ϕ disappears at Tc → $a(T) = a_0(T - T_c)$

Fourier analysis on

$G_2(y) = \langle\phi(0)\phi(y)\rangle$

$$\langle |\phi_k|^2 \rangle = Y \int G_2(y) e^{-ik(y)} dy$$

$$\langle |\phi_k|^2 \rangle = \frac{NT}{Y} \frac{1}{a(T) + A(T)k^2}$$

Susceptibility

$$\chi_k = \frac{\partial \phi_k}{\partial h} \propto \left(\frac{\partial^2 (g - g_0)}{\partial \phi^2{}_k} \right)^{-1} = \frac{1}{a_0(T - T_c)(1 + k^2\xi^2)}$$

Susceptibility in long wavelength limit

$$\chi_{k=0} = \frac{1}{a_0(T - T_c)} \propto \frac{\xi}{T} G_2(0)$$

1-D two point correlation function

$$G_2(y) = \frac{NT}{2Y^2 A(T)} \xi(T) e^{-|y|/\xi(T)}$$

Correlation length

$$\xi(T)^2 \equiv \frac{A(T)}{a_0(T - T_c)}$$

Product between correlation length and amplitude can also be a good indicator for T~Tc

Strategy to determine Tc

Step1.
Search for increase of correlation length and susceptibility (amplitude x correlation length) determined by exponential form in T>Tc→T~Tc

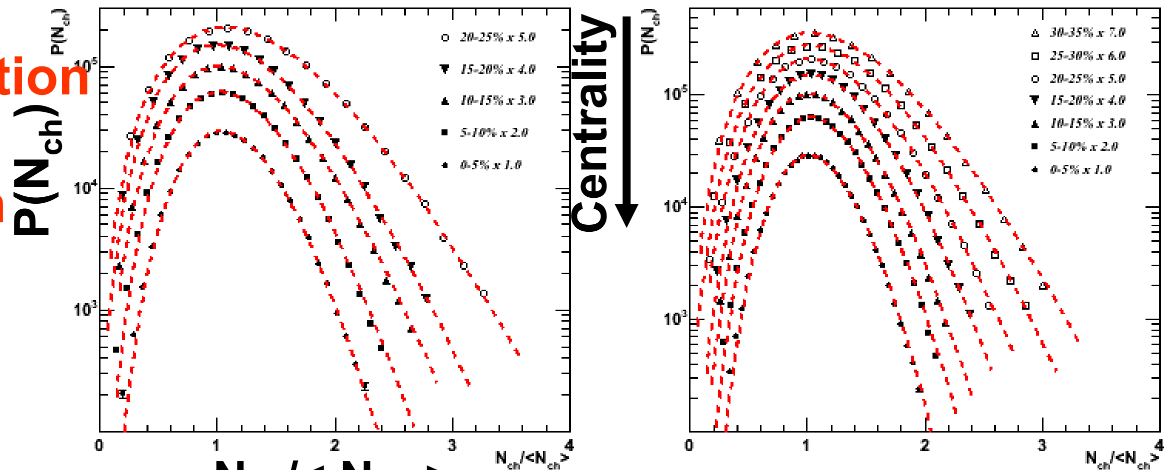
Step2.
Search for transition of two point correlation from exponential to power law form which needs higher order terms in the free energy density. This would be a stronger indication of T=Tc.

Density measurement: inclusive $dN_{ch}/d\eta$

Negative Binomial Distribution (NBD) perfectly describes multiplicities in all collision systems and centralities at RHIC.

Cu+Cu@62.4GeV

Cu+Cu@200GeV

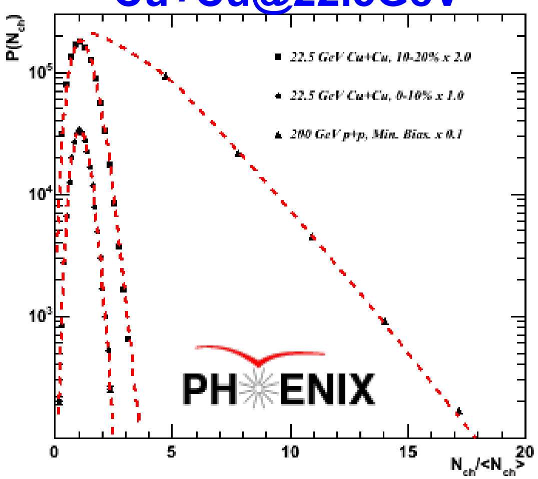


$N_{ch}/< N_{ch} >$

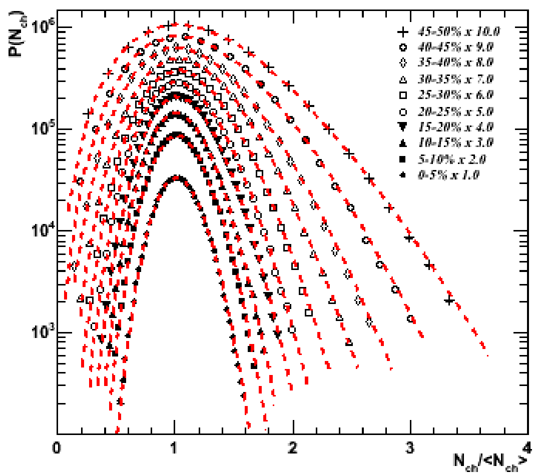
PHENIX

p+p@200GeV

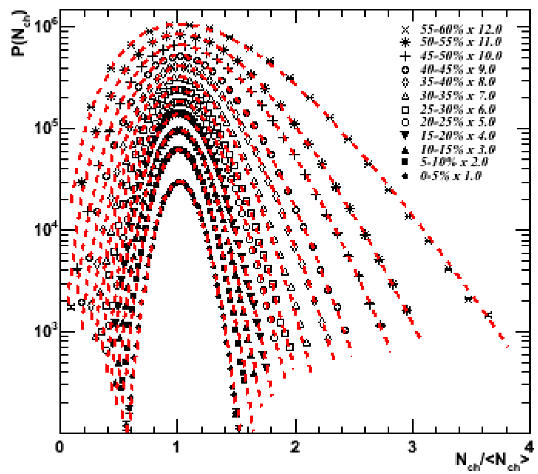
Cu+Cu@22.5GeV



Au+Au@62.4GeV



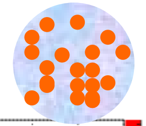
Au+Au@200GeV

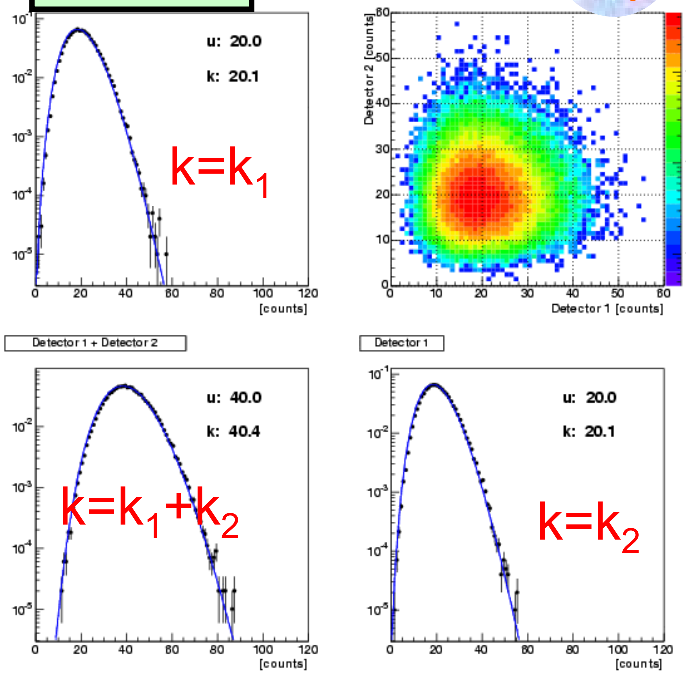


Two point correlation via NBD

source 1

Uncorrelated sources

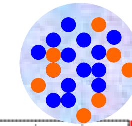


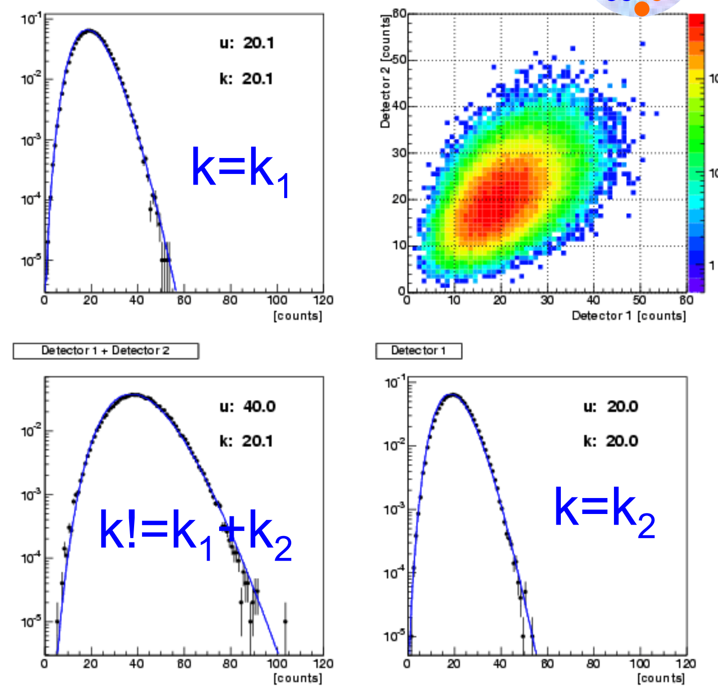


source 1+2

source 2

Correlated sources





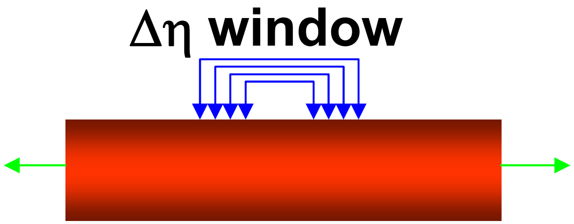
NBD $$P_n^{(k)} = \frac{\Gamma(n+k)}{\Gamma(n-1)\Gamma(k)} \left(\frac{\mu/k}{1+\mu/k} \right)^n \frac{1}{(1+\mu/k)^k}$$

k=1 Bose-Einstein
k=∞ Poisson

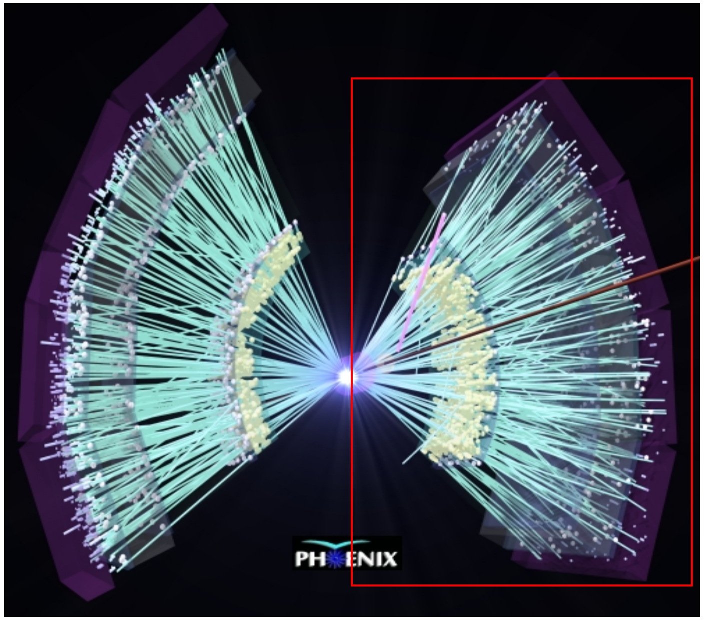
$$\frac{\sigma^2}{\mu^2} = \frac{1}{\mu} + \boxed{\frac{1}{k}} \qquad \mu \equiv \langle n \rangle$$

1/k corresponds to integral of two point correlation

Differential multiplicity measurements



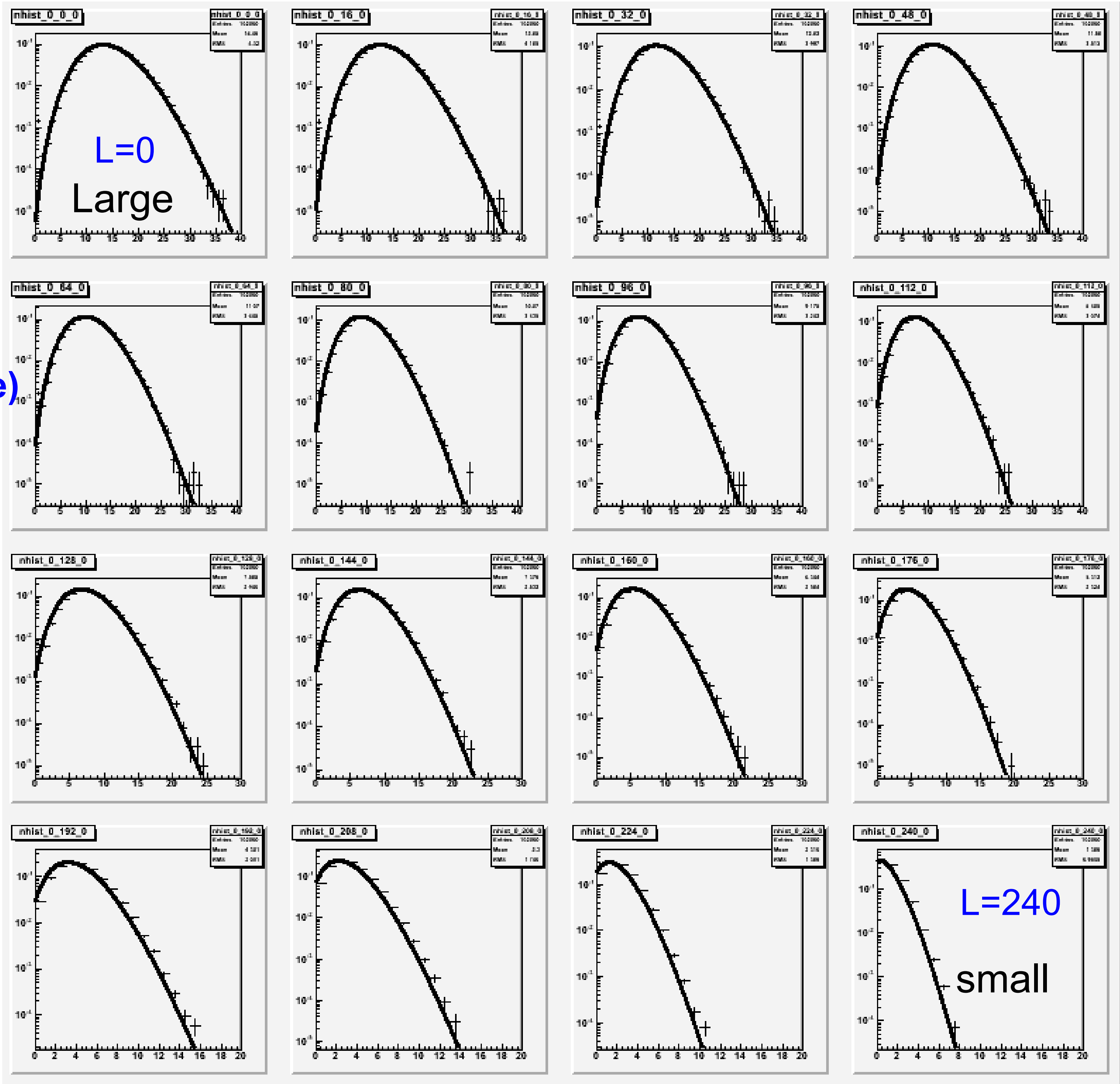
Δ η <0.7 integrated over Δ ϕ < π /2 and pT>0.1GeV



Zero magnetic field to enhance low pt statistics per collision event.

