Have we seen anything beyond (N)NLO DGLAP at HERA?

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Abstract
The evidence from HERA for parton saturation, and other low-x effects beyond the conventional DGLAP formalism, is recalled and critically reviewed in the light of new data and analyses presented at the conference.

In the mid-90's the original surprise of the HERA Neutral Current e^+p scattering data was the strong rise of the structure function F_2 at low-x. This was taken to imply a strong rise of the gluon density at low-x which was widely interpreted as implying the possibility of gluon saturation and the need for non-linear terms in the parton evolution equations. Even somewhat more conservative interpretations suggested the need to go beyond the DGLAP formalism at small-x, resumming $ln(1/x)$ as in the BFKL formalism.

However, at low-x linear NLO DGLAP evolution itself predicts a rise in F_2, and in the gluon and sea PDFs, provided that Q^2 is large enough. One can begin parton evolution at a low Q^2 input scale, Q_0^2, using flat (or even valence-like) gluon and sea-quark input shapes in x and the DGLAP Q^2 evolution will generate a steep low-x rise of the gluon and sea at larger $Q^2 \gg Q_0^2$. The real surprise - seen in the data of the late 90's- was that steep shapes were already observed at rather low Q^2. Traditionally values of $Q_0^2 \sim 4\text{GeV}^2$ were used, but the data already show a steep rise of F_2 at low-x for Q^2 values, $Q^2 \sim 1\text{GeV}^2$, see Fig. 1 left-hand-side. To interpret these data in terms of conventional NLO DGLAP evolution we clearly need a low starting scale and thus we are forced into using perturbative QCD at a scale for which $\alpha_s(Q^2)$ is quite large- $\alpha_s(1.0) \sim 0.35$. Even if this is considered to be acceptable, we also need to use flexible input parton shapes, which can reproduce the steepness of the data. Surprisingly enough this does NOT imply that both the gluon and the sea input are already steep at $Q^2 \sim 1\text{GeV}^2$. The sea input is indeed steep, but the gluon input is valence-like, with a tendency to be negative at low-x!- see Fig. 1 right-hand-side. (Essentially the gluon evolution must be fast in order that upward evolution can produce the extreme steepness of high-Q^2 data, however this also implies that downward evolution is fast and this results in the valence-like gluon at low-Q^2).

Thus when statements are made that HERA has established that the low-x gluon is steep one must remember that this is only true for higher Q^2, $Q^2 \gtrsim 10\text{GeV}^2$, within the DGLAP formalism. However this formalism seems to work to much lower Q^2. Let us examine how the gluon and sea PDFs are extracted from the measurements. At low-x, the sea PDF is extracted fairly directly since, $F_2(x,Q^2) \sim xq(x,Q^2)$. However the gluon PDF is extracted from the scaling violations, $\partial F_2/\partial ln(Q^2) \sim P_{qg} xg(x,Q^2)$, such that the measurement is related to a convolution of the splitting function P_{qg} and the gluon distribution. Thus if the correct splitting function is NOT that of the conventional DGLAP formalism, or if a more complex non-linear realtionship is needed, then a turn over of the data $\partial F_2/\partial ln(Q^2)$ at low-Q^2 and low-x may not imply a turn over of the gluon distribution. It was suggested that measurements of other gluon related quantities could help to shed light on this question and the longitudinal structure function, F_L, and the heavy quark structure functions, $F_2^{c\bar{c}}, F_2^{b\bar{b}}$, are obvious candidates. All of these quantities have now been measured (see talks of K. Papageorgiou and P. Thompson in these proceedings) and, within present experimental uncertainties, they can be explained by the conventional NLO DGLAP formalism (with the heavy quark results shedding more light on the complexities of general-mass-variable-flavour number schemes than on the gluon PDF).

Other measurements of more exclusive quantities can also give information on the correctness of the conventional formalism at low-x. For example HERA forward jet mesaurements (see talk of A. Savin in these proceedings). DGLAP evolution would suppress the forward jet cross-section, for

