

de Sitter vacua from an anomalous gauge symmetry

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Abstract

We find a new class of metastable de Sitter solutions in compactifications of six-dimensional supergravity motivated by type IIB or heterotic string vacua. Two Fayet-Iliopoulos terms of a local $U(1)$ symmetry are generated by magnetic flux and by the Green-Schwarz term canceling the gauge anomalies, respectively. The interplay between the induced D-term and a nonperturbative superpotential stabilizes the moduli and determines the size of the extra dimensions.

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1 Introduction

The observed accelerated expansion of the universe [1] is most easily explained by a constant vacuum energy density, the characteristic feature of de Sitter space. This has led to an intense search for de Sitter vacua in supergravity and superstring theories, which represent attractive extensions of the standard model of particle physics. Although much progress has been made, challenging questions still remain [2].

A particularly attractive proposal by Kachru, Kallosh, Linde and Trivedi [3] is based on quantized fluxes in type IIB string theory and additional nonperturbative effects stabilizing the overall volume modulus. These features are captured in the KKLT superpotential that yields an anti-de Sitter minimum. To achieve the required ‘uplift’ to de Sitter space additional ingredients are needed such as anti-D3 branes [3, 4], D -terms induced by magnetic flux [5–7] or constant Fayet-Iliopoulos terms [8]. Alternatively, additional matter fields have to be added allowing for an F-term uplift (see e.g. [9, 10]). Also extensions of the KKLT proposal, such as the Large Volume Scenario [11] and Kähler Uplift [12, 13] need additional ingredients to achieve an uplift to de Sitter vacua.

Necessary conditions for de Sitter vacua in four-dimensional supergravity theories derived from string theory compactifications have been systematically analyzed in [14]. In particular, constraints on the Kähler potential and, in case of gauged shift symmetries, on the associated Fayet-Iliopoulos (FI) terms have been derived [15]. For a related discussion of de Sitter and Minkowski vacua in string theory see, for instance, [6, 16–19].

In this Letter, we study de Sitter vacua in six-dimensional supergravity models, compactified on orbifolds with flux. Such models have been suggested as an intermediate step in string compactifications to four dimensions [20]. Our results represent an application of previous work [21, 22]. The crucial ingredients are the flux and the Green-Schwarz term of an anomalous local $U(1)$ symmetry. They lead to two Fayet-Iliopoulos terms of opposite sign, and they modify the gauge kinetic function. Together with a nonperturbative KKLT-type superpotential one naturally obtains de Sitter vacua, without any additional degrees of freedom below the flux induced mass scale. In the following we derive relations between the superpotential parameters and de Sitter vacua in moduli space and give an explicit example.

2 Conditions for de Sitter vacua

In this work we consider a simple $\mathcal{N} = 1$ supergravity model¹ that has been derived as the effective four-dimensional Lagrangian for flux compactifications of six-dimensional supergravity [21, 22]. It contains three chiral multiplets, S , T , U and the real vector multiplet V of a $U(1)$ gauge symmetry. The Kähler potential is given by

$$K = -\log(S + \bar{S} + iX^S V) - \log(T + \bar{T} + iX^T V) - \log(U + \bar{U}). \quad (1)$$

¹We follow the conventions of [23].

where $X^{S,T}$ are purely imaginary and constant Killing vectors parameterizing a gauged shift symmetry. Note that the Kähler potential is of no-scale type, i.e. $K_i K^{i\bar{j}} K_{\bar{j}} = 3$, where i ($\bar{j}$) denotes the derivative with respect to the i -th chiral ($\bar{j}$ -th anti-chiral) multiplet and $K^{i\bar{j}}$ is the inverse Kähler metric.

To completely define the gauge sector one further has to specify the gauge kinetic function H , whose real part h corresponds to the effective gauge coupling. In the following it will be crucial that, in addition to the classical term linear in S , the gauge kinetic function contains a second part linear in T ,

$$H = h_S S + h_T T. \quad (2)$$

A T -dependence is known to arise due to quantum corrections [24, 25]. Following the convention $S = \frac{1}{2}(s + ic)$, $T = \frac{1}{2}(t + ib)$, one has $h = \frac{1}{2}(h_S s + h_T t)$. Finally, we have to specify the superpotential. It has to be gauge invariant and can therefore only depend on the linear combination

$$Z = -iX^T S + iX^S T \equiv \frac{1}{2}(z + i\tilde{c}), \quad (3)$$

where in a first step, we ignore the shape modulus U . For reasons discussed above we consider a KKLT-type potential,

$$W(Z) = W_0 + W_1 e^{-aZ}, \quad (4)$$

where, w.l.o.g. we choose $a > 0$, W_0 and W_1 to be real.