NBD fits at each window size in CuCu@200

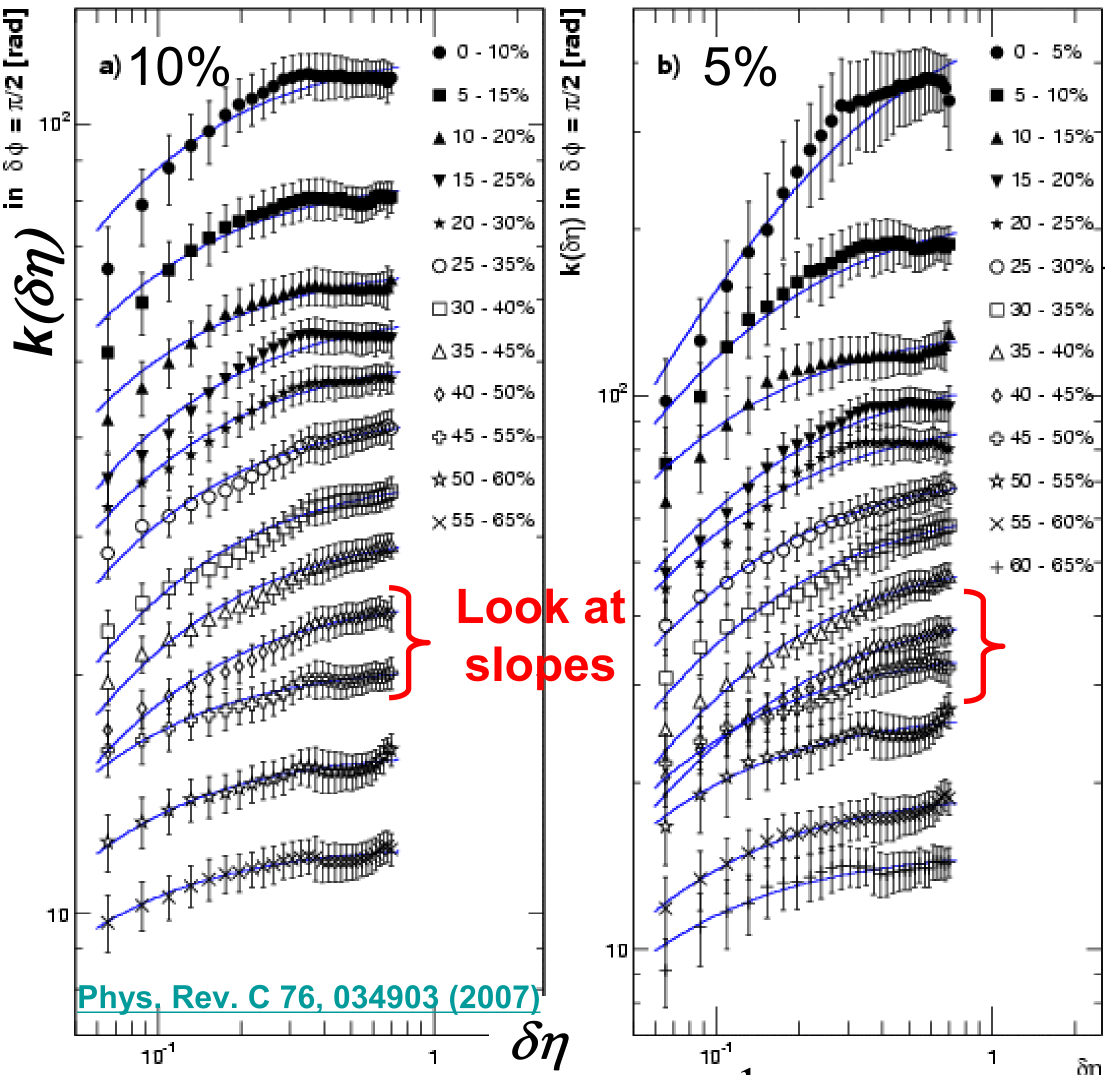
Level (window size)
$L=2^8(1-\delta\eta/\Delta\eta_{PHENIX})$



16 fit examples in most left edge in top 10% events out of $2^8/2*(1+2^8)$ times NBD fits

Extraction of αξ product

Fit with approximated functional form



Approximated functional form

$$k(\delta\eta) = \frac{1}{2\alpha\xi/\delta\eta + \beta} \quad (\xi \ll \delta\eta)$$

Parametrization of two particle correlation

$$C_2(\eta_1,\eta_2) \equiv \rho_2(\eta_1,\eta_2) - \rho_1(\eta_1)\rho_1(\eta_2)$$

$$\frac{C_2(\eta_1,\eta_2)}{\bar{\rho}_1^2} = \alpha e^{-\delta\eta/\xi} + \beta$$

β absorbs rapidity independent bias: Npart fluctuation and reaction plane rotation and v2

Exact relation with NBD *k*

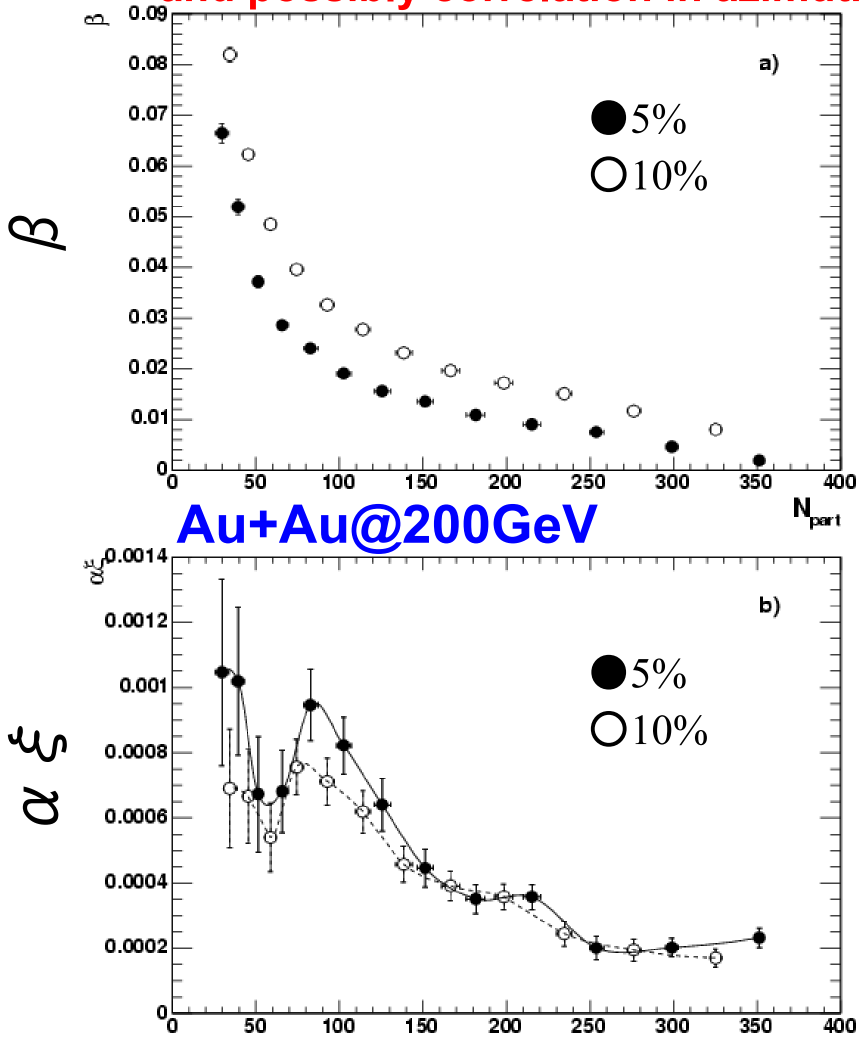
$$k^{-1}(\delta\eta) = \frac{\langle n(n-1)\rangle}{\langle n\rangle^2} - 1$$

$$= \frac{\int_0^{\delta\eta}\int_0^{\delta\eta} C_2(\eta_1,\eta_2)\,d\eta_1 d\eta_2}{\delta\eta^2 \bar{\rho}_1^2}$$

$$= \frac{2\alpha\xi^2(\delta\eta/\xi - 1 + e^{-\delta\eta/\xi})}{\delta\eta^2} + \beta$$

$\alpha\xi$, β vs. Npart

Dominantly Npart fluctuations
and possibly correlation in azimuth



Phys. Rev. C 76, 034903 (2007)

β is systematically shift to lower values as the centrality bin width becomes smaller from 10% to 5%. This is understood as fluctuations of Npart for given bin widths

$\alpha\xi$ product, which is monotonically related with $\chi_{k=0}$ indicates the non-monotonic behavior around Npart ~ 90.

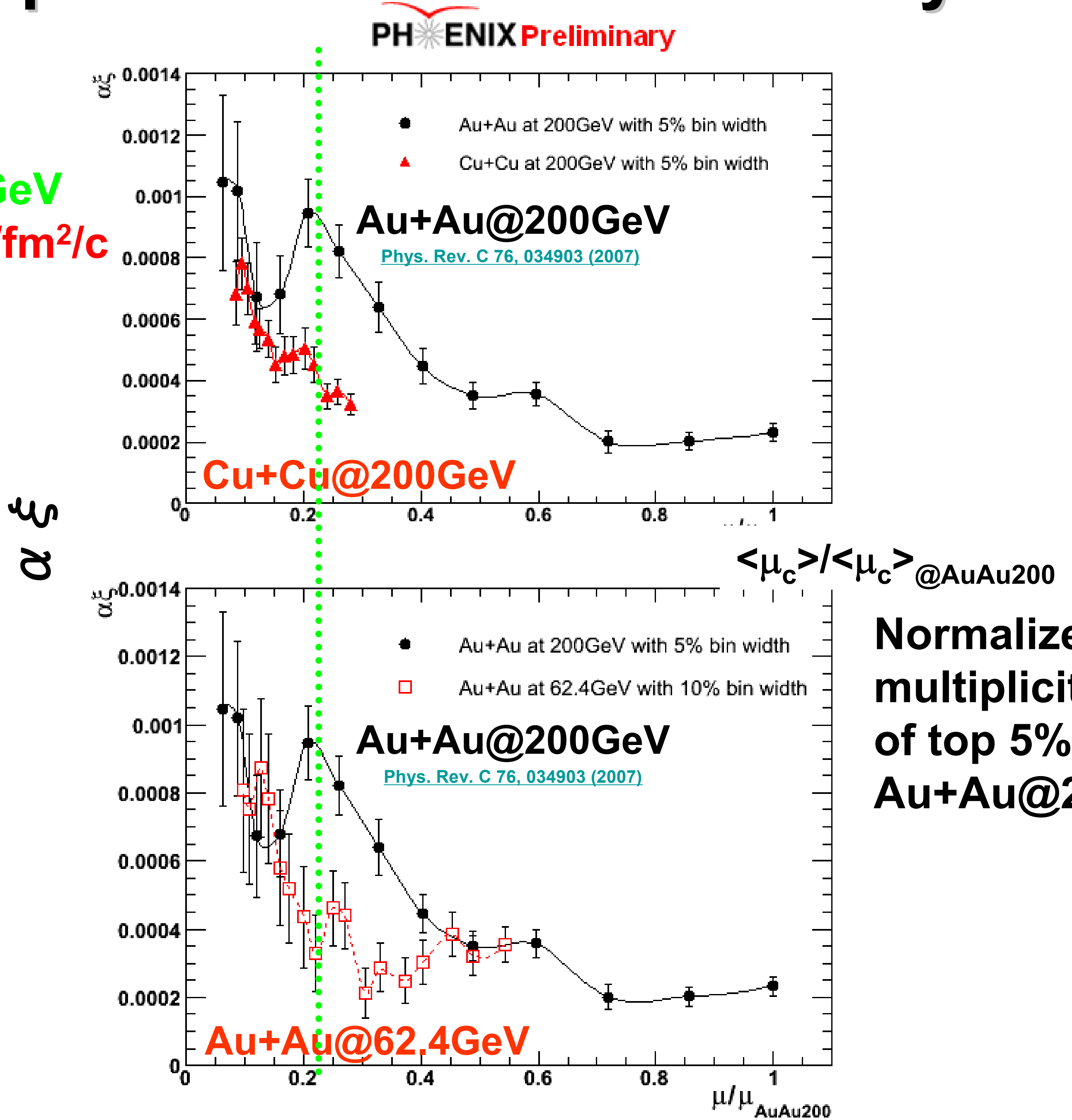
$$\alpha\xi = \chi_{k=0} T / \bar{\rho}_1^{\,2} \propto \bar{\rho}_1^{\,-2} \frac{T}{|T - T_C|}$$

Significance with Power + Gaussian:
3.98 σ (5%), 3.21 σ (10%)
Significance with Line + Gaussian:
1.24 σ (5%), 1.69 σ (10%)

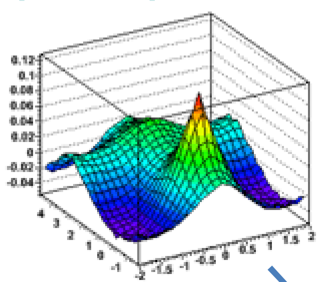
Comparison of three collision systems

Npart~90 in AuAu@200GeV

$\varepsilon_{BJ}\tau$~2.4GeV/fm^2/c



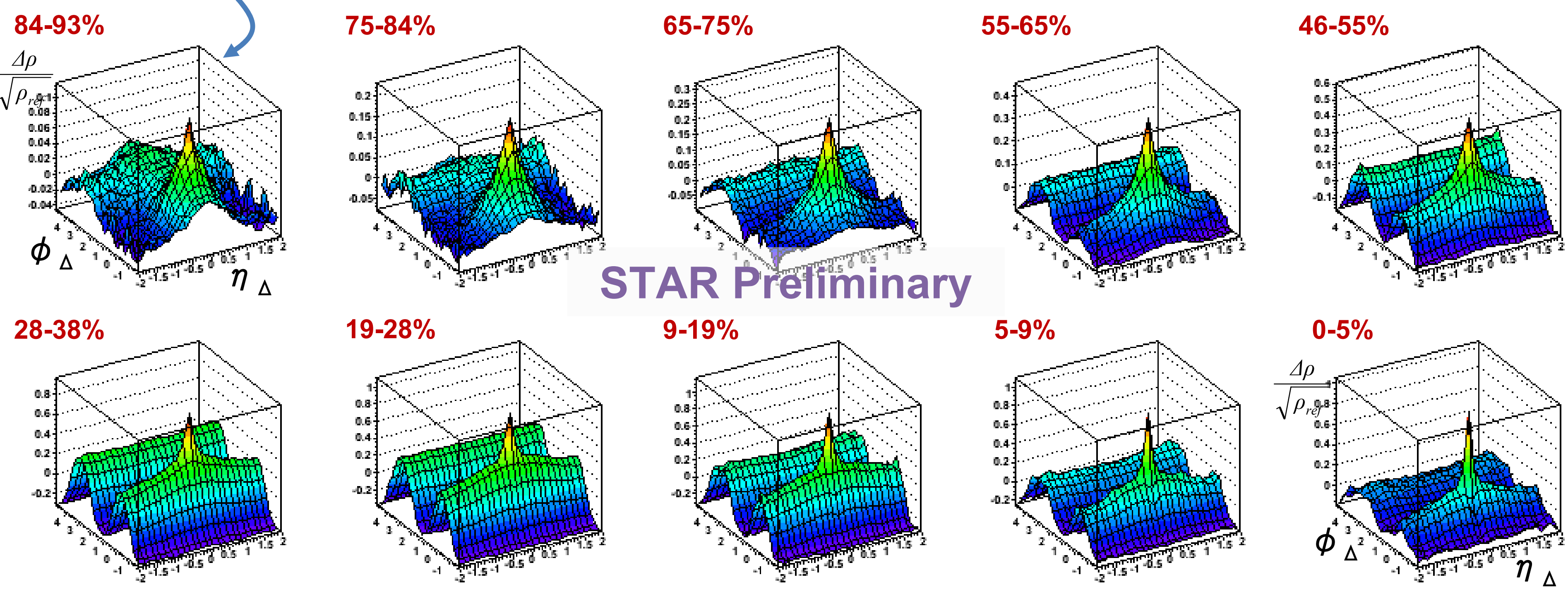
Normalized mean multiplicity to that of top 5% in Au+Au@200GeV



How about STAR?