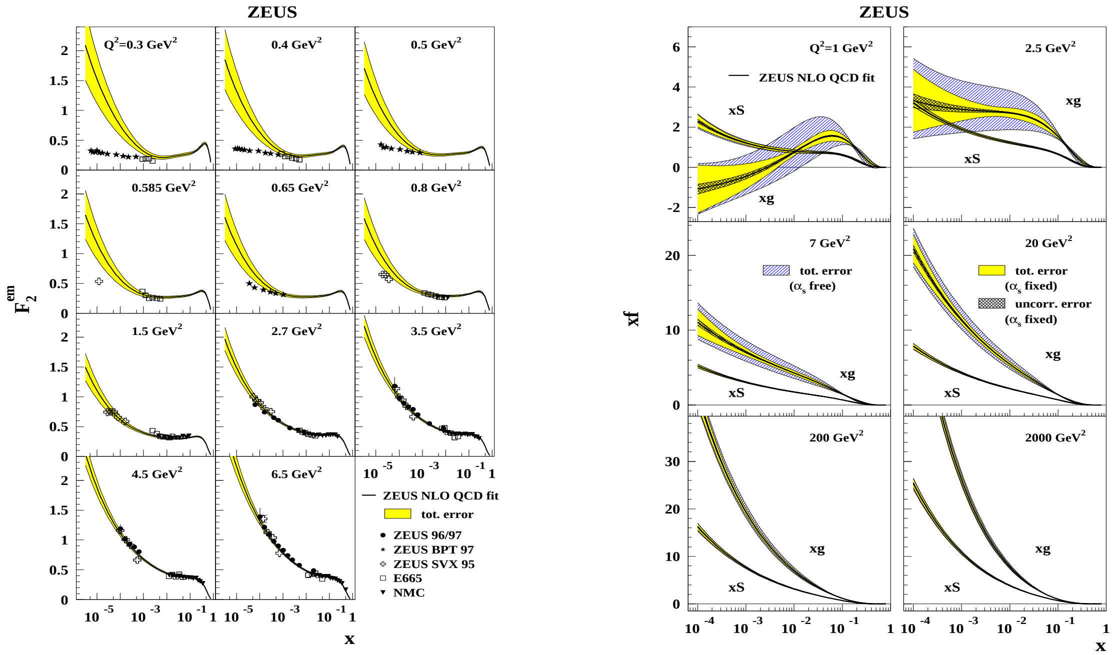

Fig. 1: Left plot: F_2 vs x for various low Q^2 values. Right plot: Sea and gluon PDF distributions extracted from a global PDF fit including these data.

jets with $P_t^2 \sim Q^2$ and low-x, because LO DGLAP evolution has strong k_t ordering, from the target to the probe, and thus it cannot produce such events. The rate is also suppressed for NLO DGLAP. However BFKL evolution has no k_t ordering and thus a larger cross-section for such events at both LO and NLO. The data do indeed show an enhancement of forward jet cross-sections wrt conventional NLO DGLAP calculations. However this cannot be regarded as a definitive indication of the need for BFKL resummation because conventional calculations at higher order, $O(\alpha_s^3)$, do describe the data.

However, as we have already mentioned, even though conventional calculations do give reasonable fits to data, the peculiar behaviour of the low-x, low-Q^2 gluon gives us cause for some concern. Thorne and White have performed an NLL BFKL resummation and matched it to NLO DGLAP at high-x in order to perform a global PDF fit. When this is done the gluon shape deduced from the scaling violations of F_2 is a lot more reasonable and a good fit is found to global DIS data, see the talk of C.White in these proceedings. A similar improvement to the gluon shape is got by introducing a non-linear term into the evolution equations, as done by Eskola et al [1]- but although this work has been widely used to give non-linear PDFs one must remember that it is limited to leading order.

These analyses make us suspect that the conventional formalism could be extended, but they are still not definitive. A different perspective comes from considering the low-x structure function data in terms of the virtual-photon proton cross-section: at low-x, $\sigma(\gamma^* p) \sim 4\pi\alpha^2 F_2/Q^2$. The data are presented in this way in Fig. 2 left-hand-side. A rise of $F_2(x) \sim x^{-\lambda}$, implies a rising cross-section with W^2, the centre-of mass energy of the photon-proton system, $\sigma(W^2) \sim (W^2)^\lambda$ (since $x = Q^2/W^2$ at low-x). However, the real-photon proton cross-section (and all high energy hadron-hadron cross-sections) rises slowly as $(W^2)^{\alpha-1}$, where, $\alpha = 1.08$, is the intercept of the soft-Pomeron Regge trajectory. Thus the data on virtual-photon proton scattering are showing something new - a faster rise of cross-section than predicted by the soft-Pomeron which has served us well for many years. In Fig. 2 right-hand-side we show the slope of this rise, $\lambda = (\alpha - 1)$, as calculated from the data, $\lambda = \partial ln F_2/\partial ln(1/x)$. One can see a change in behaviour at $Q^2 \sim 0.8\text{GeV}^2$ as we move out of the non-perturbative region -where the soft pomeron intercept gives a reasonable description of the data -to larger Q^2. Does this imply that we need a hard Pomeron as well?

