AdS/CFT Correspondence in Heavy Ion Collisions†

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Abstract

We construct a model of high energy heavy ion collisions as two ultra-relativistic shock waves colliding in AdS_5. The metric in the forward light cone after the collision is constructed perturbatively through expansion in graviton exchanges. We conclude that shock waves corresponding to physical energy-momentum tensors of the nuclei must completely stop almost immediately after the collision in AdS_5, which, on the field theory side, corresponds to complete nuclear stopping due to strong coupling effects, likely leading to Landau hydrodynamics. We propose using zero-net energy shock waves, which continue moving along their light cones after the collision, as a possible way to model the collision which may lead to Bjorken hydrodynamics at late proper times.

1 General Setup: Expansion in Graviton Exchanges

Our goal is to describe the isotropization (and thermalization) of the medium created in heavy ion collisions assuming that the medium is strongly coupled and using AdS/CFT correspondence to study its dynamics. We want to construct a metric in AdS_5 which is dual to an ultrarelativistic heavy ion collision as pictured in Fig. 1. Throughout the discussion we will use Bjorken approximation of the nuclei having an infinite transverse extent and being homogeneous (on the average) in the transverse direction, such that nothing in our problem would depend on the transverse coordinates x^1, x^2.

We start with a metric for a single shock wave moving along a light cone [2]:

$$ds^2 \,=\, \frac{L^2}{z^2}\,\left\{-2\,dx^+\,dx^- + \frac{2\,\pi^2}{N_c^2}\,\langle T_{--}(x^-)\rangle\,z^4\,dx^{-\,2} + dx_\perp^2 + dz^2\right\}. \tag{1}$$

Here $x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$, z is the coordinate describing the 5th dimension such that the boundary of the AdS space is at $z=0$, and L is the curvature radius of the AdS space. According to holographic renormalization [3], $\langle T_{--}(x^-)\rangle$ is the expectation value of the energy-momentum tensor for a single ultrarelativistic nucleus moving along the light-cone in x^+-direction in the gauge theory.

The metric in Eq. (1) is an exact solution of Einstein equations in AdS_5: $R_{\mu\nu} + \frac{4}{L^2}\,g_{\mu\nu} = 0$. It can also be represented perturbatively as a single graviton exchange between the source nucleus

† This talk was based on [1].
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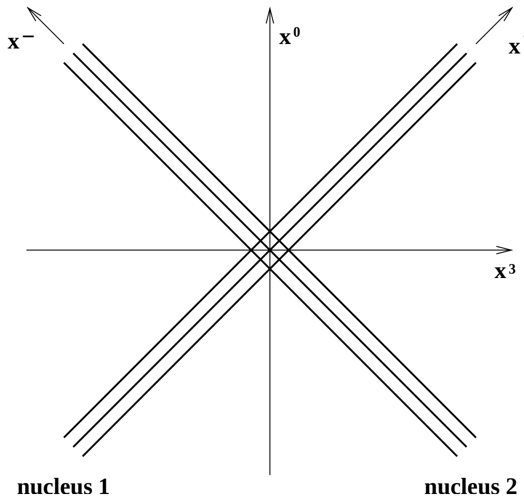

Fig. 1: The space-time picture of the ultrarelativistic heavy ion collision in the center-of-mass frame. The collision axis is labeled x^3, the time is x^0.

at the AdS boundary and the location in the bulk where we measure the metric/graviton field. This is shown in Fig. 2, where the solid line represents the nucleus and the wavy line is the graviton propagator. Incidentally a single graviton exchange, while being a first-order perturbation of the empty AdS space, is also an exact solution of Einstein equations. This means higher order tree-level graviton diagrams are zero (cf. classical gluon field of a single nucleus in covariant gauge in the Color Glass Condensate (CGC) formalism [4]).

