A bottom-up strategy for reconstructing the underlying MSSM parameters at the LHC

J.-L. Kneur
Lab. Physique Thorique et Astroparticules, UMR5207 CNRS, Univ. Montpellier 2, France
DOI: http://dx.doi.org/10.3204/DESY-PROC-2009-01/84

Abstract
We illustrate a particular "bottom-up" reconstruction of MSSM parameters at the LHC for both general and constrained MSSM, starting from a limited set of particle mass measurements, using gluino/squark cascade decays and the lightest Higgs boson mass. Our method gives complementary information to more standard "top-down" reconstruction approaches and is not restricted to the LHC data properties.

1 Introduction

If new physics is seen at LHC a very first non trivial issue will be to distinguish supersymmetry from other beyond standard model (BSM) scenarios, like extra dimensions, little Higgs models etc. If evidence for low energy supersymmetry is found, the next crucial step would be to measure Minimal Supersymmetric Standard Model (MSSM) [1] basic parameters accurately enough to extract precisely the underlying SUSY-breaking mechanism. This may not be easy if only a limited part of the predicted MSSM sparticles will be discovered and some of their properties measured with the prospected LHC accuracies. Most reasonable scenarios assume that the lightest Higgs scalar h could be discovered, and some of the squarks and the gluino are copiously produced (if not too heavy) at the LHC due to their strong interactions. In addition some of the neutralinos, including the lightest supersymmetric sparticle (LSP), could be identified and have their masses extracted indirectly from detailed study of squark and gluino cascade decays (see e.g. [2]). Various analysis have been conducted [3, 4] to reconstruct the basic MSSM parameter space from the above assumed experimental measurements. A largely illustrated strategy, in a so-called "top-down" approach, is to start from a given supersymmetry-breaking model at very high grand unification (GUT) scale, predicting for given input parameter values of the superpartner spectrum at experimentally accessible energy scales, and next fitting this spectrum (with other observables like cross-sections etc) to the data to extract constraints on the model parameters (see e.g. [5] for recent elaborated fitting techniques). There is however a lively debate now on what will be the most efficient approaches, either the above "top-down", or some alternative bottom-up reconstruction methods; or more "blind" analysis, etc. Among other things there has been some concern raised about the "LHC inverse problem" i.e. the possible occurence of discrete ambiguities (potentially many) in reconstructing basic MSSM parameters [6].

Our aim here is to illustrate a recent alternative bottom-up reconstruction strategy [7], based on a rather "minimal" set of identified sparticles, within different scenarios (e.g. with GUT scale universality assumptions or not). Our approach is based on inverse mapping relations between measured masses and basic parameters. This has been investigated in the past [8, 9] but mainly at tree-level approximation and in the context of the ILC. One of the novelty here is to incorporate radiative corrections into our framework at realistic level, and very similarly to the way in which radiative corrections are included in more conventional top-down calculations. This allows to keep most advantages of the bottom-up approach. Our analysis is far from being fully realistic concerning the LHC data simulations, not using sophisticated Monte Carlo tools that are ultimately necessary. But the accent is on considering as much as possible realistic and minimal LHC sparticle identifications, using a limited set of sparticle mass measurements.

2 Experimental assumptions and strategy

At the LHC, one expects to determine quite accurately some sparticle masses (see Table 1 for the SPS1a benchmark study) from "kinematical endpoints" analysis of (2-body) cascade decays:

$$\tilde{g} \to \tilde{q}_L q \to \chi_2^0 q_f q \to \tilde{l}_R l q_f q \to \chi_1^0 l_f l q_f q \tag{1}$$

We assume in our analysis that the lightest Higgs mass m_h will be also measured with good accuracy, mainly through its $\gamma\gamma$ decay mode.

scenarios (+th assumptions)	measured mass	expected LHC accuracy (GeV)	decay or process
(minimal):	$m_{\tilde{g}}$,	7.2	$\tilde{g}$ cascade decay
S_1(MSSM),	$m_{\tilde{N}_1}$,	3.7	" "
S_2(universality)	$m_{\tilde{N}_2}$.	3.6	" "
S_4,	$m_{\tilde{q}_L}$,	3.7	" "
S'_4 (universality)	$m_{\tilde{l}_R}$	6.0	" "
$S_3 = S_1$ +:	$m_{\tilde{N}_4}$	5.1	$\tilde{q}_L \to \tilde{\chi}_4^0 +$ .. cascade
S_5,	$m_{\tilde{b}_1}$,	7.5	$\tilde{g}$ cascade decay
S'_5 (universality)	$m_{\tilde{b}_2}$	7.9	" "
$S_6 = S_2 + S'_4 + S'_5$ +:	m_h	0.25 (exp)–2 (th)	$h \to \gamma\gamma$ (mainly)

Table 1: Different scenarios S_i on the amount of sparticle mass measurements at the LHC from gluino cascade and other decays with different theoretical assumptions (see ref. [7] for more details). Mass accuracies correspond to SPS1a benchmark studies, combined from refs. [2, 3].

