Soft photon production in matter in two particle green's function consideration

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Abstract

The production of soft photons in dense matter is studied in terms of the two-particle Green's function in a non-equilibrium medium. The rate of photons is calculated and studied in detail.

1 Introduction

Production of soft photons in matter is studied in the context of the formalism of two-particle Green' functions in a non-equilibrium medium. The exact expression for such functions which determines completely the spectrum of soft photons in matter is derived in the diffusive approximation. On a basis of the calculated two-particle Green's functions the photon rate in equilibrium matter is obtained. The contribution of the bremsstrahlung, two-to-two particle process as well as inelastic pair annihilation is taken into account in the derived rate in the whole region of the emission spectrum of the soft photons which includes the :andau-Pomeranchu-Migdal (LPM) $[1, 2]$ effect range. It is shown that the consistent consideration of both the elastic and inelastic collisions of in-matter particles leads to the additional suppression of the rate of photons as compared with the results obtained earlier in studying the Landau-Pomeranchuk effect [2]. The rate of soft photons from an equilibrium hot quark-gluon plasma is studied in detail. It is shown that the rate is suppressed along all range of the energies of soft photons due to multi-particle interaction between particles in the matter. In this way, the spectral distribution of the emitted photons has a maximum which shifts to the short-wave region of the spectrum with increasing temperature of the matter.

2 Two-particle Green's functions and photon production in the matter

The probability of photon production by the current j^ν is given by the following expression:

$$d^4w = \frac{4\pi}{\omega(k)} e_\mu e_\nu^* (1+n_\gamma) \int d^4x_1 d^4x_2 \exp\left(-ik(x_2-x_1)\right) \langle j^{\mu+}(x_2) j^\nu(x_1)\rangle \frac{d^3k}{(2\pi)^3} \tag{1}$$

where $k = (\omega, \vec{k})$ and e_α are the 4-vector of momentum and the polarization vector of a photon; n_γ is the occupancy number of photon states; $j^\nu(x)$ is the current of the particles generating photons. The angle brackets mean averaging over some state of the particles in matter; x are 4-coordinates. In the absence of a photon "bath" we have $n_\gamma = 0$.

When the energy of produced photons is not too large, so that the emission of them can not change the state of the matter, the bilinear combination of the currents in the last equation can be written as follows :

$$\langle j^{\mu +}(x_2) j^{\nu}(x_1)\rangle = \alpha_E \cdot \langle i| \left(\hat{O}^{\mu}\right)_{\alpha,\beta} \left((\hat{O}^{\dagger})^{\nu}\right)_{\gamma,\delta} |j\rangle \langle {\Psi^{\dagger}}_{\gamma}(x_2)\Psi_{\delta}(x_2)\Psi_{\alpha}(x_1){\Psi^{\dagger}}_{\beta}(x_1)\rangle, \tag{2}$$

where $\langle i| \left(\hat{O}^{\mu}\right)_{\alpha,\beta} \left((\hat{O}^{\dagger})^{\nu}\right)_{\gamma,\delta} |j\rangle$ is the matrix element of some operator which is independent on 4-coordinates, $\Psi_{\alpha}(x)$ are the psi-operators in the Heisenberg picture; $\alpha, \beta, \gamma, \delta$ are the spin variables; α_E is the fine structure constant.

Thus, the problem of the calculation of the photon rate in matter is reduced to obtaining the two-particle Green's function since it is proportional to the product of four Ψ-functions.