In the following it will be important that the parameter h_T in the gauge kinetic function is negative while the classical contribution h_S is positive. Furthermore, the two Killing vectors have to be of opposite sign. As we shall see in the next section, both conditions are satisfied in flux compactifications of 6d supergravity. The line along which $h(s, t) = 0$ divides the moduli space into a physical ($h > 0$) and an unphysical region ($h < 0$). The physical region thus satisfies the condition

$$t < t_{(h)} = -\frac{h_S}{h_T} s. \quad (5)$$

The scalar potential is a sum of F - and D -term contributions,

$$V = V_F + V_D = e^K (K^{i\bar{j}} D_i W D_{\bar{j}} \bar{W} - 3|W|^2) + \frac{g^2}{2h} D^2, \quad (6)$$

where $D_i W = W_i + K_i W$ is the Kähler covariant derivative of the superpotential and g is a gauge coupling. The D -term is given in terms of the Killing vectors,

$$D = iK_i X^i = -\frac{i}{s} X^S - \frac{i}{t} X^T. \quad (7)$$

Due to their opposite sign $D = 0$ defines a second line in (s, t) space given by

$$t_{(D)} = -\frac{X^T}{X^S} s. \quad (8)$$

If this line is not part of the physical region in moduli space, i.e. $|X^T/X^S| \geq |h_S/h_T|$, the D -term potential is positive definite. This is the situation we want to study in the following.

Solutions of the equations of motion leading to Minkowski or de Sitter vacua have to fulfill,

$$\partial_S V = 0, \quad \partial_T V = 0, \quad V = \epsilon \geq 0, \quad (9)$$

which yield three relations between the F -term and D -term contributions to the potential. It is convenient to use the linear combinations $\partial_{\pm} = s\partial_S \pm t\partial_T$ instead of $\partial_{S,T}$. Derivatives of the superpotential then produce the factors $\partial_- Z = stD$ and $\partial_+ Z = stE$, where

$$E = iK_T X^T - iK_S X^S = -\frac{i}{t} X^T + \frac{i}{s} X^S. \quad (10)$$

In terms of D and E the scalar potential can be written in the compact form

$$V = \frac{st}{2}(D^2 + E^2)A - EB + \frac{g^2}{2h}D^2, \quad (11)$$

with

$$A = |\partial_Z W|^2 = a^2 W_1^2 e^{-az}, \quad (12)$$

$$B = 2\text{Re}(\partial_Z W \overline{W}) = -2aW_1 \left(W_1 e^{-az} + W_0 e^{-\frac{a}{2}z} \cos\left(\frac{a}{2}\tilde{c}\right) \right). \quad (13)$$

Note that the term $-3|W|^2$ in the scalar potential (6) has disappeared due to the no-scale structure of the Kähler potential. Choosing opposite signs for W_0 and W_1 , the axion $\tilde{c}$ is stabilized at zero, and in the following we set the cosine term to one. The orthogonal axion gives a mass to the $U(1)$ vector field via the Stueckelberg mechanism [21].

In order to find minima we invert the problem and solve for the superpotential parameters in terms of s and t . In this way we can see which superpotential parameters lead to minima in the (s, t) plane for realistic parameter ranges. The conditions for extrema (9) yield three relations between the superpotential parameters and the position

(s, t) of the extrema in moduli space. A straightforward calculation leads to

$$\begin{aligned} A &= -\frac{1}{2h^2st(1-\rho^2)} \left(h_T t \rho + h(2-\rho+\rho^2) + h^2 \frac{2\epsilon}{E^2} \right), \\ B &= -\frac{E}{4h^2(1-\rho^2)} \left(h_T t \rho(1+\rho^2) + h(2-\rho+\rho^2-\rho^3+3\rho^4) + h^2 \frac{8\epsilon}{E^2} \right), \\ a &= -\frac{2E(1-\rho^2)}{st} \frac{h_T t \rho + h(2-\rho-3\rho^2)}{E^2(h_T t \rho(1+\rho^2) + h(2-\rho+5\rho^2-\rho^3-\rho^4)) + 8h^2\rho^2\epsilon}, \end{aligned} \quad (14)$$

where we have introduced the ratio $\rho = D/E \in (-1, 1)$. The quantities on the r.h.s. of these equations, h , E and ρ , only depend on the values of the moduli fields s and t , whereas A , B and a are related to the superpotential by the definitions (12) and (13).