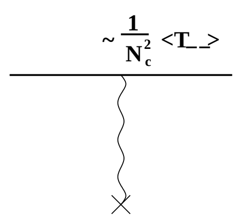

Fig. 2: A representation of the metric (1) as a graviton (wavy line) exchange between the nucleus at the boundary of AdS space (the solid line) and the point in the bulk where the metric is measured (denoted by a cross).

Now let us try to find the geometry dual to a collision of two shock waves with the metrics like that in Eq. (1). Defining $t_1(x^-) \equiv \frac{2\pi^2}{N_c^2} \langle T_{1\,--}(x^-)\rangle$ and $t_2(x^+) \equiv \frac{2\pi^2}{N_c^2} \langle T_{2\,++}(x^+)\rangle$ we write the metric resulting from such a collision as

$$ds^2 = \frac{L^2}{z^2} \left\{ -2\,dx^+\,dx^- + dx_\perp^2 + dz^2 + t_1(x^-)\,z^4\,dx^{-\,2} + t_2(x^+)\,z^4\,dx^{+\,2} \right. \\ \left. + \text{higher order graviton exchanges} \right\} \qquad (2)$$

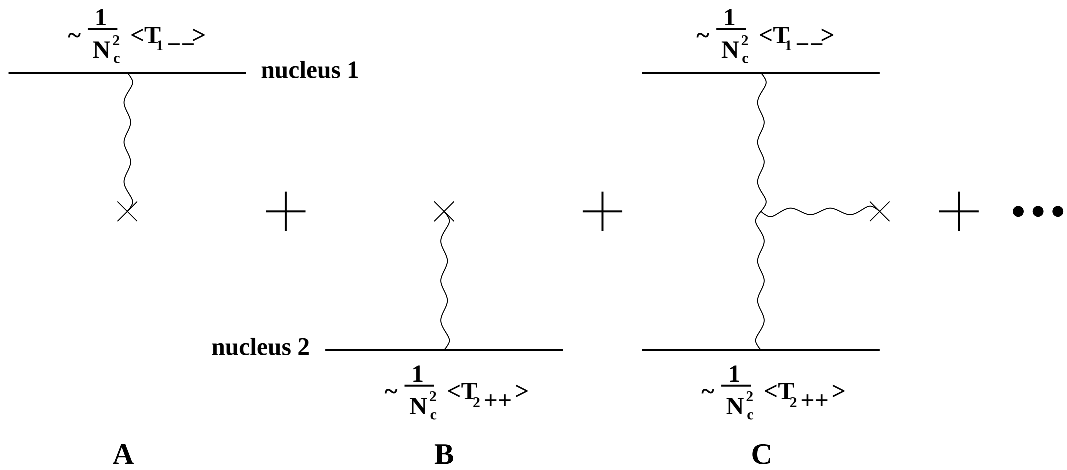

Fig. 3: Diagrammatic representation of the metric in Eq. (2). Wavy lines are graviton propagators between the boundary of the AdS space and the bulk. Graphs A and B correspond to the metrics of the first and the second nucleus correspondingly. Diagram C is an example of the higher order graviton exchange corrections.

The metric of Eq. (2) is illustrated in Fig. 3. The first two terms in Fig. 3 (diagrams A and B) correspond to one-graviton exchanges which constitute the individual metrics of each of the nuclei, as shown in Eq. (1). Our goal below is to calculate the next order correction to these terms, which is shown in the diagram C in Fig. 3. Fig. 3 illustrates that construction of dual geometry to a shock wave collision in AdS_5 consists of summing up all tree-level graviton exchange diagrams, similar diagrammatically to the classical gluon field formed by heavy ion collisions in CGC [5]. While classical gluon fields lead to free-streaming final state [6], their AdS graviton "dual" is likely to lead to a hydrodynamic final state for the gauge theory like the one found in [2].