3 Analytic inverse mapping from masses to basic parameters

In the unconstrained MSSM there are three naturally separated sectors (at tree level):
- the gauginos/Higgsinos sector involving the basic MSSM parameters M_1, M_2, μ, and $\tan\beta$;
- the squarks/slepton sector involving μ, $\tan\beta$, and soft scalar terms $\tilde{m}_{q_L}$, $\tilde{m}_{q_R}$, $\tilde{m}_{e_L}$,...;
- the Higgs sector involving μ, $\tan\beta$, M_{H_u}, M_{H_d}, M_A.

In each sector one can derive simple analytic inversions (at tree-level), i.e. linear or quadratic equations [7] that express basic MSSM parameters as function of sparticle masses. Our precise strategy evidently depends on the available input masses (as it is also the case in a top-down approach). We proceed step by step in the three sectors rather than doing "all at once" fits.

3.1 Incorporating radiative corrections

Radiative corrections (RC) to sparticle masses evidently spoil the above simple inverse mapping picture, by introducing highly non-linear dependence on all parameters, so that "brute force" inversion is untractable. However to very good approximation, RC keep a tree-level form, e.g. in ino sector: $\mu \to \mu + \Delta\mu$, $M_1 \to M_1 + \Delta M_1$,.. (where $\Delta\mu$, ΔM_1, ΔM_2 depend on other sector: squarks, sleptons, etc), such that it preserves analytic inversion. Moreover the leading RC for $\tilde{g}$ involve $\tilde{q}$ of cascade (and reciprocally), thus depending on already known parameters. Once some of the MSSM parameters are determined, one can eventually assume universality (SUGRA) relations *within* loops as a reasonable approximation in many cases. In our analysis we solve the analytical (tree-level) inversion equations for various input/output choices, after incorporating leading RC relating pole to running masses in the above manner. We then vary mass input within errors (with uniform "flat prior" or Gaussian distributions) to determine constraints on output basic MSSM parameters within different asumptions on e.g. soft term universality at high scale.

3.2 Gaugino/Higgsino sector from Neutralino masses

Brute inversion of the neutralino mass matrix would be cumbersome and need all four neutralino mass input. More interestingly, one can extract two relations [7,8] involving only the two relevant neutralino mass input, to be used differently depending on input/output choice:

$$P_{12}^2 + (\mu^2 + m_Z^2 - M_1 M_2 + (M_1 + M_2) S_{12} - S_{12}^2) P_{12} + \mu m_Z^2 M_{12} \sin 2\beta - \mu^2 M_1 M_2 = 0 \quad (2)$$

$$\begin{aligned}(M_1 + M_2 - S_{ij}) P_{12}^2 + (\mu^2 (M_1 + M_2) + m_Z^2 M_{12} - \mu \sin 2\beta)) P_{12} \\ + \mu (m_Z^2 M_{12} \sin 2\beta - \mu M_1 M_2) S_{12} = 0 \end{aligned} \quad (3)$$

with $M_{12} \equiv c_W^2 M_1 + s_W^2 M_2$, $S_{12} \equiv \tilde{m}_{N_1} + \tilde{m}_{N_2}$, $P_{12} \equiv \tilde{m}_{N_1} \tilde{m}_{N_2}$. In unconstrained MSSM this determines M_1, M_2 for given μ, $\tan\beta$ $m_{\tilde{N}_1}, m_{\tilde{N}_2}$ input [7], up to a possible twofold ambiguity, $M_1 < M_2$ or $M_1 > M_2$, due to the use of only mass input. If a third neutralino mass $m_{\tilde{N}_4}$ can be measured, it gives a simple analytic determination of μ independently of $\tan\beta$, again with discrete ambiguities on the $M_1, M_2, |\mu|$ relative ordering in unconstrained MSSM. Resulting bounds on $M_1, M_2, |\mu|$ for input accuracies of Table 1 are illustrated in Table 2. In addition one can check specific SUSY-breaking models by comparing these bounds with the M_1, M_2 determination from M_3, e.g. from mSUGRA GUT universality or different M_i relations in other models. Alternatively for any M_i relations assumed, one can determine μ and $\tan\beta$ from the very same Eqs. (2),(3): the corresponding constraints for universal $M_i(Q_{GUT})$ are given in Table 2.

4 Squark, slepton parameter (first two generations)

From the expression of sfermion masses in unconstrained MSSM, e.g for $\tilde{u}_1, \tilde{e}_2$:

$$\begin{aligned} m_{\tilde{u}_1}^2 &= m_{\tilde{u}_L}^2 + (\frac{1}{2} - \frac{2}{3} s_W^2) m_Z^2 \cos 2\beta \\ m_{\tilde{e}_2}^2 &= m_{\tilde{e}_R}^2 - s_W^2 m_Z^2 \cos 2\beta \end{aligned} \quad (4)$$