Analyzed 1.2M minbias 200 GeV Au+Au events, and 13M 62 GeV minbias events (not shown) Included all tracks with p_T > 0.15 GeV/c, | η | < 1, full ϕ

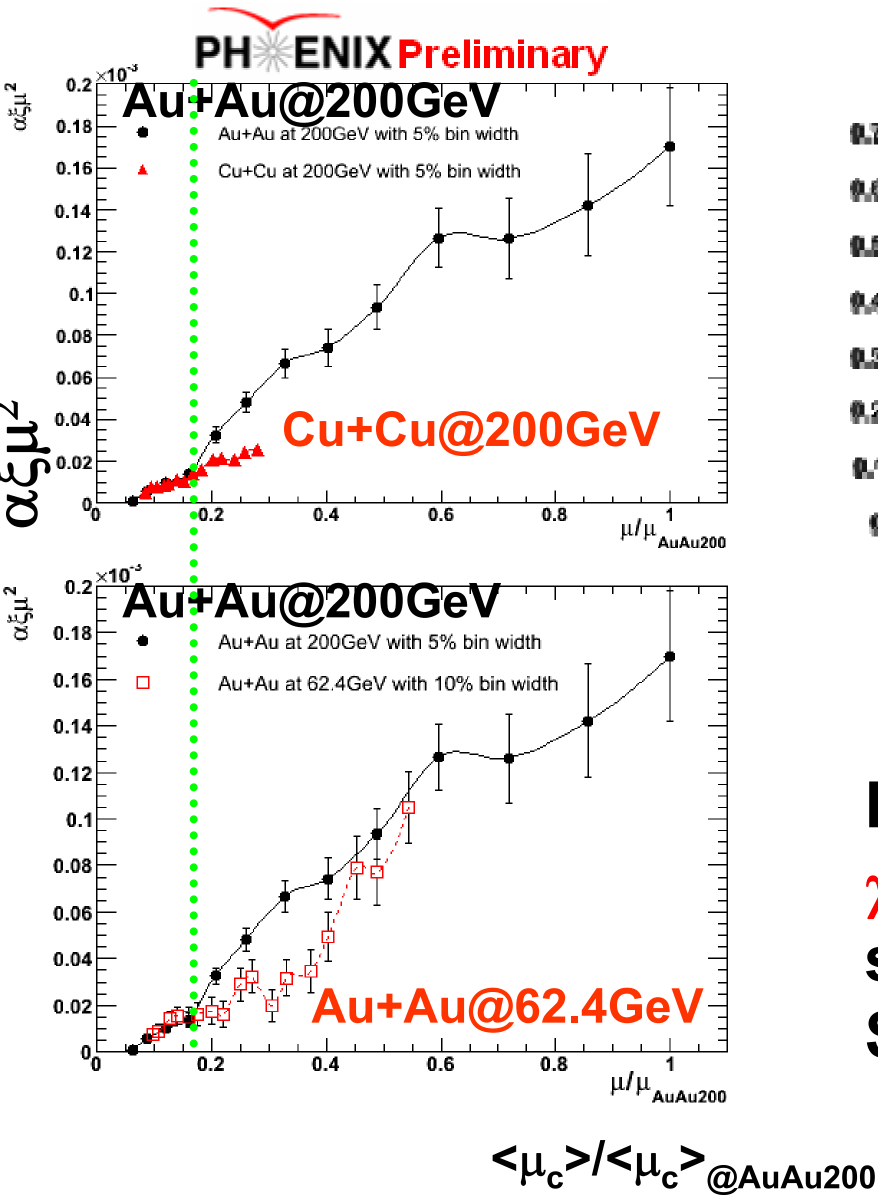
note: 38-46% not shown

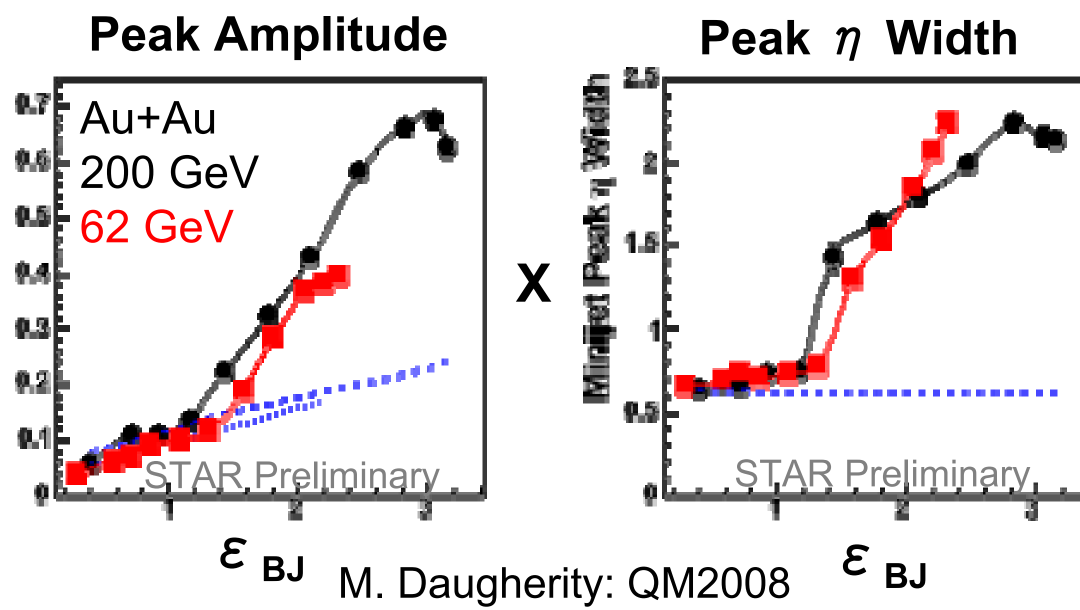


We see the evolution of correlation structures from peripheral to central Au+Au

Slide from M. Daugherity, STAR Collaboration presented at QM08

Similarity to STAR mini jet results at low p_T





Equivalent quantity;
$\chi T \propto \alpha\xi\mu^2 \propto$ amplitude x width
shows similar trends to what STAR sees.

Summary

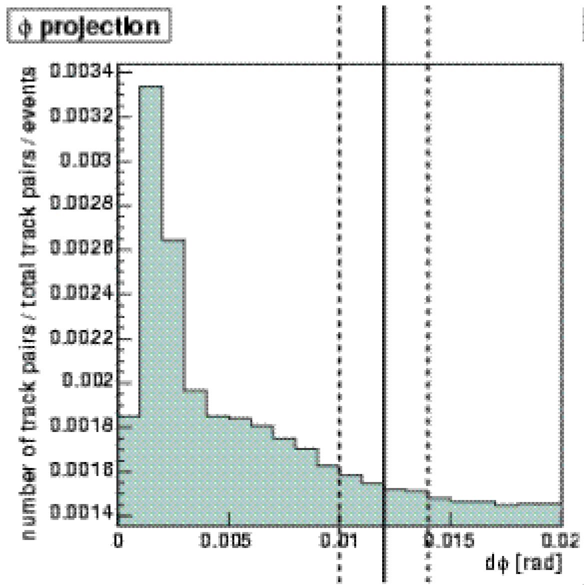
1. RHIC created strongly coupled high temperature & opaque state with partonic d.o.f. This is the very beginning of the scientific program on quantitative understanding of the QCD phase structure.

2. Correlation function derived from GL free energy density up to 2nd order term in the high temperature limit (exponential form) is consistent with what was observed in NBD k vs $\delta\eta$ in three collision systems. The $\alpha\xi$ as a function of Npart indicates a possible non monotonic increase at Npart~90 in Au+Au@200GeV. However, transitions from exponential to power law function were not seen.

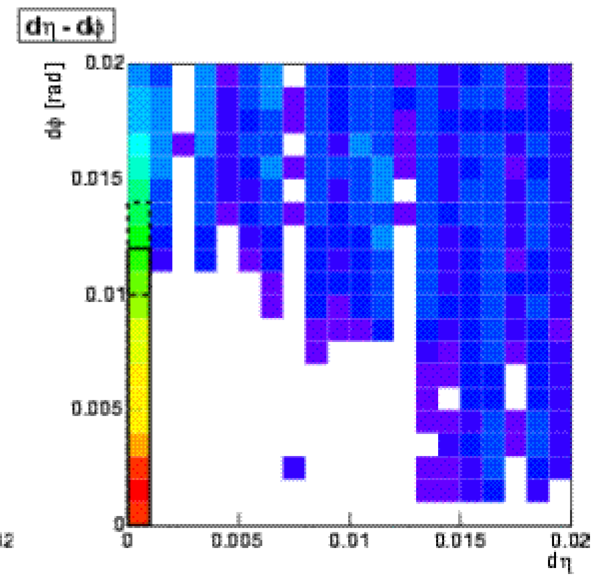
3. Centrality dependence of the product between susceptibility and temperature ($\chi T \propto \alpha\xi\mu^2$) is qualitatively consistent with what STAR observed by amplitude and width parameter in η correlations with low pT particles.

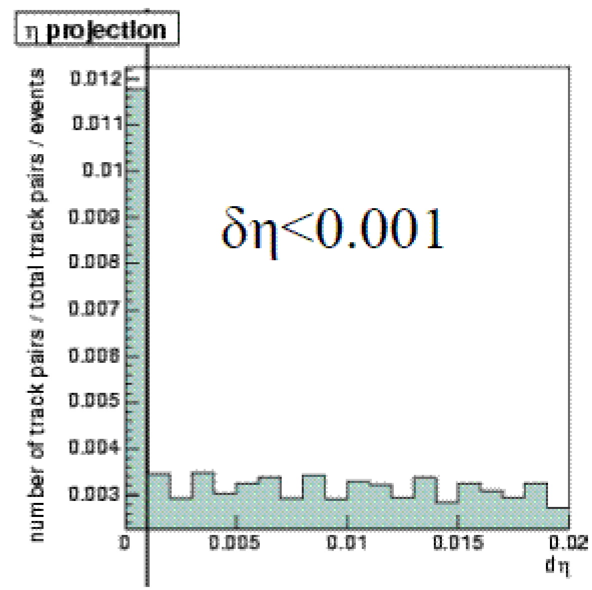
Buck up

Trivial correlations (ghost & $\gamma\rightarrow ee$)

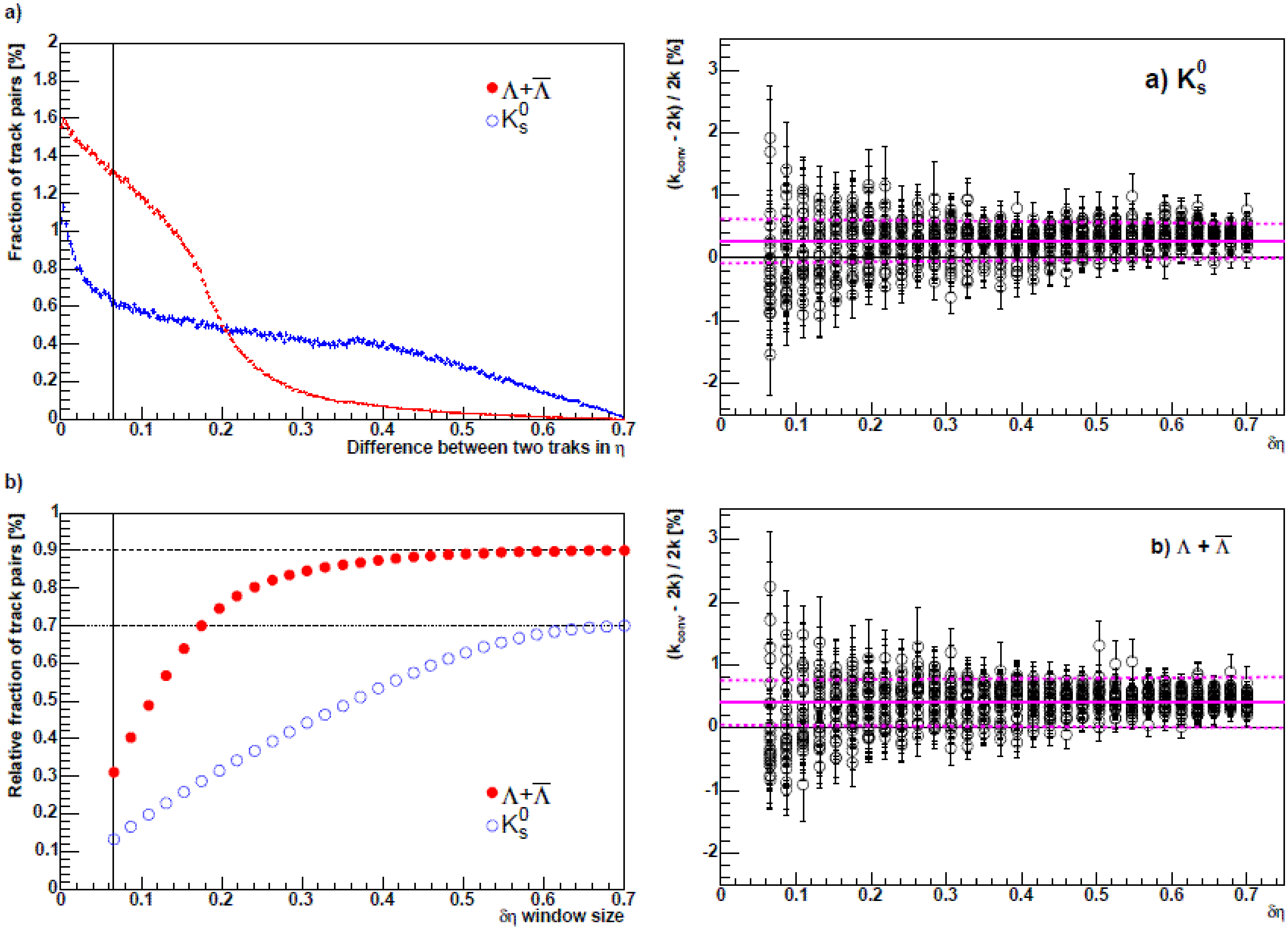


δφ<0.012±0.002

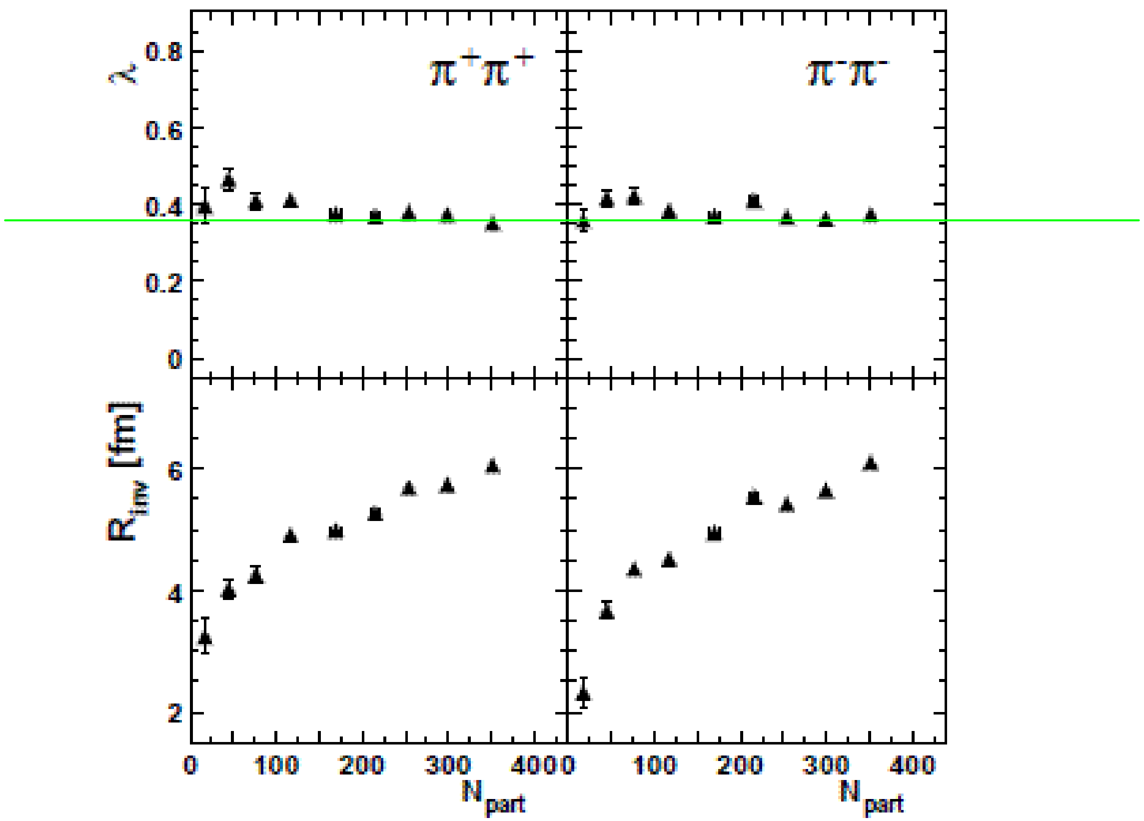




Trivial correlations (weak decays)