We assume that the matter is such that the in-matter particles are ultrarelativistic ones and their spins are equal to $1/2$. Then, the influence of scattering in the matter on the spin states of the particles is negligible [1, 2]. Expanding the correlator $\langle {\Psi^{\dagger}}_{\gamma}(x_2)\Psi_{\delta}(x_2)\Psi_{\alpha}(x_1){\Psi^{\dagger}}_{\beta}(x_1)\rangle$ over the whole set of plane waves, we can write the expression for the probability of photon production dW per unit volume as follows (see Eqs.(1)-(3)):

$$\begin{aligned} d^4W = \overline{d^4w} \quad = \quad & \frac{4\pi\alpha_E}{\omega(k)} \frac{d^3k}{(2\pi)^3} \left\{ \frac{(1+n_{\gamma})}{2} \cdot \right. \\ & \int \frac{d^4p_1 d^4p_2 d^4p_3 d^4p_4}{(2\pi)^8} \delta^4(p_2 - p_1 - k) \cdot \delta^4(p_3 - p_4 - k) \\ & Tr\left[e_{\mu} e_{\nu} \left\langle \overline{u}^{\beta}(p_2)\overline{u}^{\gamma}(p_3) \middle| \left(\hat{O}^{\mu}\right)_{\alpha,\beta} \left((\hat{O}^{\dagger})^{\nu}\right)_{\gamma,\delta} \middle| u^{\alpha}(p_1) u^{\delta}(p_4) \right\rangle \right. \\ & \left. \cdot K^{-+,-+}_{\alpha\gamma,\beta\delta}(p_1;p_2|p_3;p_4) \right], \end{aligned} \tag{3}$$

where $p_i = (p_i^0, \vec{p}_i)$ are the 4-momentum of the radiating particle, s is its spin, $u^{\alpha}(p)$ are the Dirac spinors. The line over d^4w means the averaging and summing over the corresponding spin states of the particles in the matter. In the case of the generation of photons by fermions with the spin $s = 1/2$ the operator $\hat{O}^{\nu}$ is the corresponding Dirac matrix.

The function $K^{-+,-+}_{\alpha\gamma,\beta\delta}(p_1;p_2|p_3;p_4)$ is the so-called time-unordered two-particle Green's function $K(1(-),2(-)|3(+),4(+))$ in the momentum representation. Thus, the problem of the calculation of the photon production in matter is reduced to obtaining the non-chronological (time-unordered) two-point Green's functions $K(p_1(-);p_4(-)|p_3(+);p_2(+))$ [3].

3 Two-particle Green's functions in non-equilibrium matter in the diffusive approximation

According to [3] the Green's function $K^{ac,bd}_{\alpha\gamma,\beta\delta}(p_1;p_2|p_3;p_4)$ satisfies the Bethe-Salpeter-like equation which has the following form in the momentum representation ($\hbar = c = 1$) in the case of the Fermi statistics:

$$
\begin{aligned}
K^{ac,bd}_{\alpha\gamma,\beta\delta}(p_1;p_2|p_3;p_4) \quad = \quad & (2\pi)^4 \Big\{ G^{ac}_{\alpha\gamma}(p_1) G^{bd}_{\beta\delta}(p_2)\delta(p_1-p_3) - \\
& G^{ac}_{\alpha\delta}(p_1) G^{bd}_{\beta\gamma}(p_2)\delta(p_1-p_4) \Big\} - \Big\{ G^{aa_1}_{\alpha\alpha_1}(p_1) G^{bb_1}_{\beta\beta_1}(p_2) \\
& \int dq \Gamma^{a_1a_2,b_1b_2}_{\alpha_1\alpha_2,\beta_1\beta_2}(p_1;p_2,|p_1-q;p_2-q) \cdot \\
& K^{a_2c,b_2d}_{\alpha_2\gamma,\beta_2\delta}(p_1-q,;p_2-q|p_3;p_4) \Big\}.
\end{aligned}
\tag{4}
$$

where the Roman letters are minus or plus sign, the Greek letters mean spin variables; where $\Gamma^{\dots}_{\dots}(\dots)$ is the exact two-particle vertex function consisting of all diagrams that can not be cut by a vertical line so that this line only intersects two lines which correspond to the exact or free one-particle Green's functions; $G^{ab}_{\alpha\beta}(p_1=p_3)$ is the exact 2-point Green's function in the momentum representation [4].