Eqs. (14) can now be used to identify consistency conditions for the existence of de Sitter vacua and to construct explicit solutions. A similar discussion for pure F -term breaking has recently been carried out in [26]. From Eq. (12) we see that A has to be positive, and by definition $a > 0$. Furthermore, $B > 0$ is needed for a cancellation of the positive contributions in the potential (11), thus allowing for a vanishing or small cosmological constant. From Eqs. (14) one reads off that these conditions can be satisfied if $h_T < 0$. The reason is that the terms proportional to h , which would give the unwanted sign, can be made small due to the opposite sign of the two contributions $h_S s$ and $h_T t$. Given a solution for a chosen extremum at (s, t) , one then has to examine whether the obtained values for h , a , W_0 and W_1 are physically meaningful.² Let us finally emphasize again that the obtained Minkowski and de Sitter minima are a consequence of the opposite signs of the moduli contributions to both the gauge kinetic function and the D -term in combination with the nonperturbative superpotential.

In the de Sitter vacua constructed this way supersymmetry is broken by a D -term since, as discussed above, $D > 0$ in the physical region of the moduli space. Vanishing F -terms, i.e. $D_S W = D_T W = 0$, would imply $D\partial_Z W = 0$, which is not possible for $D > 0$ and a KKLT-type superpotential. Hence, as expected, supersymmetry is also broken by F -terms and the Goldstino is a mixture of the gaugino and modulini, with $m_{3/2} \sim \langle F \rangle \sim \langle D \rangle$. It is well known that no-scale models do not allow metastable de Sitter vacua for arbitrary superpotentials [14]. In the above analysis this no-go theorem is avoided by means of a gauged shift symmetry, a possibility which has already been discussed previously in [15].

3 An example

In [22] we derive the low-energy effective action for a six-dimensional supergravity model with a $U(1)$ gauge field and a charged bulk matter field, compactified on an

²We have checked that these conclusions remain valid if the modulus U is included in the analysis, with an additional term $W_2 \exp(-a'U)$, in the superpotential (4). However, the resulting equations are considerably more cumbersome than Eqs. (14). They will be given in [22].

orbifold $T^2/\mathbb{Z}_2$ with magnetic flux. The antisymmetric tensor field couples to the $U(1)$ gauge field via a Chern-Simons term. The four-dimensional effective action involves the Kähler potential (1) with a gauged shift symmetry in S and T . The Killing vectors are given by

$$X^T = -i\frac{f}{\ell^2}, \quad X^S = -ig^2\frac{N+1}{(2\pi)^2}, \quad (15)$$

where f is the quantized flux, $f = -4\pi N < 0$, $N \in \mathbb{N}$, and ℓ is a length scale in Planck units. X^S is a consequence of the Green-Schwarz term which is needed to cancel bulk and fixed point anomalies. It represents a quantum correction which, to our knowledge, has been neglected in previous analyses of moduli stabilization. Note that the two Killing vectors contribute to the D -term with opposite signs: $-iX^T > 0$, $-iX^S < 0$.

The orbifold compactification with N flux quanta generates $N+1$ massless Weyl fermions, as discussed in [21]. Their scalar superpartners ϕ_i obtain masses from the D -term potential

$$V_D = \frac{g^2}{2h} \left(\sum_{i=1}^{N+1} |\phi_i|^2 + D \right)^2, \quad (16)$$

with D given by Eq. (7). The stabilization of the charged matter fields ϕ_i at the origin requires $D > 0$, as assumed in the previous section. Note that all matter fields ϕ_i have the same $U(1)$ charge since they originate from the same bulk field. The size of D determines the mass splitting within matter multiplets and therefore the scale of supersymmetry breaking [27]. Furthermore, we want to point out that the quantity E introduced above is always positive, $E = iX^S/s - iX^T/t > 0$, and $\rho = D/E \in (0, 1)$.

As discussed in the previous section, the existence of de Sitter minima crucially depends on the gauge kinetic function for which one obtains [22]

$$h_S = 2 \quad h_T = -\frac{2g^2\ell^2}{(2\pi)^3}. \quad (17)$$

At the classical level, the gauge kinetic function only depends on S . The contribution proportional to T is a direct consequence of the Green-Schwarz term, and it is remarkable that the sign of h_T is indeed negative. Note that for X^T , X^S , h_S and h_T as given in Eqs. (15) and (17), $h > 0$ always implies $D > 0$ for $N \geq 1$. Hence, for nonzero flux, the matter fields are always stabilized at the origin.