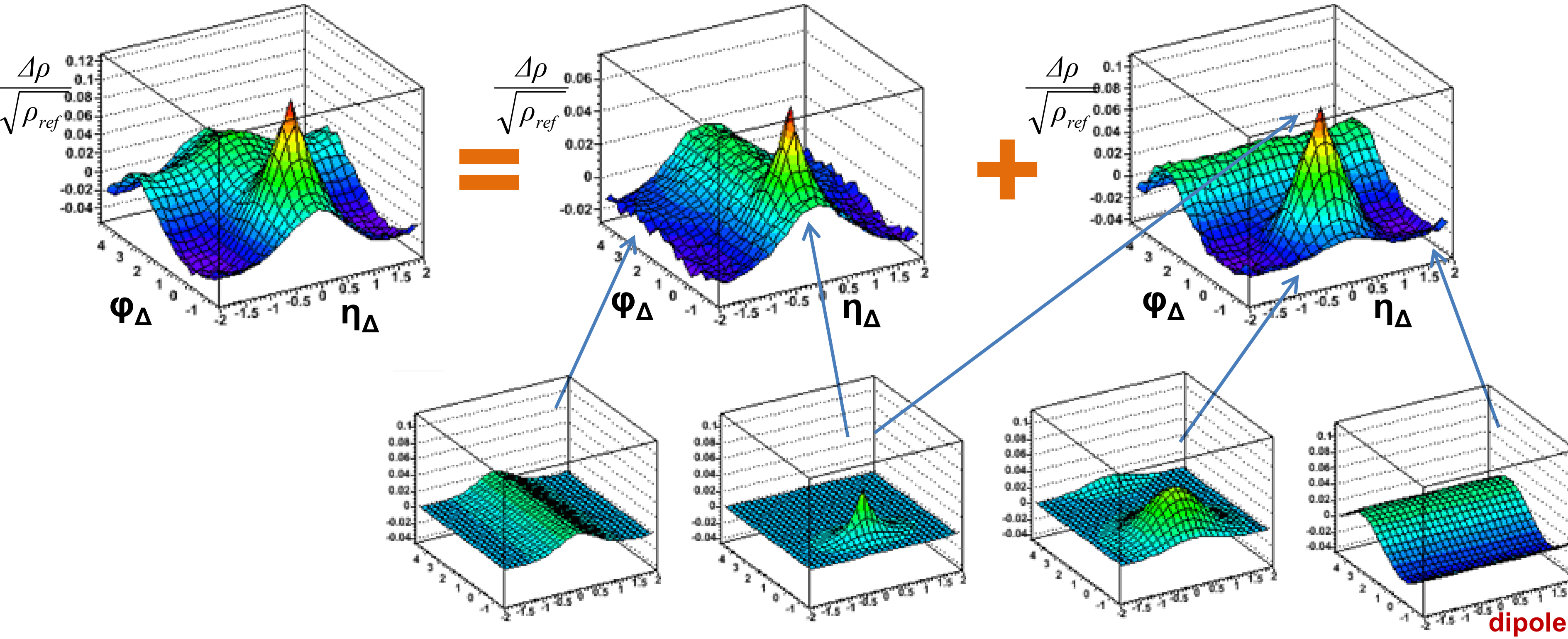
Trivial correlations (HBT)



Fit Function (in 5 Easy Pieces)

Proton-Proton fit function

STAR Preliminary



longitudinal fragmentation
1D gaussian

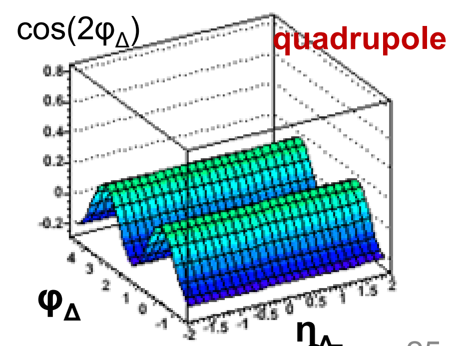
HBT, res., e+e-
2D exponential

Minijet Peak
2D gaussian

Away-side
-cos(φ)

Au-Au fit function

Use proton-proton fit function + $\cos(2\varphi_\Delta)$ quadrupole term (“flow”).
This gives the ***simplest possible*** way to describe Au+Au data.

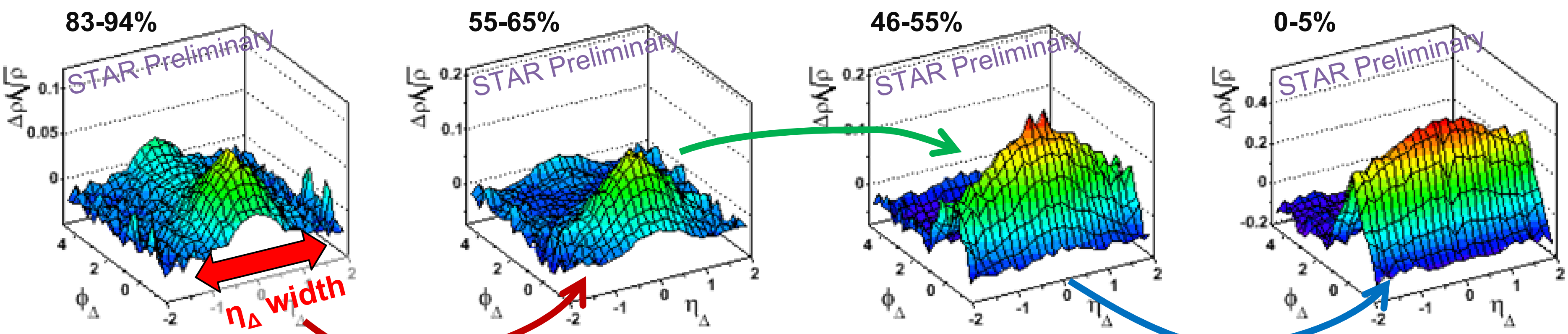


Note: from this point on we’ll include entire momentum range instead of using soft/hard cuts

Transition

Does the transition from narrow to broad η_Δ occur quickly or slowly?

data - fit (except same-side peak)



Shape changes little from peripheral to the transition

Large change within ~10% centrality

Smaller change from transition to most central

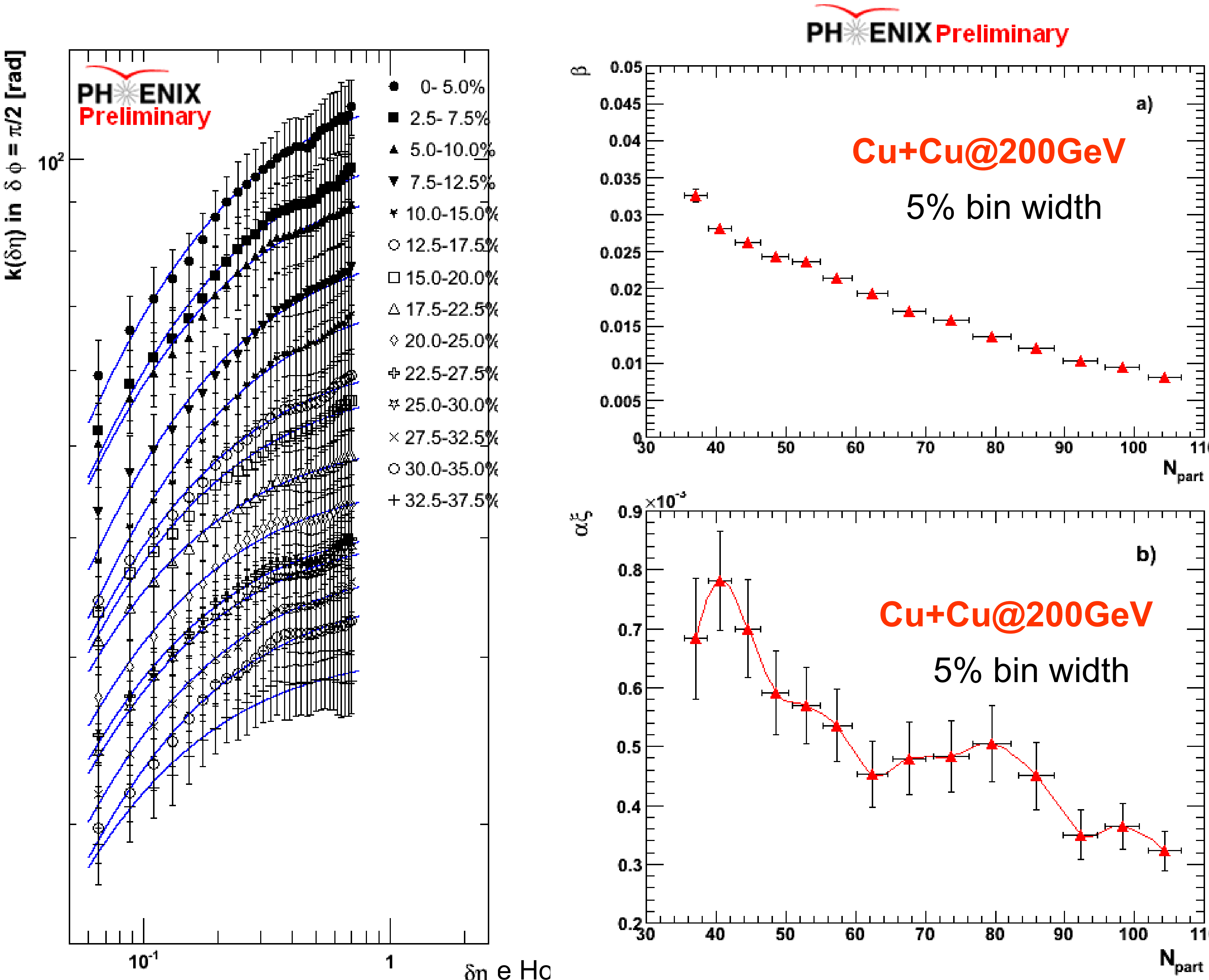
Low-p_T manifestation of the "ridge"

The transition occurs quickly

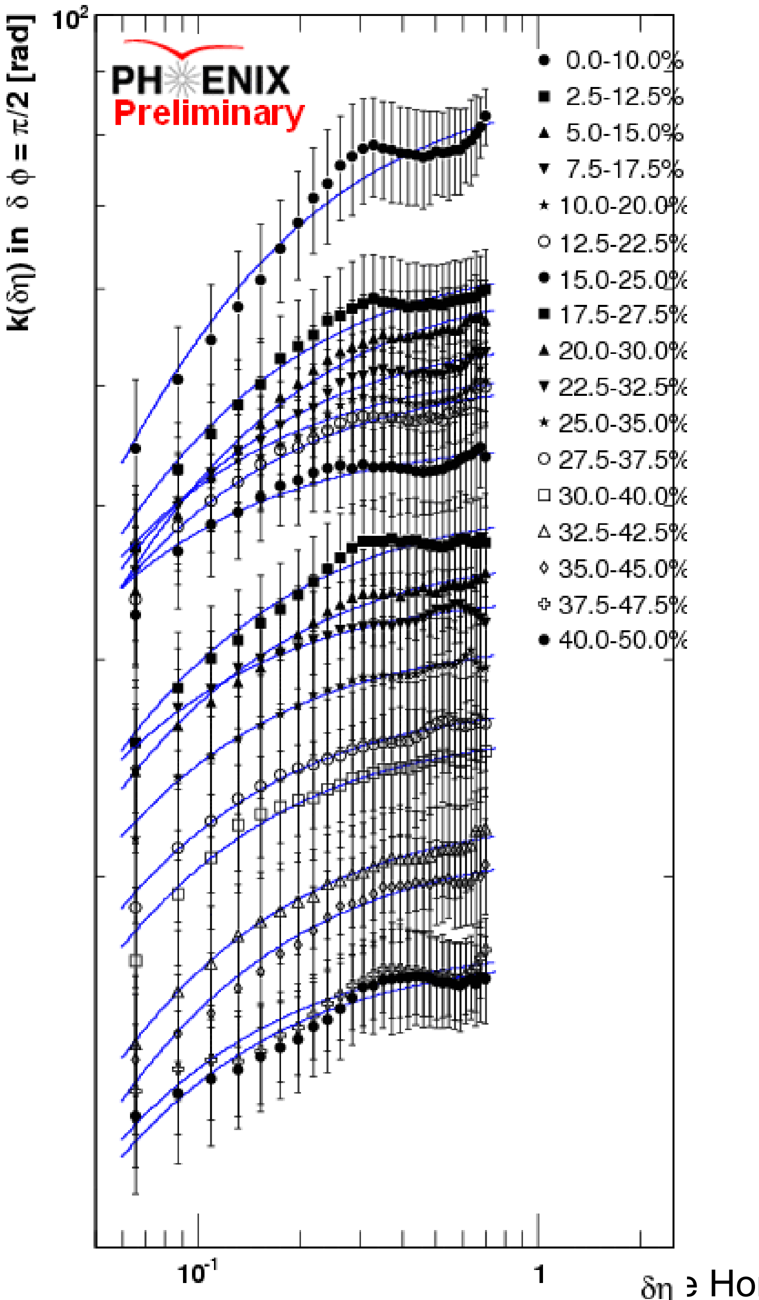
Slide from M. Daugherity, STAR Collaboration presented at QM08