Dipole models have given us a way to look at virtual-photon proton scattering which can model

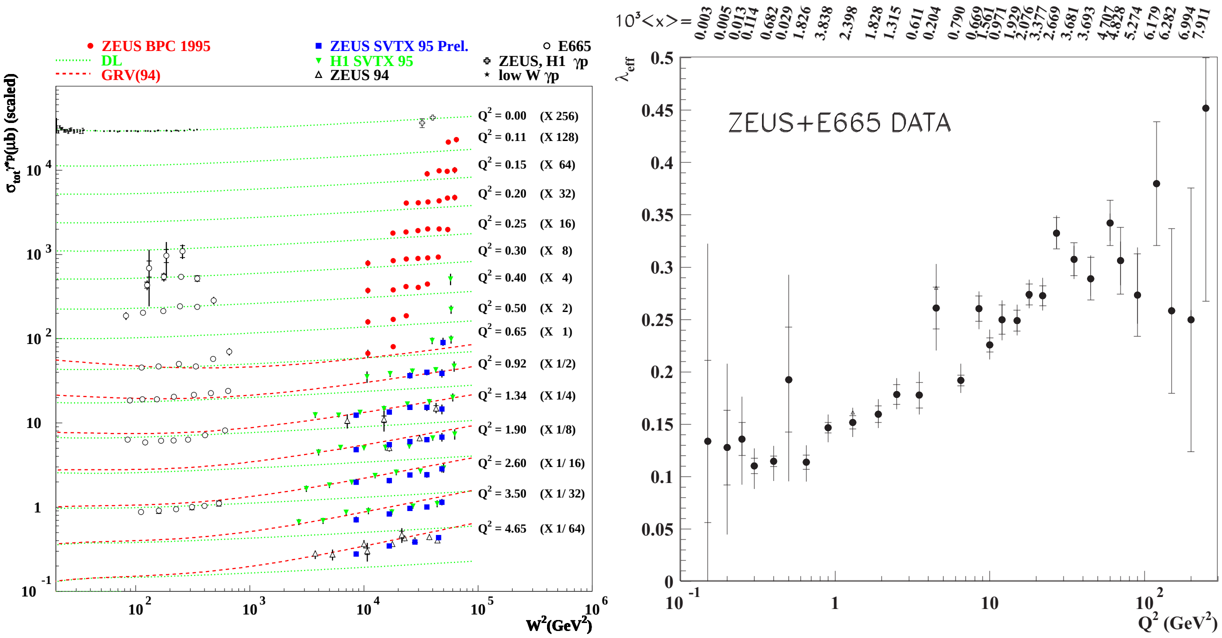

Fig. 2: Left plot: the photon-proton cross-section vs W^2 for various virtualities of the photon. Right plot: the slope $\lambda = \partial ln F_2/\partial ln(1/x)$.

the transition from the non-perturbative to the perturbative region. The interaction can be viewed as the virtual photon breaking up into a quark-antiquark pair and this pair, or dipole, then interacts with the proton. At low-x, the lifetime of the $q\bar{q}$ pair is longer than the dipole-proton scattering time, such that the physics is contained in the modelling of the dipole-hadron cross-section. There are many dipole models but the simplest Golec-Biernat Wusthoff model [2] contains the essential features: $\sigma = \sigma_0(1 - exp(-r^2/(2R_0^2))$, where r is the transverse size of the dipole and R_0 is the transverse separation of the gluons in the target, $R_0^2 = 1/Q_0^2(x/x_0)^\lambda$, where $x^\lambda \sim 1/(xg(x))$, is inverse to gluon density. Thus for small dipoles, $r < 1/Q$ and large Q^2, one obtains $\sigma \sim r^2 \propto 1/Q^2$ and Bjorken scaling (sophistications to the model correct this to give logarithmic scaling violation), whereas for large dipoles and small Q^2, one obtains $\sigma \sim \sigma_0$, ie a constant cross-section which corresponds to the correct photo-production limit. The reason that such dipole models have attracted attention in recent years is that the dipole-proton cross-section can be written in terms of a single scaling variable, τ, $\sigma = \sigma_0(1 - exp(-1/\tau)$, where $\tau = Q^2R_0^2 = Q^2/Q_0^2(x/x_0)^\lambda$, rather than in terms of the two variables x, Q^2. This is known as geometric scaling, and evidence for it is shown by the low-x ($x < 0.01$) data in Fig. 3. Note that only low-x data show this scaling. Geometrical scaling is predicted by many theoretical approaches to the low-x regime which involve saturation and, $Q_s^2 = 1/R_0^2$, is interpreted as a saturation scale below which non-linear dynamics applies.