In the diffusive approximation the last equation is reduced to the corresponding differential equation which can be solved in the small angle approach with respect to elastic scattering of particles in matter.

When small angle scattering occurs it is convenient to introduce the angle vectors $\vec{\eta}$ and $\vec{\theta}$ $[1,2]$ which are connected with the velocity $\vec{v}$ of a particle and the wave vector of a photon $\vec{k}$ by means of the formulae:

$$
\begin{aligned}
\vec{v} &= \vec{v}_0\left(1-\frac{\vec{\eta}^2}{2}\right) + v_0\vec{\eta}; \qquad \vec{v}_0 \perp \vec{\eta}; \qquad |\vec{\eta}| \ll 1 \\
\vec{k} &= \frac{k\vec{v}_0}{v_0}\left(1-\frac{\vec{\theta}^2}{2}\right) + v_0\vec{\theta}; \qquad \vec{v}_0 \perp \vec{\theta}; \qquad |\vec{\theta}| \ll 1,
\end{aligned}
\tag{5}
$$

Then, the solution of the equation for the unordered two-particle Green's function in the momentum representation can expressed by the following formulae:

$$
\begin{aligned}
& K^{-+,-+}_{\alpha\gamma,\beta\delta}(p\,;p-k|p';p'-k) = \\
& (2\pi)^4 \int d^2\vec{\zeta} \Big\{ \delta_{\alpha\gamma}\delta_{\beta\delta} \cdot \delta(p-p') \cdot \delta\left(p^0 - E(p) + \mu\right) \cdot F(p\,,p',\vec{\eta},\vec{\zeta}) \\
& \frac{\Gamma \cdot \left[n(p^0)\cdot\theta(p^0) + \left(1-n(p^0)\right)\cdot\theta(-p^0)\right] \cdot \left[\left(1-n(p^0)\right)\cdot\theta(p^0) + n(p^0)\cdot\theta(-p^0)\right] \delta\left(\vec{\eta}\right)}{\left(\Gamma^2 + \left(E(p)-E(p-k)-\omega\right)\right)^2\right)} \Big\},
\end{aligned}
$$

where $F(p\,,p',\vec{\eta},\vec{\zeta})$ is equal to:

$$F(p\ ,p',\vec{\eta},\vec{\zeta}) = \frac{a}{\pi < \theta_s^2 >} \int\limits_0^{+\infty} \frac{d\tau}{\sinh(a\tau)} \exp\left\{ - \frac{a\cdot\left(\vec{\eta}-\vec{\zeta}\right)^2}{<\theta_s^2> \coth(a\tau)} + \right. \tag{6}$$

$$\left. \frac{2a}{q}\left(\vec{\eta}-\vec{\zeta}\right)\cdot\left(\vec{\theta}-\vec{\zeta}\right)\tanh\left(\frac{a\tau}{2}\right) - \frac{2a}{q}\left(\vec{\theta}-\vec{\zeta}\right)^2\tanh\left(\frac{a\tau}{2}\right) - \imath\Omega\tau(1-v) \right\}\cdot\delta\left(\vec{\zeta}\right),$$

where Γ is the width with respect to inelastic processes.

Substituting Eqs.(6), (7) into the formulae (3) and carrying out the needed integrations, we derive the probability of the photon production in the absence of the photon "bath" ($n_\gamma = 0$):

$$\begin{aligned} \frac{dW}{d\omega} \quad = \quad & \frac{\alpha_E}{\pi}\int \frac{d^3\vec{p}}{(2\pi)^3}\cdot\int\limits_0^{+\infty} dz \exp\left(-\frac{z\left(\omega/<\theta_s^2>\right)^{1/2}}{2\gamma^2}\right)\cdot\left\{\sin\left(\frac{z\left(\omega/<\theta_s^2>\right)^{1/2}}{2\gamma^2}\right)\right. \\ & \left.\cdot\coth(z) - \frac{1}{z}\right\}\cdot\frac{\Gamma\cdot[n(E(p))\cdot\theta(E(p)) + (1-n(E(p)))\cdot\theta(-E(p))]}{\gamma^2\pi\left(\Gamma^2 + (E(p)-E(p-k)-\omega))^2\right)} \\ & [(1-n(E(p-k)))\cdot\theta(E(p-k)) + n(E(p-k))\cdot\theta(-E(p-k))] \end{aligned} \tag{7}$$