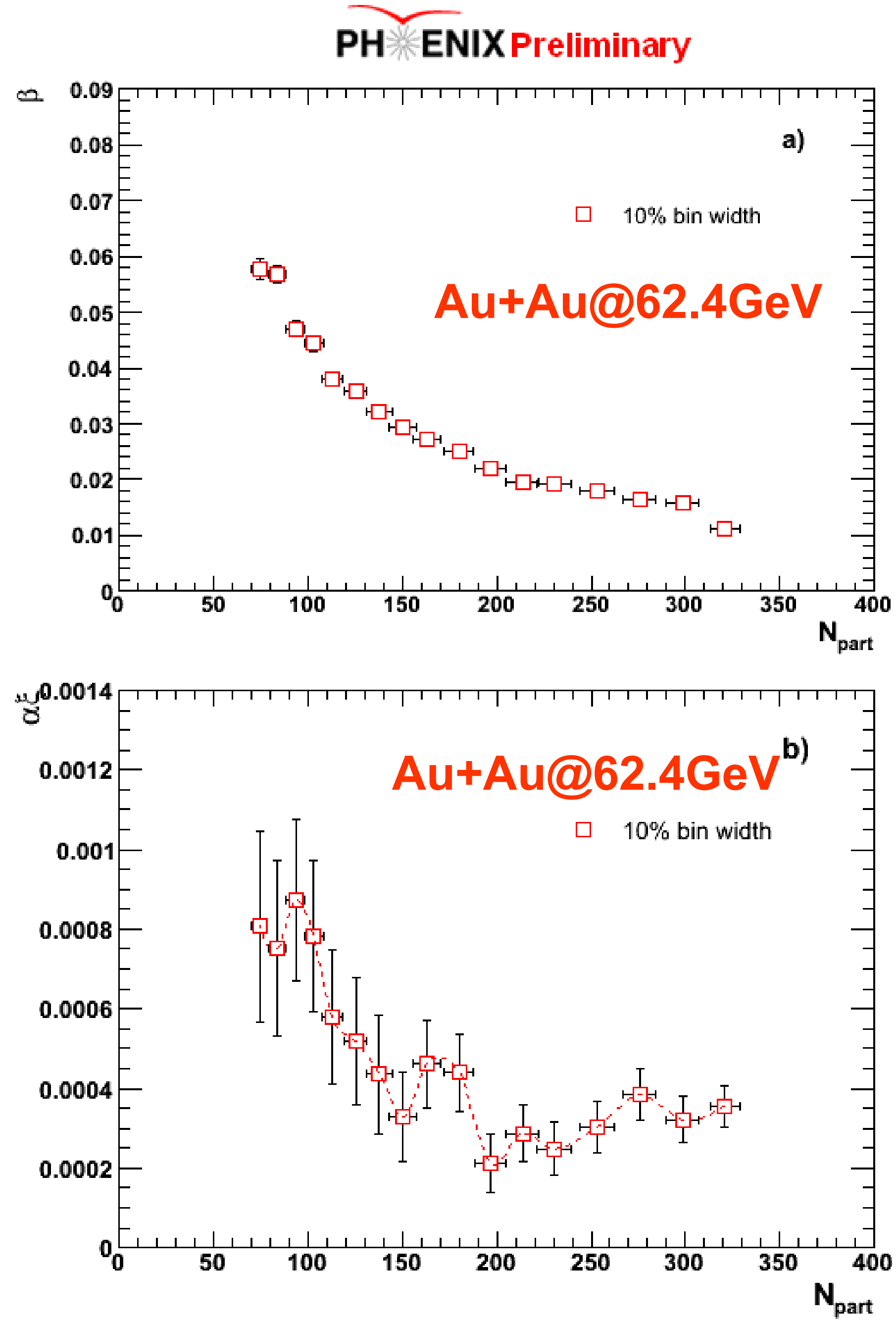
26

Analysis in smaller system: Cu+Cu@200GeV



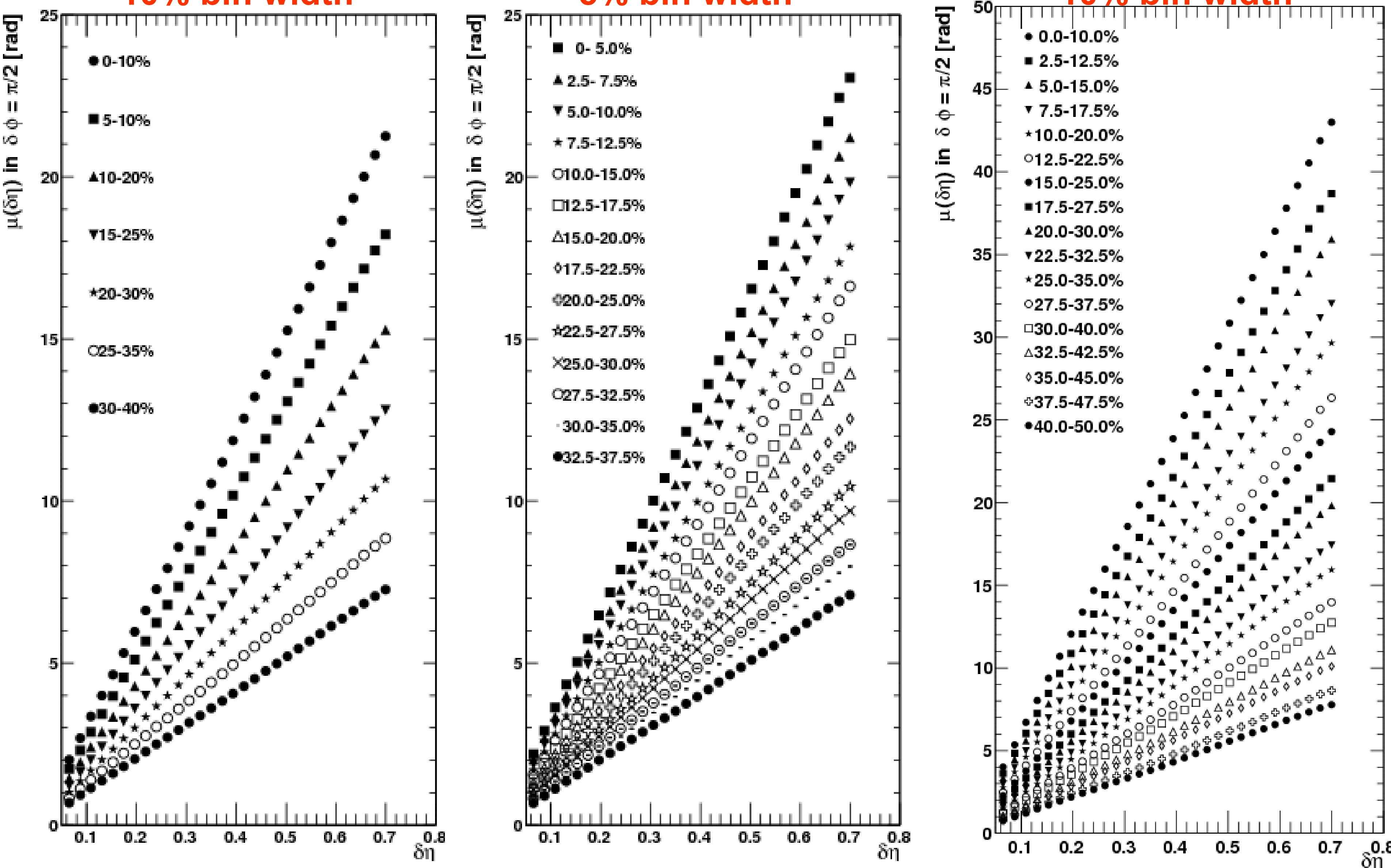
Analysis in lower energy: Au+Au@62.4GeV





Corrected mean multiplicity $<\mu_c>$

Cu+Cu@200GeV 10% bin width | Cu+Cu@200GeV 5% bin width | Au+Au@62.4GeV 10% bin width



Simple base line: Participant Superposition Model

- **In a Participant Superposition Model, multiplicity fluctuations are given by:**

$$\omega_N = \omega_n + \langle N \rangle \omega_{Np}$$

where $\omega = \sigma^2/\mu$. ω_N = *total fluctuation*, ω_n = *fluctuation in each source (e.g. hadron-hadron collision),* ω_{Np} = *fluctuation in number of sources (participants),* <N>=*mean multiplicity per wounded nucleon.*

- **After correcting for fluctuations due to impact parameter,** $\omega_N = \omega_n$ **is independent of centrality.**

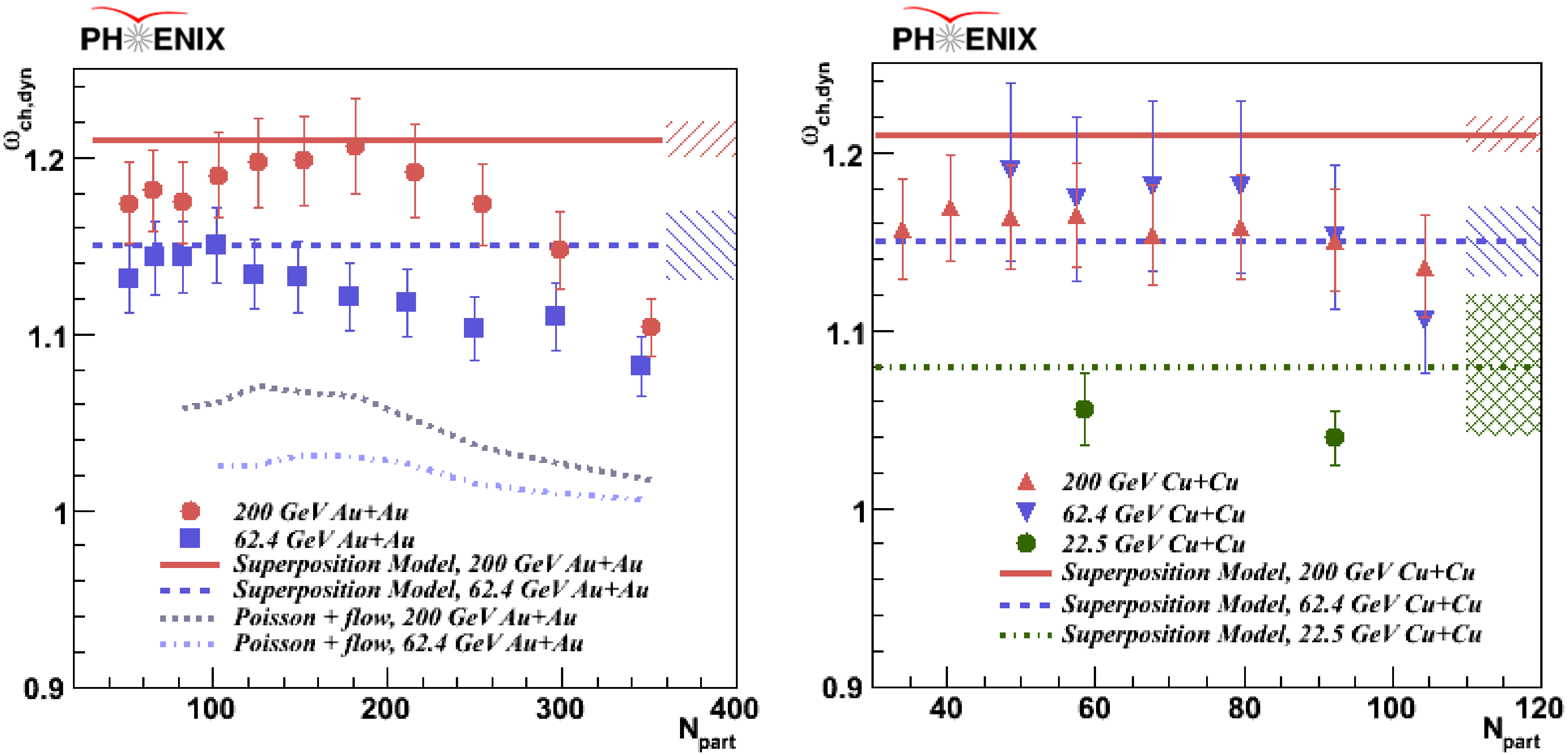
- **Multiplicity fluctuations are also dependent on acceptance:**

$$\omega_n = 1 + f(\omega_n - 1)$$

where $f = N_{accepted}/N_{total}$. ω_n = **fluctuations from each source in** 4π

Multiplicity Fluctuation Results

Bottom line: Near the critical point, the multiplicity fluctuations should exceed the superposition model expectation → No significant evidence for critical behavior is observed.



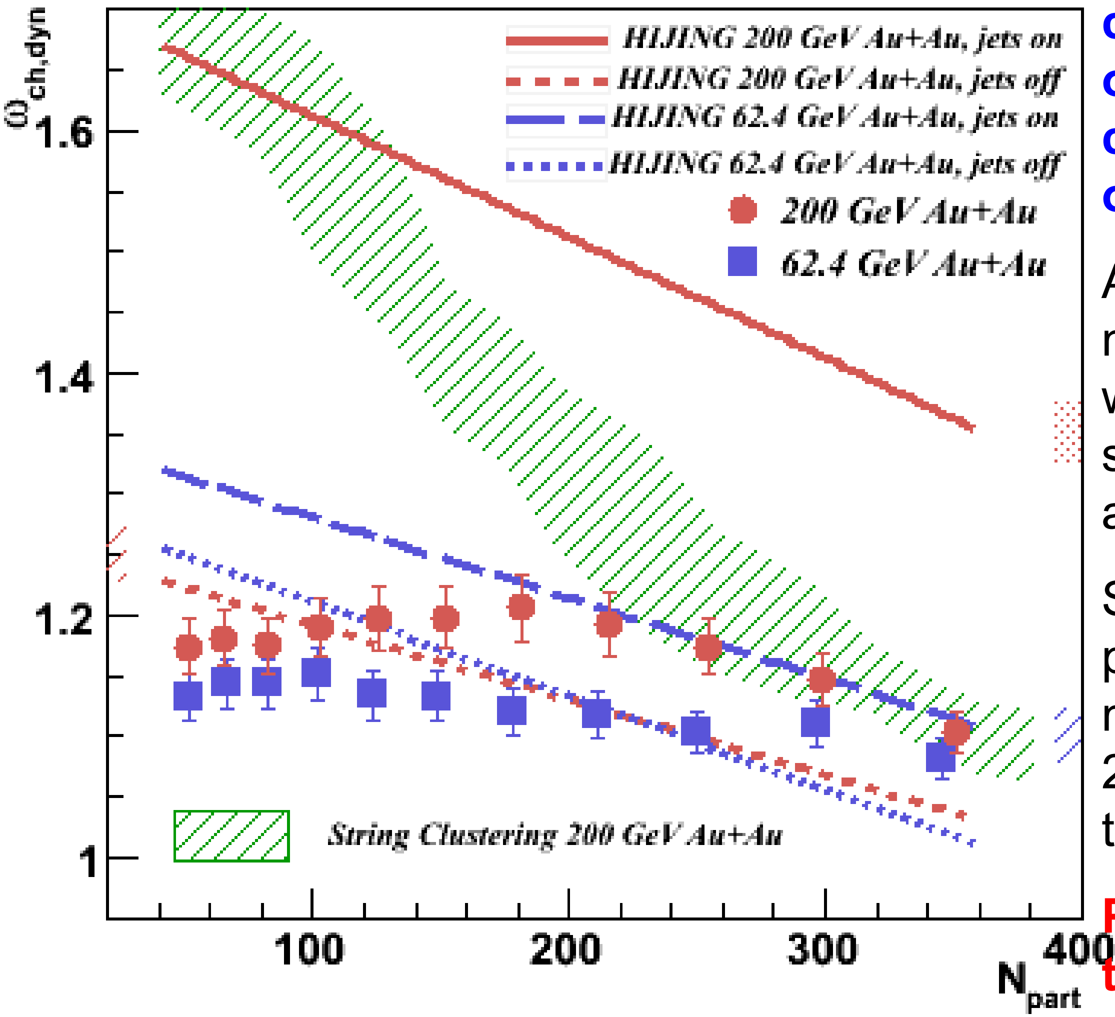
Centrality dependence is dominated by elliptic flow

Superposition model at 200 GeV taken from PHENIX measurements of 200 GeV p+p. The results agree with UA5 measurements in PHENIX's pseudorapidity window.