2 Perturbative Solution of Einstein Equations

To solve Einstein equations perturbatively in graviton exchanges we write

$$g_{\mu\nu} \, = \, g^{(0)}_{\mu\nu} + g^{(1)}_{\mu\nu} + g^{(2)}_{\mu\nu} + \ldots \, . \tag{3}$$

Here $g^{(0)}_{\mu\nu}$ is the metric of the empty AdS_5 space with non-zero components

$$g^{(0)}_{+-} = g^{(0)}_{-+} = -\frac{L^2}{z^2}, \quad g^{(0)}_{ij} = \delta_{ij} \, \frac{L^2}{z^2}, \ \ i,j = 1,2, \qquad g^{(0)}_{zz} = \frac{L^2}{z^2}. \tag{4}$$

$g^{(1)}_{\mu\nu}$ is the first perturbation of the empty AdS_5 space due to the two nuclei

$$g^{(1)}_{--} = t_1(x^-) \, L^2 \, z^2, \quad g^{(1)}_{++} = t_2(x^+) \, L^2 \, z^2 \tag{5}$$

with all the other components zero.

We want to find the next non-trivial correction $g^{(2)}_{\mu\nu}$. By the choice of Fefferman-Graham coordinates one has $g_{z\mu} = g_{\mu z} = 0$ exactly for $\mu \neq z$ and $g_{zz} = L^2/z^2$. Hence the non-trivial components of $g_{\mu\nu}^{(2)}$ are those for $\mu, \nu = 0, \ldots, 3$. Due to translational and rotational invariance of the nuclei in the transverse direction $g_{ij}^{(2)} \sim \delta_{ij}$. We thus parametrize the unknown components of $g_{\mu\nu}^{(2)}$ as

$$\begin{aligned} g_{--}^{(2)} &= \tfrac{L^2}{z^2}\, f(x^+, x^-, z), \quad g_{++}^{(2)} = \tfrac{L^2}{z^2}\, \tilde{f}(x^+, x^-, z), \\ g_{+-}^{(2)} &= -\tfrac{1}{2}\, \tfrac{L^2}{z^2}\, g(x^+, x^-, z), \quad g_{ij}^{(2)} = \tfrac{L^2}{z^2}\, h(x^+, x^-, z)\, \delta_{ij} \end{aligned} \tag{6}$$

with f, $\tilde{f}$, g and h some unknown functions. Imposing causality we require that functions f, $\tilde{f}$, g and h are zero before the collision, i.e., that before the collision the metric is given only by the empty AdS space and by the contributions of the two nuclei (5). Also, according to general properties of $g_{\mu\nu}$ outlined in Sect. 1 (see [3]), we demand that f, $\tilde{f}$, g and h go to zero as z^4 when $z \to 0$.

Linearizing Einstein equations in f, $\tilde{f}$, g, and h we solve the obtained system of differential equation to obtain [1]

$$h(x^+, x^-, z) \;=\; h_0(x^+, x^-)\, z^4 + h_1(x^+, x^-)\, z^6 \tag{7}$$

where h_0 and h_1 are determined by the causal solutions of the following equations

$$(\partial_+ \partial_-)^2\, h_0(x^+, x^-) \;=\; 8\, t_1(x^-)\, t_2(x^+), \tag{8}$$

$$\partial_+ \partial_-\, h_1(x^+, x^-) \;=\; \frac{4}{3}\, t_1(x^-)\, t_2(x^+). \tag{9}$$

f, $\tilde{f}$, and g are easily expressed in terms of $h(x^+, x^-, z)$ from Eq. (7) (see [1]).

3 Nuclear Stopping and How One May Avoid It

Imagine a collisions of two shock waves whose energy-momentum tensors are given by smeared delta-functions

$$t_1(x^-) \;=\; 2\,\pi^2\, \frac{\mu}{a}\, \theta(x^-)\, \theta(a - x^-), \;\; t_2(x^+) \;=\; 2\,\pi^2\, \frac{\mu}{a}\, \theta(x^+)\, \theta(a - x^+). \tag{10}$$