We are now ready to apply the general analysis of the previous section to our example. For simplicity, we set $\epsilon = 0$ in Eqs. (14) and look for Minkowski vacua. Obviously, there exist very similar solutions for de Sitter vacua with $\epsilon \gtrsim 0$. Having GUT scale extra dimensions and three generations in mind [28], we choose $\ell = 50$, $g = 0.2$ and $N = 3$. Using Eqs. (14) and writing $az \equiv a_S(s + \kappa t)$, we find for a

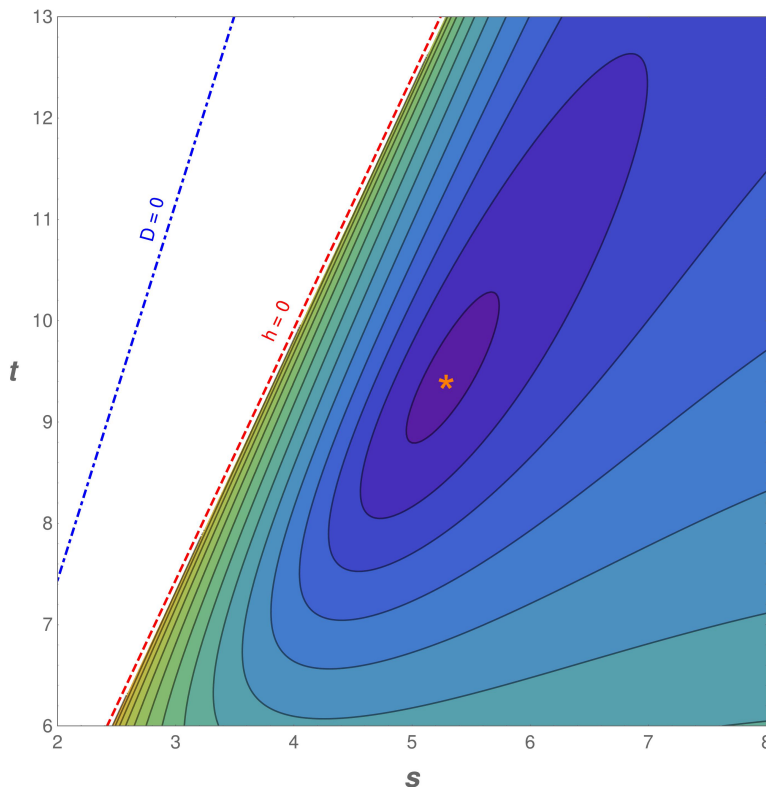


Figure 1: Minkowski minimum for the superpotential parameters (18). h vanishes along the red dashed line (5) and D vanishes along the blue, dashed-dotted line (8). The minimum is at $(s, t) = (5.3, 9.4)$ as indicated by the orange star.

Minkowski minimum at $(s, t) \simeq (5.3, 9.4)$ typical superpotential parameters,

$$a_S \simeq 0.63, \quad W_0 \simeq -9.1 \times 10^{-4}, \quad W_1 \simeq 6.9 \times 10^{-4}. \quad (18)$$

For the effective gauge coupling we obtain $g_{\text{eff}} = gh^{-1/2} \simeq 0.16$, and the size of the compact dimension is $V_2 = \ell^2 \sqrt{st}/(2g^2) = (7 \times 10^{15} \text{GeV})^{-2}$, implying GUT scale supersymmetry breaking. Note that also larger extra dimensions can be obtained without fine tuning for larger values of h , i.e. for smaller gauge couplings and smaller superpotential parameters W_0, W_1 . The masses of all moduli are of order 10^{14}GeV . In Fig. 1 the scalar potential is shown close to the Minkowski minimum. It diverges as s and t approach the critical line $h = 0$.

4 Discussion

We have presented a new class of metastable de Sitter vacua. They arise in no-scale supergravity models with a gauged shift symmetry involving two moduli fields, S and T . This leads to two Fayet-Iliopoulos terms with different moduli dependence. Also

the gauge kinetic function has contributions linear in S and T . The superpotential of KKLT-type depends on the gauge invariant linear combination of S and T . As we have shown, de Sitter or Minkowski vacua are naturally obtained if the T -dependent contribution to the gauge kinetic function is negative.

It is remarkable that the low-energy effective action of six-dimensional supergravity with flux, compactified on an orbifold, fulfills the sufficient conditions for de Sitter vacua. The T -dependent FI term is generated by magnetic flux, whereas the S -dependent FI term and the T -dependent contribution to the gauge kinetic function are due to the Green-Schwarz term needed to cancel the $U(1)$ gauge anomaly. The Green-Schwarz-term leads to the negative sign of the T -dependent part in the gauge kinetic function.

Flux and anomaly cancellation by the Green-Schwarz mechanism are generic ingredients of string compactifications. We therefore believe that our discussion of de Sitter vacua is relevant far beyond the six-dimensional supergravity example discussed in this Letter. It will be interesting to study applications for the heterotic string as well as for type IIB string theory and F-theory. Furthermore, it is intriguing that typical values for the superpotential parameters and the gauge coupling lead to GUT scale extra dimensions and a related large supersymmetry breaking scale. This is a challenge for the electroweak hierarchy problem, analogous to the cosmological constant problem which is encoded in the fine tuning of the superpotential parameters.

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