Superposition model at 22 GeV taken from NA22 measurements in PHENIX's pseudorapidity window.

Superposition model at 62 GeV taken from interpolation of UA5 results in PHENIX's pseudorapidity window.

String Percolation Model



String percolation: strings form clusters of geometrically overlapping strings and each cluster emits particles depending on the number strings.

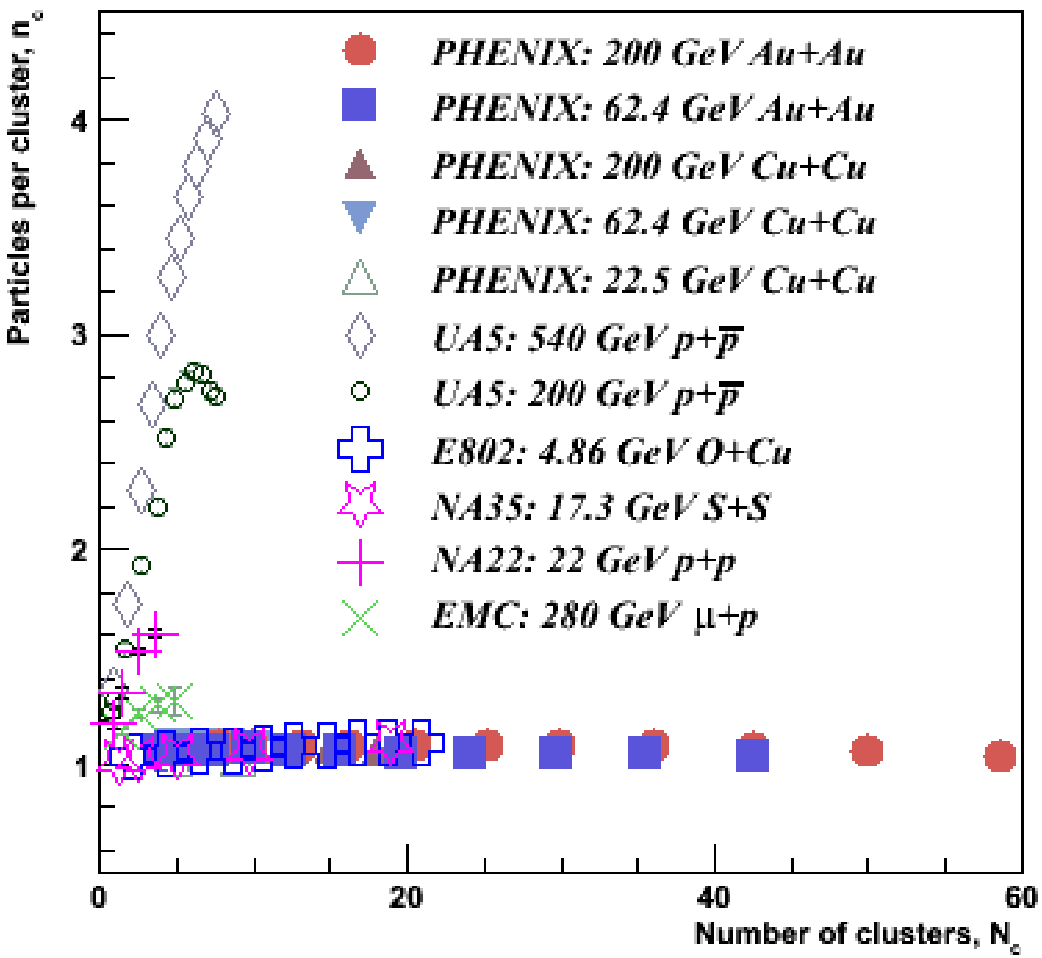
As the centrality increases, the number of clusters decreases along with the variance of the number of strings per cluster, which results in a decrease of scaled variance.

Shown in green are the direct predictions of the string percolation model (PRC72,024907(2005)) for 200 GeV Au+Au, scaled down to the PHENIX acceptance.

Percolation still does not explain the plateau in the most peripheral Au+Au collisions.

CLAN Model

A. Giovannini et al., Z. Phys. C30 (1986) 391.



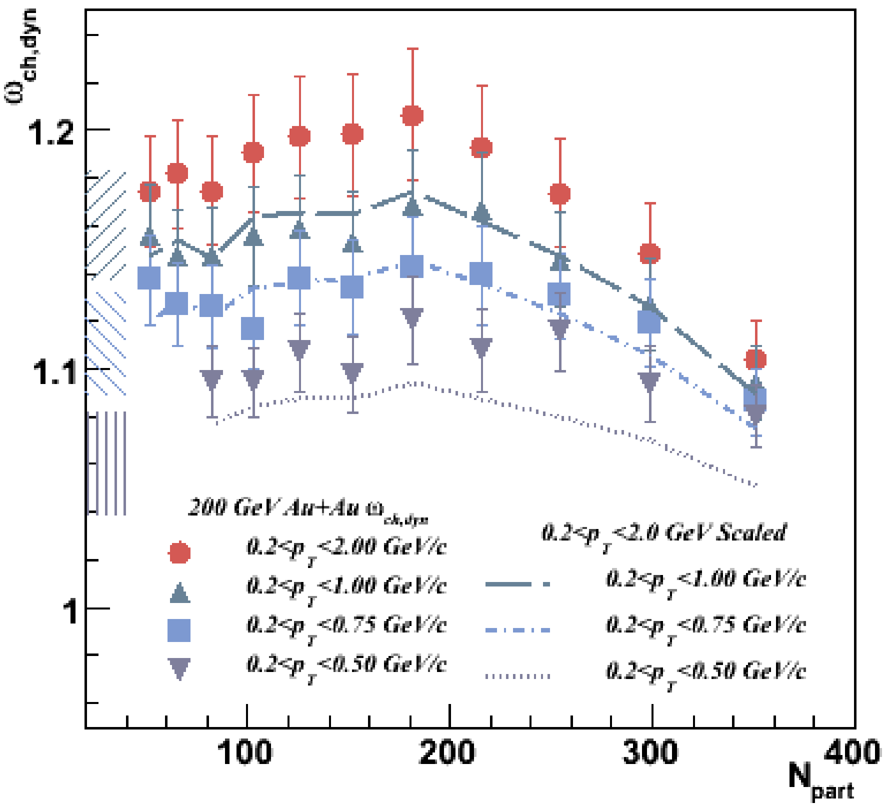
The CLAN model was developed to attempt to explain the reason that p+p multiplicities are described by NBD rather than Poisson distributions.

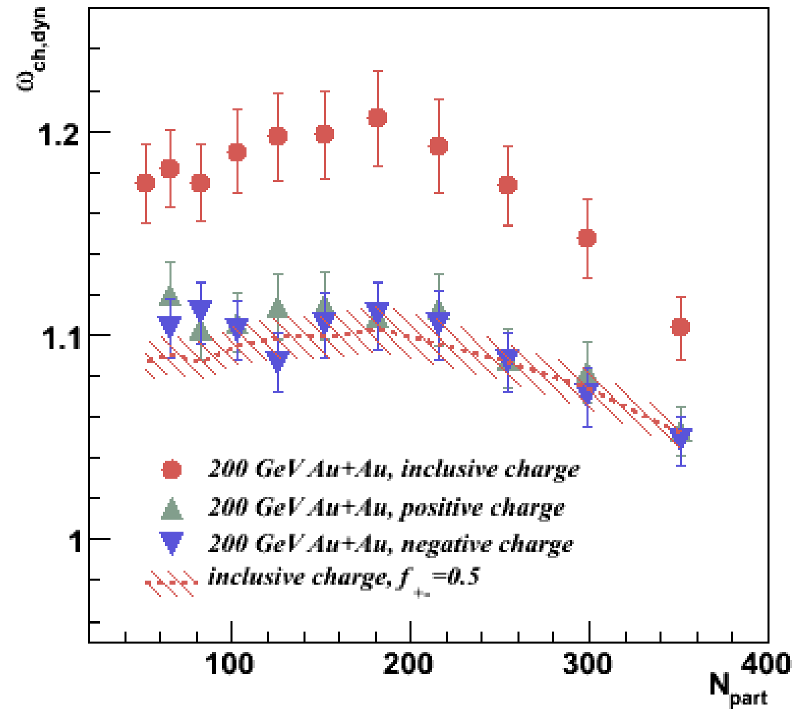
Hadron production is modeled as independent emission of a number of hadron clusters, N_c, each with a mean number of hadrons, n_c. These parameters can be related to the NBD parameters:

$N_c = k_{NBD} \log(1 + \mu_{ch}/k_{NBD})$ and
$\langle n_c \rangle = (\mu_{ch}/k_{NBD})/\log(1 + \mu_{ch}/k_{NBD})$.

A+A collsions exhibit weak clustering characteristics, independent of collision energy.

Charge and p_T-Dependence





$\omega_{+-} = 1 + f(\omega_{inclusive} - 1)$

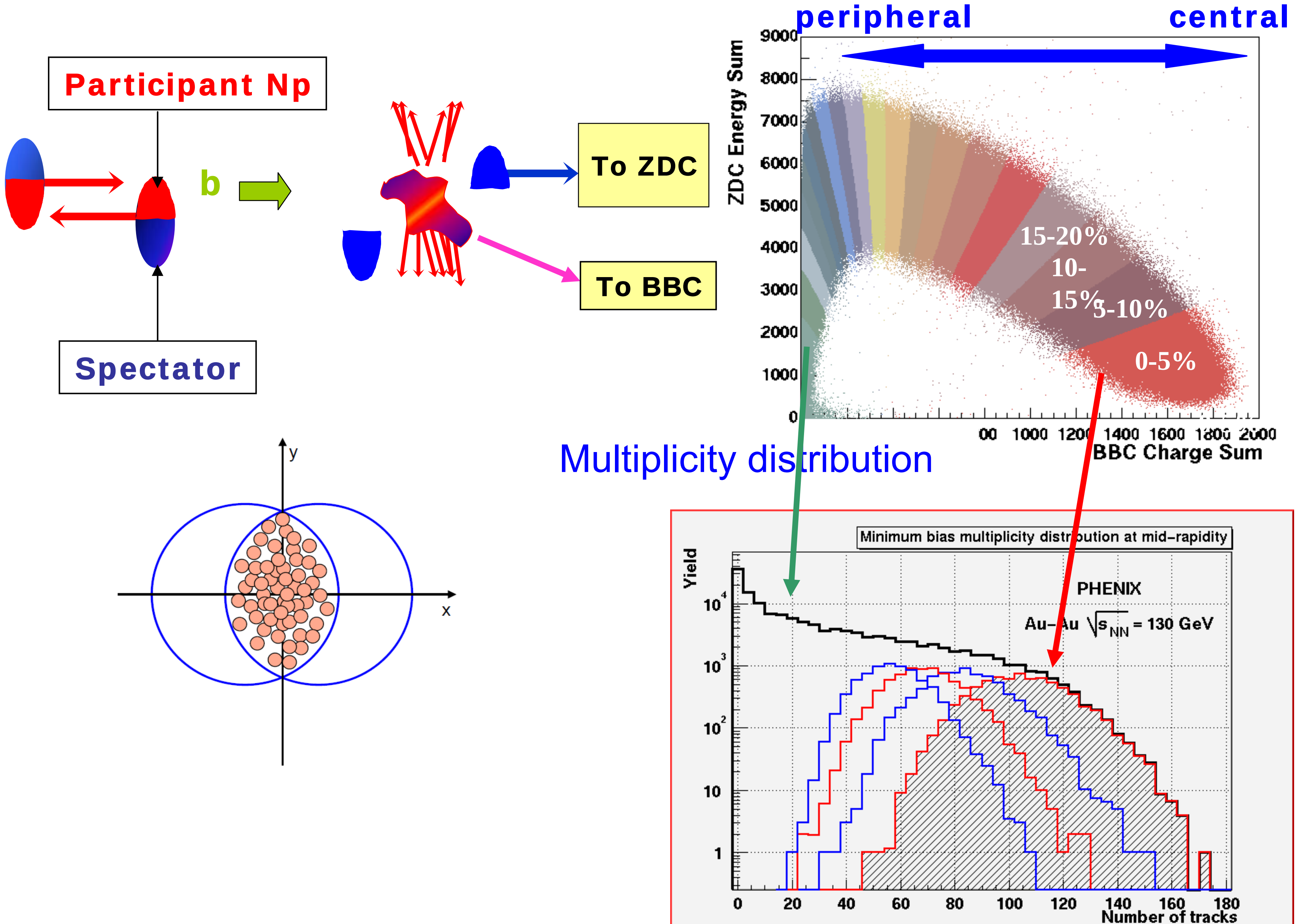
where f=0.5.

If pT-dependence is random, the scaled variance should scale with <N> in the same manner as acceptance:

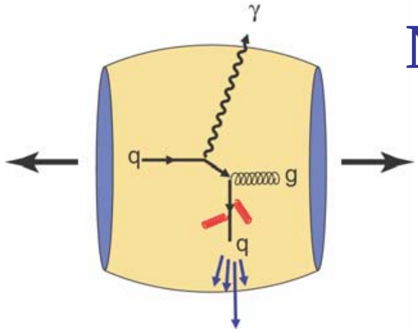
$\omega_{pT} = 1 + f(\omega_{pT,max} - 1)$

Within errors, no charge dependence of the fluctuations is seen for 200 GeV Au+Au.

Number of participants, Np and Centrality



Is medium dense enough?