Here for a shock wave moving in the x^+-direction $\mu \propto p^+\, \Lambda^2\, A^{1/3}$ and $a \;\propto\; R\, \frac{\Lambda}{p^+} \;\propto\; \frac{A^{1/3}}{p^+}$, where the nucleus of radius R has A nucleons in it with N_c^2 valence gluons each. p^+ is the light cone momentum of each nucleon and Λ is the typical transverse momentum scale. Using the solution found in Sect. 2 along with holographic renormalization we find the "$--$" component of the energy-momentum tensor of a shock wave after the collision at $x^- = a/2$ and for $x^+ \gg a$:

$$\langle T_{--}(x^+ \gg a, x^- = a/2) \rangle \;=\; N_c^2\, \frac{\mu}{a} - 4\,\pi^2\, N_c^2\, \mu^2\, x^{+\,2}. \tag{11}$$

The first term on the right of Eq. (11) is due to the original shock wave while the second term describes energy loss due to graviton emission.

Eq. (11) shows that $\langle T_{--} \rangle$ of a nucleus becomes *zero* at light-cone times

$$x^+ \;\sim\; \frac{1}{\sqrt{\mu\, a}} \;\sim\; \frac{1}{\Lambda\, A^{1/3}}. \tag{12}$$

Indeed zero $\langle T_{--}\rangle$ would mean a complete *stopping* of the shock wave and the corresponding nucleus. At larger x^+ the energy-momentum tensor component in Eq. (11) becomes negative: one can show that higher order graviton exchanges become important at this light cone time likely preventing $\langle T_{--}\rangle$ from becoming negative. As the shock wave can loose all of its energy by emitting a single graviton as shown in Fig. 3C, it is highly unlikely that higher order graviton exchanges/emissions would prevent the shock wave from stopping. We thus conclude that the collision of two nuclei at strong coupling leads to a necessary stopping of the two nuclei shortly after the collision. If the nuclei stop completely in the collision, the strong interactions between them are almost certain to thermalize the system, probably leading to Landau hydrodynamics [7].

However, in the real-life heavy ion collisions the nuclei interact weakly at the early stages of the collisions and continue moving along their light cones after the collision. While finding a dual theory describing these weak coupling effects in the framework of the AdS/CFT correspondence is very hard, we suggest mimicking them by using zero-net energy shock waves with

$$t_1(x^-) \,=\, \Lambda_1^2\, \delta'(x^-), \;\; t_2(x^+) \,=\, \Lambda_2^2\, \delta'(x^+) \tag{13}$$

in the shock waves metric of Eq. (2). $\delta'(x)$ denotes the derivative of a delta-function and Λ_1 and Λ_2 are the transverse momentum scales describing the two nuclei. One can then show [1] that the lowest order non-trivial graviton exchange of Fig. 3C leads to the following energy density ϵ and transverse p and longitudinal p_3 pressure components for the produced medium at early times:

$$\epsilon(\tau) \,=\, \frac{N_c^2}{\pi^2}\, 4\, \Lambda_1^2\, \Lambda_2^2, \;\; p(\tau) \,=\, \frac{N_c^2}{\pi^2}\, 4\, \Lambda_1^2\, \Lambda_2^2, \;\; p_3(\tau) \,=\, -\frac{N_c^2}{\pi^2}\, 4\, \Lambda_1^2\, \Lambda_2^2. \tag{14}$$

(One can prove [1] that graviton expansion of Fig. 3 corresponds to expansion in $\Lambda_1^2\,\tau^2$ and $\Lambda_2^2\,\tau^2$ for the energy-momentum tensor of the gauge theory: hence the lowest order diagram (Fig. 3C) gives the dominant contribution to $T_{\mu\nu}$ at early times.) One can see from Eq. (14) that the energy density of the strongly coupled medium starts out as a constant at early times, a conclusion which has been reached earlier in [8]. The energy-momentum tensor components in Eq. (14) are also similar to those found in CGC at early times [9], and may serve as a starting point for a possible evolution of the strongly-coupled system towards Bjorken hydrodynamics [10].

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