where $\gamma = E/M$ is the Lorenz-factor of the particle; $E(\vec{p})$ is the energy of a particle, M is its mass.

The products of the first and last terms in the square brackets in Eq.(8) are the contribution to the probability of photon production due to the particle-particle and antiparticle-antiparticle bremsstrahlung in matter. The products of the other terms in the square bracket results in the photon production via the annihilation of off-shell particles and antiparticles and on-shell antiparticles and particles, respectively.

4 Photon production in a hot equilibrium quark-gluon plasma

We illustrate the applicability of the developed method of the calculation of the photon rate in matter and consider a hot quark-gluon plasma. We assume that the plasma is in equilibrium at temperature $T \geq 300MeV$ and consists of light quarks mainly. In this case the quarks are ultrarelativistic ones, and they are scattered on small angles. The small angle elastic scattering in a hot quark-gluon plasma can be described by the t-channel-exchange diagrams [5]. In this case the mean square of the angle per unit path length is $<\theta_s^2> = 8.5\cdot L_c\cdot{\alpha_s}^2\cdot\frac{T^3}{p^2}$ [5], where L_c is the Coulomb logarithm depending on ${\alpha_s}^2; T$ and p. Owing to the logarithm we set L_c as the constant of the order of unit.

Taking into account the flavor degeneracy in Eq.(8) we derive the following for the energy being escaped from the quark-gluon plasma via the photon emitted by the light quarks:

$$\frac{d\varepsilon}{d\omega} = 2\frac{\omega\cdot W}{d\omega} \tag{8}$$

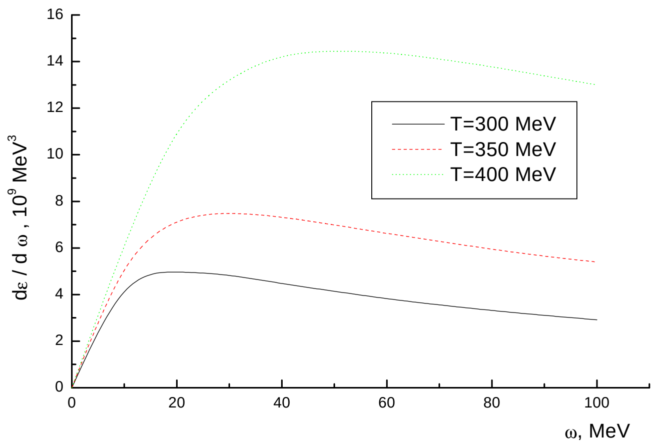

Fig. 1: Dependence of emission energy on the energy of photons at the fixed matter temperature.

The results of the numerical calculation of the photon rate according to Eq.(9) are presented in Figs.1. It follows from Figs.1. that the emission energy increases with increasing the temperature of the matter at any fixed frequency. In this way, the maximum of the spectral distribution of the emission energy shifts to the short wave range of the spectrum with increasing the temperature of the medium.

5 Conclusion

The photon production in matter in terms of the two-particles Green's functions in non-equilibrium matter is considered in the paper. The developed method of the calculation of photon rate allows us to take properly into account the contribution of all mechanism of forming the emission spectrum such as the particle (antiparticle) and antiparticle (particle) bremsstrahlung, particles and off-shell-antiparticle annihilation, two-to-two process. As an illustration of the applicability of the developed method, the energy emitted from a hot equilibrium quark-gluon plasma due quark emission is calculated for various temperatures of the matter.

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