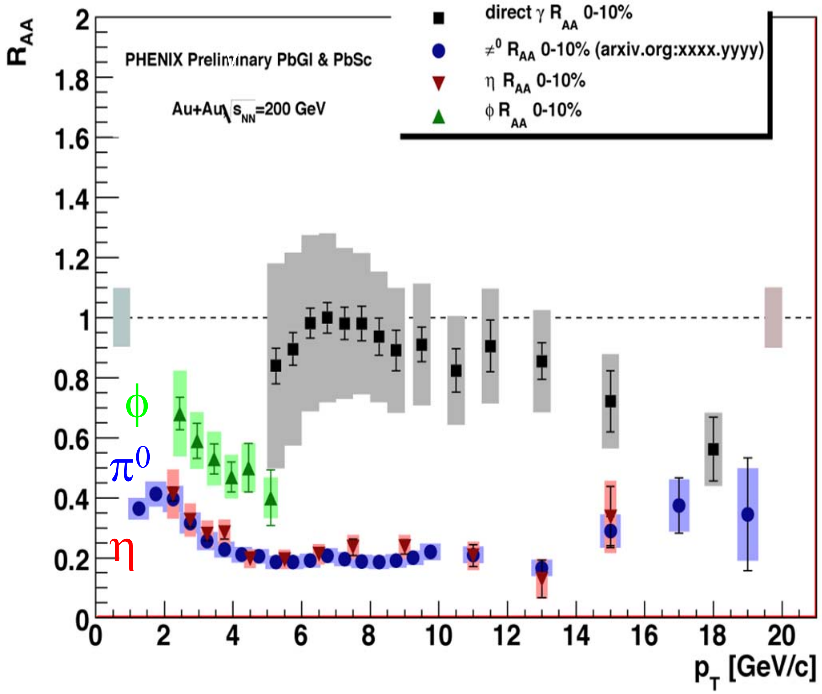


Nuclear Modification Factor

$$R_{AA} \equiv \frac{d^2N^{AA}/dydp_T}{d^2N^{pp}/dydp_T \cdot \langle N_{coll}^{AA} \rangle}$$

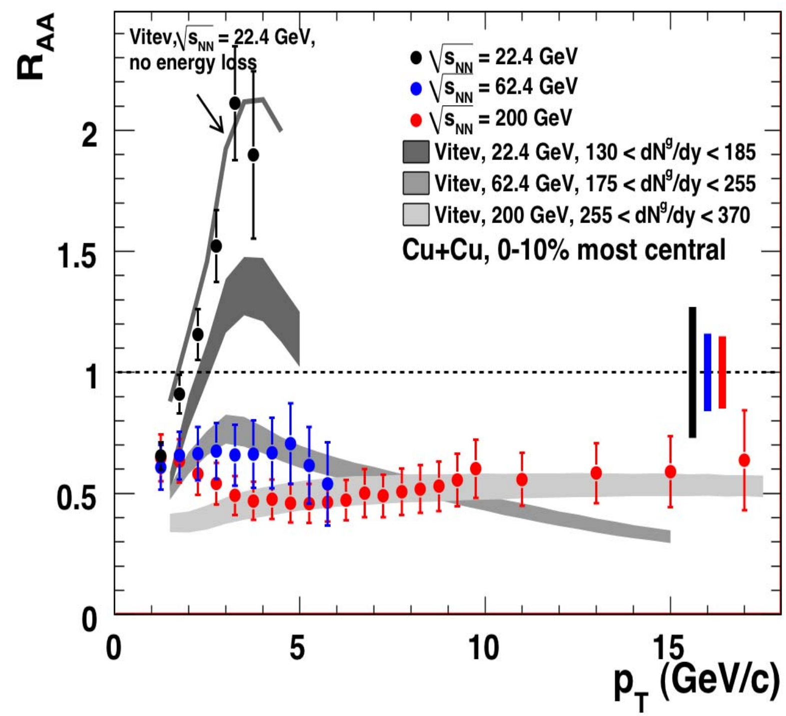
Particle Species

π^0 Au+Au 200 GeV (Run 4)



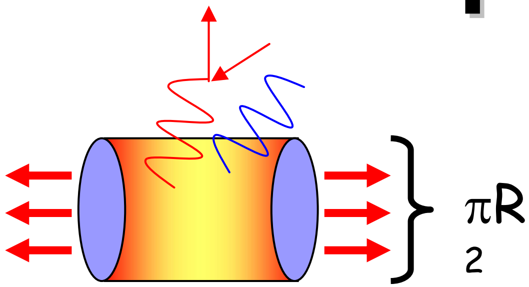
Energy

π^0 Cu+Cu 22,62,200 GeV (Run 5)



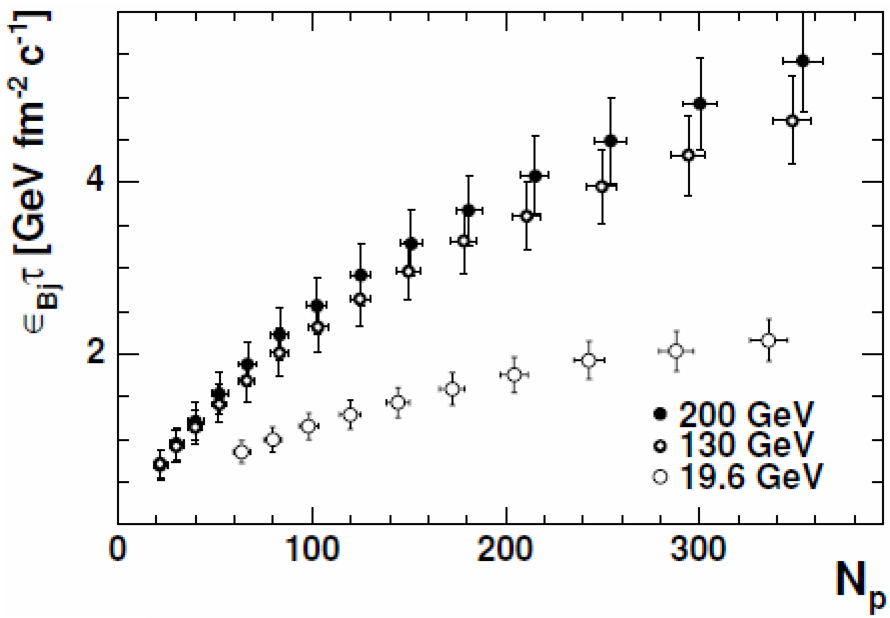
arXiv:0801.4555

Is initial temperature high enough?



$$dz = \tau_0 dy$$

$$\varepsilon_{Bj} = \frac{1}{\pi R^2} \frac{1}{\tau_0} \frac{dE_T}{dy}$$

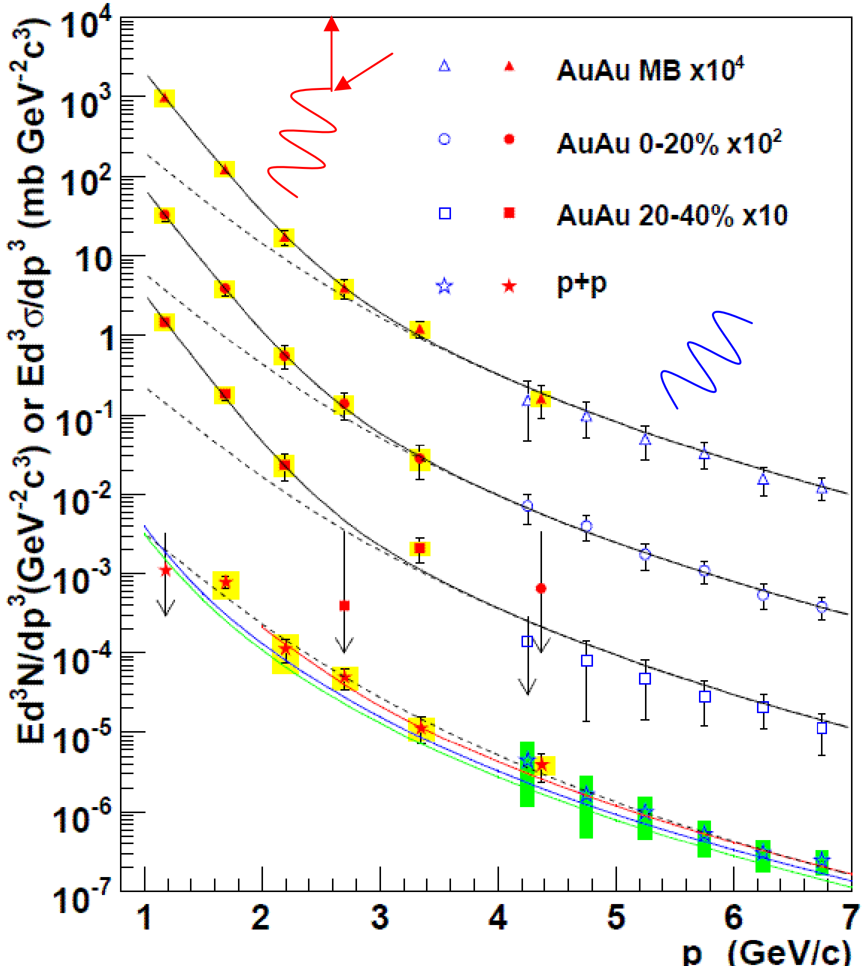


PHYSICAL REVIEW C 71, 034908 (2005)

In central Au+Au collision

T=221±23(stat)±18(sys)

Lattice result T_c~170MeV

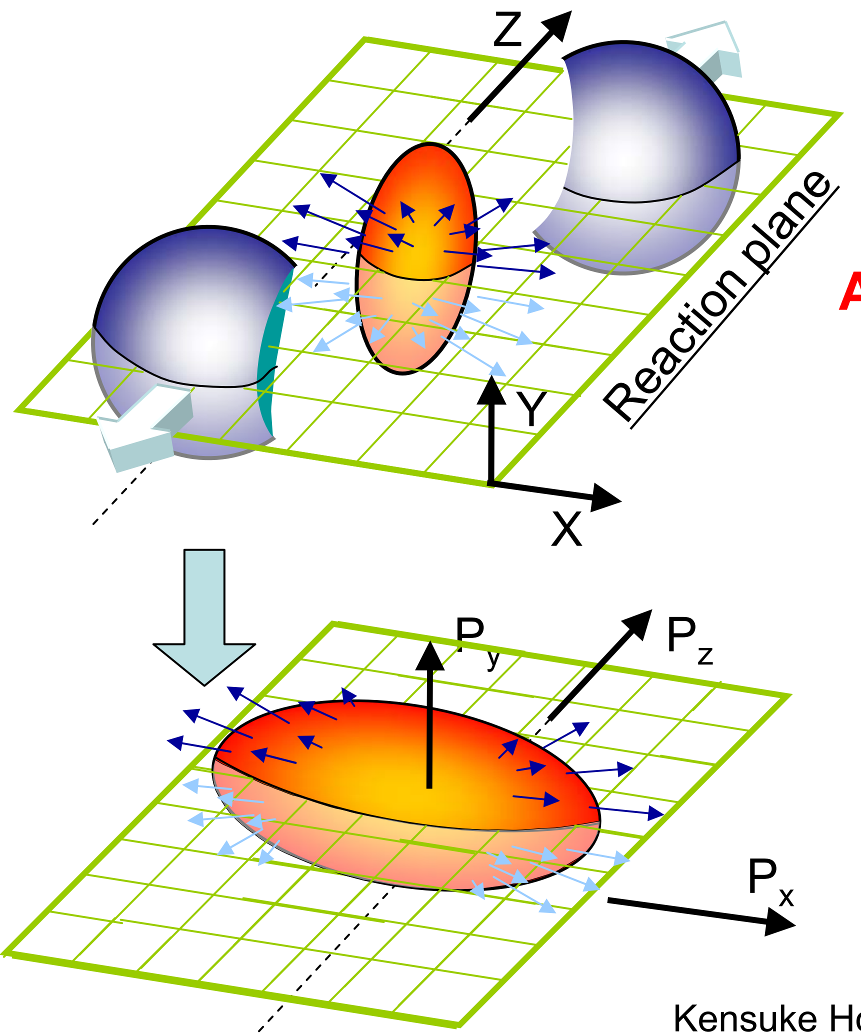


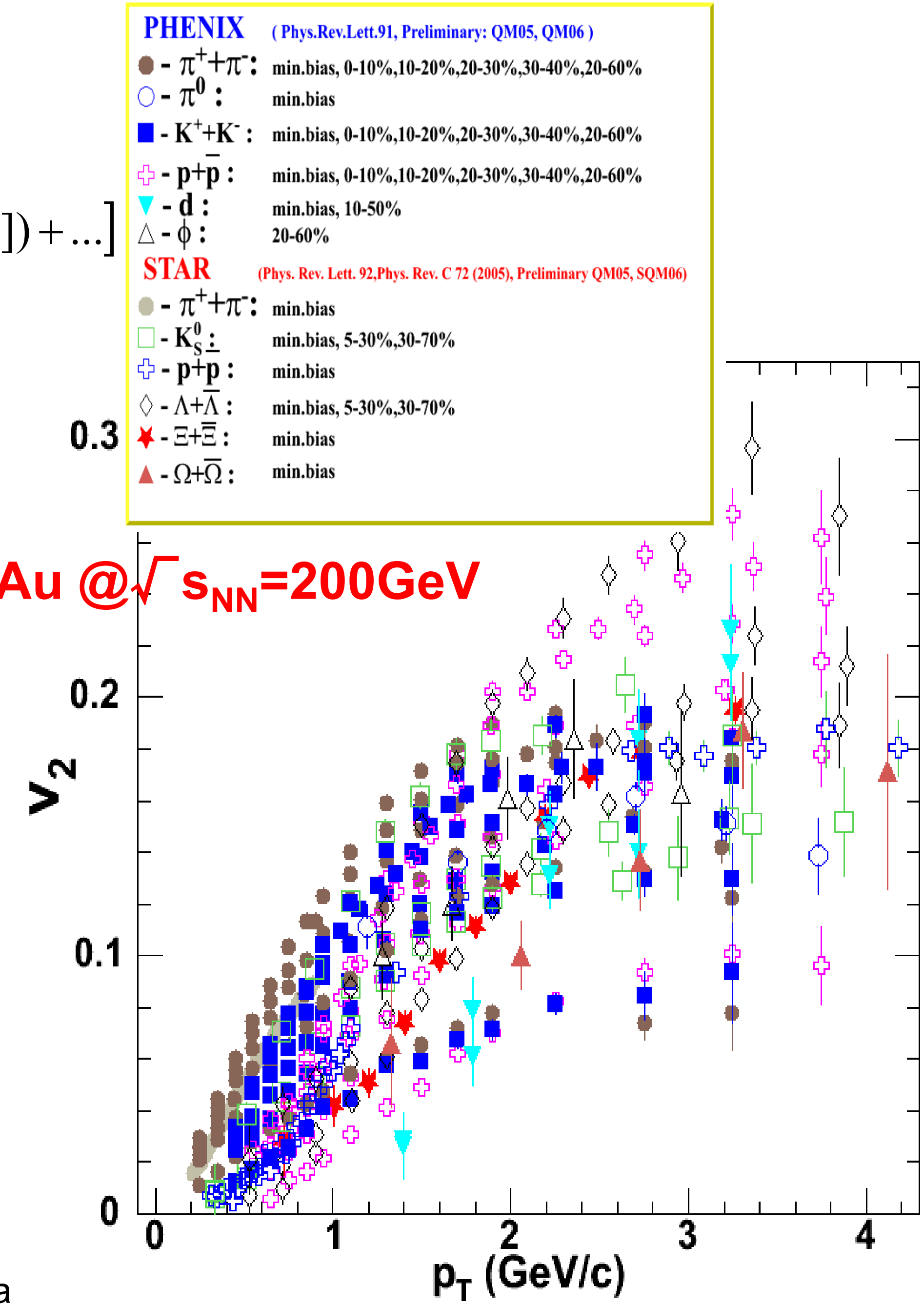
arXiv:0804.4168v1 [nucl-ex] 25 Apr 2008

Is bulk collective motion seen?

$$E\frac{d^3N}{d^3p} =$$

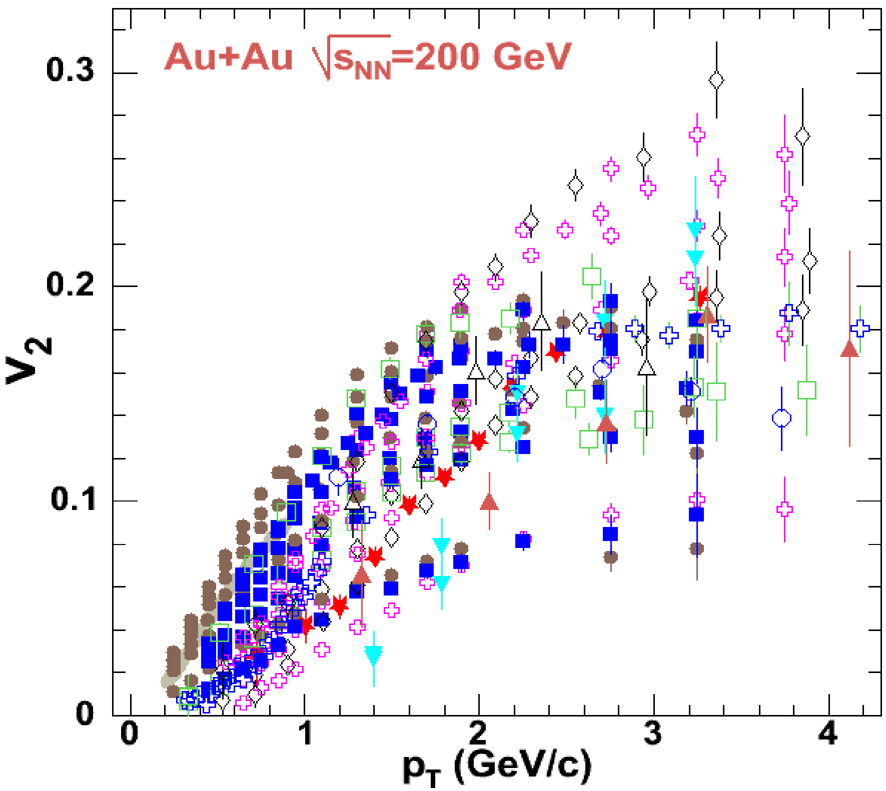
$$\frac{1}{\pi}d^2\frac{N}{dp_T^2dy}\left[1+2v_1\cos(\varphi-\Psi_R)+2v_2\cos(2[\varphi-\Psi_R])+\ldots\right]$$