we can take linear combinations to eliminate the $\tan\beta$ dependence, obtaining in this way constraints on the relevant soft scalar terms independently of $\tan\beta$. Moreover the RG evolution in this sector only depends (at one-loop) on gaugino M_i and gauge couplings, so that to good approximation and without further assumptions than the available input from (1) we can determine $m_0^{q,l}$ at GUT scale (upon assuming now squark-slepton universality):

$$86\,\text{GeV} \lesssim m_0^{q,l} \lesssim 112\,\text{GeV} \tag{5}$$

5 Third generation squark and Higgs sectors with universality assumptions

We can determine the sbottom parameters m_{Q3_L}, m_{b_R} with quite good accuracy both from sbottom masses and/or from (5) if assuming scalar universality (see Table 2). For the Higgs parameters reconstruction, in unconstrained MSSM the prospects at LHC are not optimistic if assuming solely the input from Table 1. In contrast universality assumptions relate $m_0^{q,l}$ to scalar terms m_{H_d}, m_{H_u}, thus predicting m_A value:

$$\bar{m}_A^2(m_0) = m_{H_d}^2 + m_{H_u}^2 + 2\mu^2 = \frac{\bar{m}_h^2(m_Z^2 - \bar{m}_h^2)}{m_Z^2 \cos^2 2\beta - \bar{m}_h^2} + RC(m_t, X_t, \cdots) \tag{6}$$

where the second equality is a naive (tree-level) relation defining m_A from m_h: this is clearly unrealistic since very important RC enter this relation, sketchily denoted here as $RC(m_t, X_t, \cdots)$. Those RC involve essentially running-to-pole m_h, m_A mass corrections and as is well-known depend strongly on the top mass and stop parameters (with $X_t \equiv A_t - \mu/\tan\beta$), among other MSSM parameters. The naive Eq. (6) nevertheless defines our strategy: For m_h accuracy from Table 1 and m_A determined from squark/slepton with universality assumptions, Eq. (5), we can put some constraints on e.g. $RC(m_t, X_t)$ and/or $\tan\beta$. For the Higgs sector RC we use actually (elaborated) approximations of one- and two-loop expressions [10] which differ from the full one-loop + leading two-loop results [11] by 1-2 GeV, i.e. of the order of theoretical uncertainties.

Finally, once the parameters are determined at low scale, we evolve them to GUT scale with bottom-up renormalization group evolution (RGE)[1], studying error propagation from low to high energy, which can be important for some parameters notably in the scalar sector.

6 Conclusion

We presented a quite simple-minded bottom-up approach essentially based on analytic inverse mapping from sparticle masses to basic MSSM parameters. It incorporates radiative corrections at realistic level but is certainly not yet very elaborated as compared to the state-of-the art in more standard top-down simulation tools. From assumptions in Table 1, not surprisingly the constraints (summarized in Table 2) are quite good for the gaugino/Higgsino and squark/slepton soft terms, even for unconstrained MSSM, while the determination of other parameters like $\tan\beta$ notably is much less accurate. Those results compare reasonably well with more standard top-down fitting results [5], but this bottom-up approach also provides complementary information with a clear handle e.g on discrete reconstruction amibiguities, or other possible obstacles. This could hopefully suggest new strategies, helping to distinguish from other BSM scenarios since it exhibits theoretical constraints (e.g. correlations) specific to MSSM and not automatically foreseen by "global" fit approaches.

[1] An appropriate bottom-up RGE option is publically available for the SuSpect [12] code versions ≥ 2.40.

Table 2: Combined constraints on some MSSM basic parameters from bottom-up reconstruction. * indicates discrete reconstruction amibiguities.

Assumptions	Parameter	Constraint (GeV)	SPS1a
gen. MSSM,	$M_1(Q_{EWSB})^\star$	∼95–115	101.5
$m_{\tilde{g}}, m_{\tilde{N}_1}, m_{\tilde{N}_2}$.	$M_2(Q_{EWSB})^\star$	∼175–220	191.6
	$M_3(Q_{EWSB})$	∼580–595	586.6
	$\mu(Q_{EWSB})$	∼280–750	357
+ m_{N_4}	$\mu(Q_{EWSB})^\star$	∼350–372	357
$\tilde{q}, \tilde{l}$-universality	$m_0^{q,l}(Q_{GUT})$	∼90–112	100
M_i-universality	$M_i(Q_{GUT})$	∼ 245–255	250
$\tilde{b}_1, \tilde{b}_2$ +universality	$\tan\beta(Q_{EWSB})$	∼3–28	9.74
	$m_{Q3_L}(Q_{EWSB})$	∼490–506	497
	$m_{b_R}(Q_{EWSB})$	∼512–530	522
mSUGRA	m_0	∼90-112	100
	$m_{1/2}$	∼245–255	250
	$-A_0$	∼ -100-350	100
	$\tan\beta(m_Z)$	∼ 5.5–28	10

Whatever the approach, the parameter determination will be clearly improved if using the most sophisticated analysis, both experimental and theoretical. This probably involves new developments in calculating parameter-to-mass relations (as well as all possible signals) at higher order accuracy, using new observables, but also exploiting all possible low energy constraints and the crucial interplay with dark matter observables.

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