Note that the power $\lambda \sim 0.3$, which describes the gluon density, $xg(x) \sim x^{-\lambda}$, within many dipole-models, is fitted to the data. It cannot be trivially related to the measured slope, $\partial ln F_2/\partial ln(1/x)$, at any Q^2, and it is not justified by the steep slopes of the gluon distribution observed at HERA- because such steep slopes are not in fact observed but are derived within the DGLAP formalism- which is explicitly not the formalism of most dipole models- and a steep slope $\lambda \gtrsim 0.3$ is only found for $Q^2 \gtrsim 10\text{GeV}^2$. However the saturation scale for HERA data is much lower, $Q_s^2 \sim 1 - 2GeV^2$. Thus the steep slope of the gluon in the dipole models must be regarded as an input assumption.

Geometric scaling is not unique to non-linear approaches, it can be derived from solutions to the linear BFKL equation [3] and even from the DGLAP equation [4]. But note that such solutions do not extend into the low-Q^2 region and cannot give a picture of the transition from low to high-Q^2, as the dipole models do. Moreover, dipole models provide explanations for the constant ratio of the diffractive to the total cross-section data at HERA, and geometric scaling has also been observed in diffractive processes including vector meson production and deeply virtual compton scattering, see the

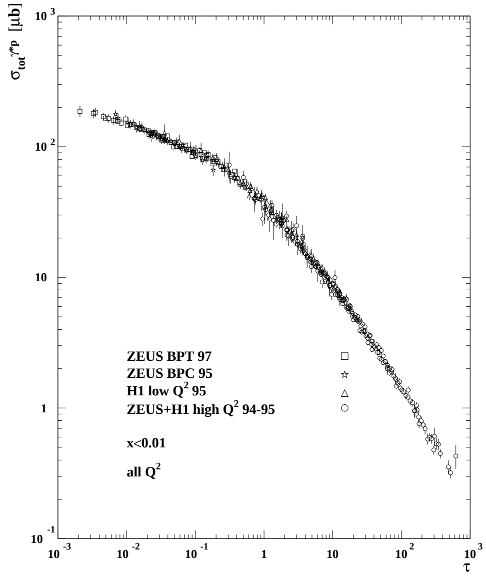

Fig. 3: $\sigma(\gamma^* p)$ vs the scaling variable $\tau = Q^2/Q_s^2$

talk of R. Yoshida in these proceedings. These observations give hints that there is some truth to the dipole picture of saturation even though data at HERA are not definitive.

Even if the evidence for saturation at HERA is taken seriously the saturation scale is only, $Q_s^2 \sim 1{-}2\mathrm{GeV}^2$, such that the region of non-linear dynamics largely coincides with the strongly-coupled region (where α_s is large). That is why there is interest in results from RHIC, where the nuclear environment enhances the high-density of the partons by $A^{1/3}$, such that saturation scales are higher, see the talk of A. Dainese in these proceedings. But what of the LHC? Clearly ALICE data will be interesting, but even proton-proton data can be searched for signs of saturation if the large rapidity region is considered, since small x values are then accessed. For example, low-mass Drell-Yan data at LHCb can access $x \sim 10^{-6}$, see the talk of T. Shears in these proceedings.

If our conventional picture of DGLAP evolution in the HERA x region is significantly wrong then this will have implications even for classic Standard Model predictions, such as W and Z production in the central region of CMS and ATLAS. These bosons are produced at low-x, $5 \times 10^{-4} < x < 5 \times 10^{-2}$, in the central rapidity region, $-2.5 < y < 2.5$ and they are produced with enormous rate (even a modest 100 pb^{-1} luminosity produces 10^6 W events) such that very early low luminosity running could show up discrepancies with our predictions. Whereas rapidity spectra may not be much affected by unconventional Q^2 evolution [5], it should be fruitful to examine the boson p_t spectra, since lack of p_t ordering could affect these significantly [6].

In summary, it is unclear that HERA data have actually given any evidence for BFKL evolution, non-linear evolution or saturation, but there are hints in many places. The contribution of A. deRoeck to this discussion considers the possibilities for further progress at HERA, the LHC and at future facilities.

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