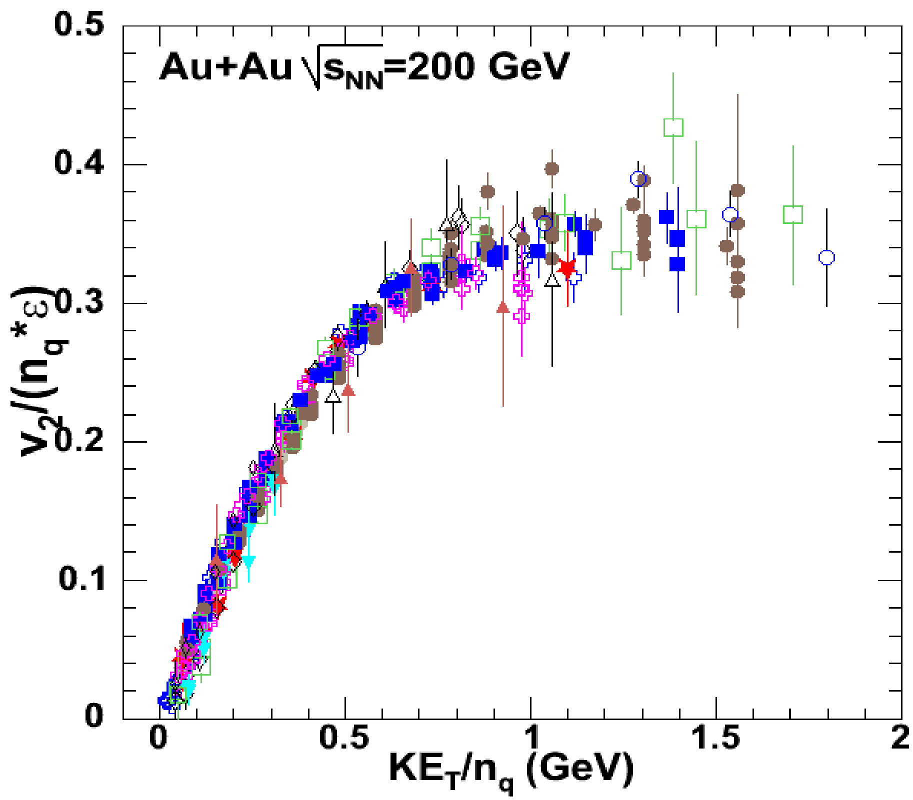


Kensuke Homma

Any partonic degree of freedom?



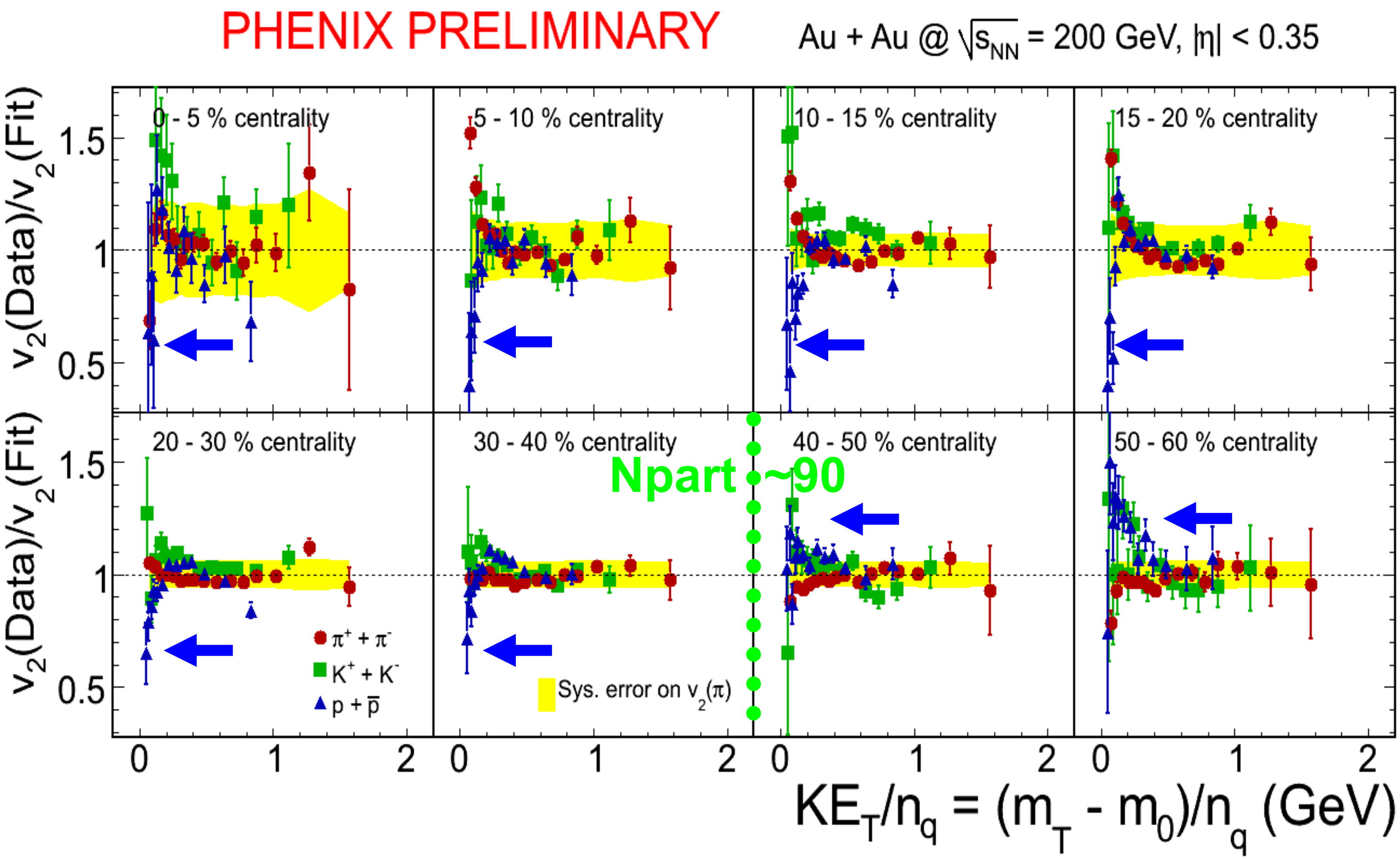
Constituent quark number, n_q scaling



$$KE_T=m(\gamma_T-1)=m_T-m$$

Are there symptoms in other observables at around the same Npart?

Deviation from scaling at low KE_T region ?

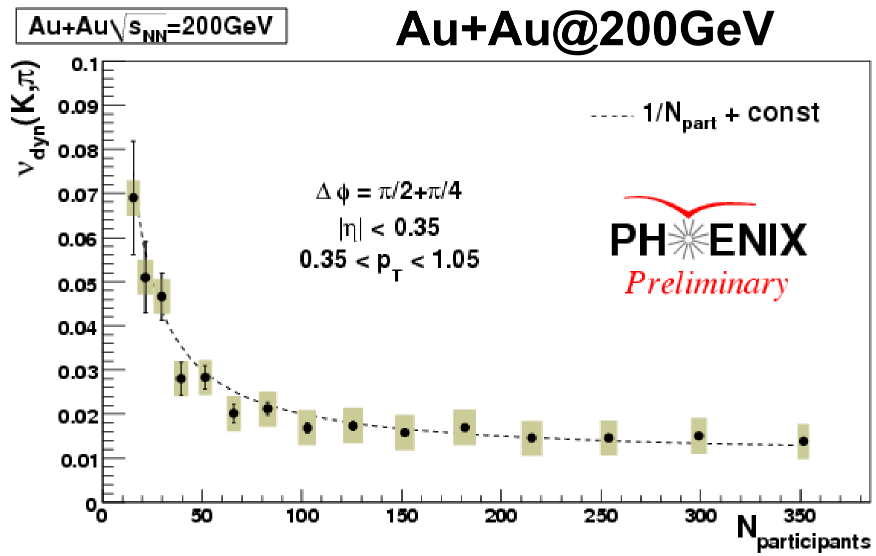


In lower KE_T, there seems to be different behaviors between baryon and mesons. The transition is at Npart~90.

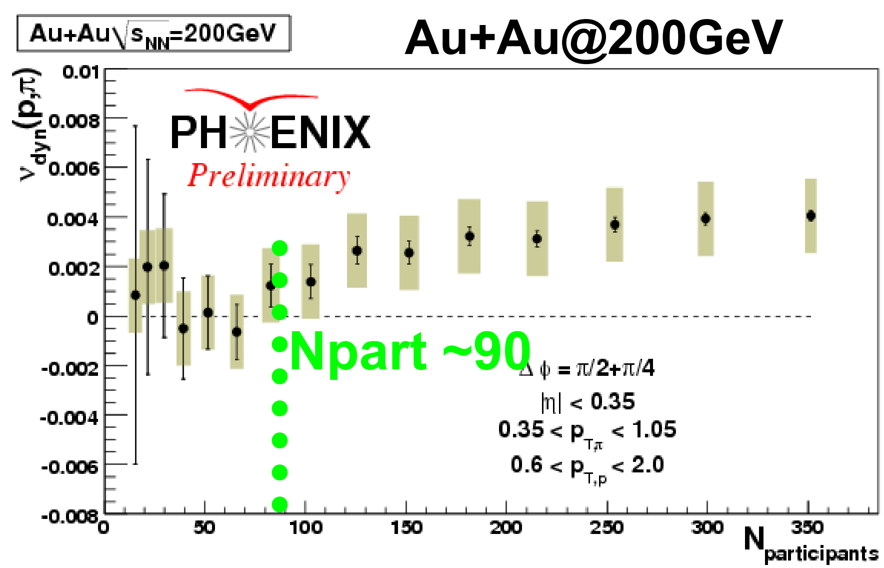
Low mass sigma field may repulse pion and attract proton?

Meson-meson and baryon-meson fluctuations

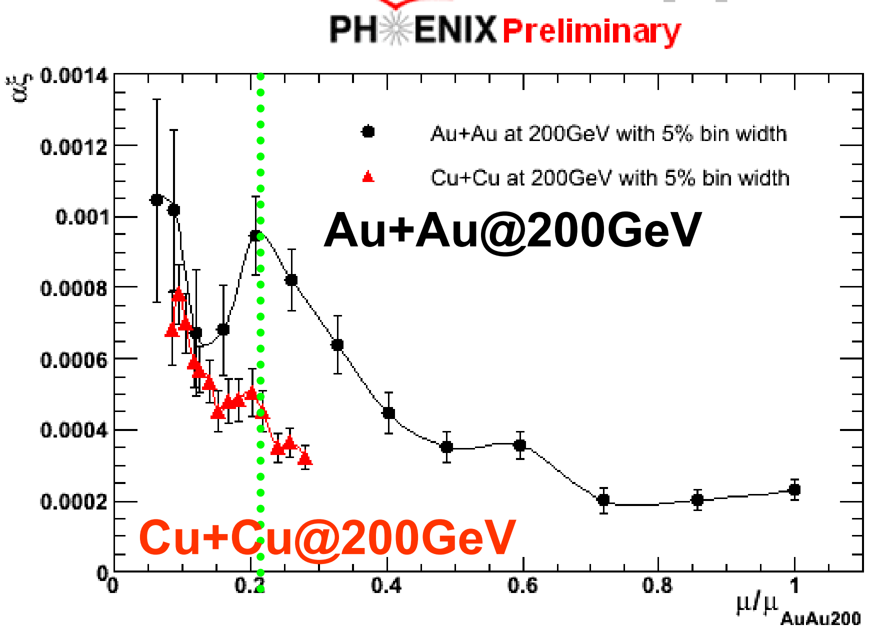
$$\nu_{dyn}(K,\pi) = \frac{\langle \pi(\pi-1)\rangle}{\langle \pi\rangle^2} + \frac{\langle K(K-1)\rangle}{\langle K\rangle^2} - 2\frac{\langle \pi\, K\rangle}{\langle \pi\rangle\langle K\rangle}$$



$$\nu_{dyn}(p,\pi) = \frac{\langle \pi(\pi-1)\rangle}{\langle \pi\rangle^2} + \frac{\langle p(p-1)\rangle}{\langle p\rangle^2} - 2\frac{\langle \pi\, p\rangle}{\langle \pi\rangle\langle p\rangle}$$

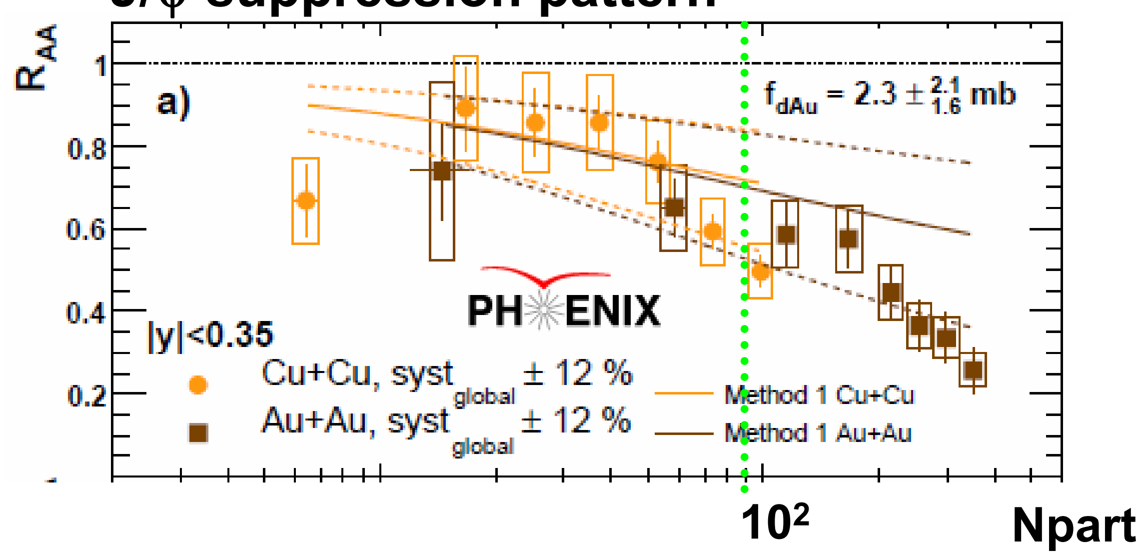


How about <c$\bar{c}$> suppression?



Npart~90 in AuAu@200GeV
$\varepsilon_{BJ}\tau$~2.4GeV/fm2/c

J/ψ suppression pattern



arXiv:0801.0220v1 [nucl-ex]