Proton EDM Simulations Using Fourth-Order Runge-Kutta Integration

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High intensity, polarized protons can be stored in an all-electric ring to probe the proton electric dipole moment. The expected sensitivity level is $10^{-29}e\cdot$cm, better than any planned hadronic EDM experiment by one to two orders of magnitude. The protons at their magic momentum of 0.7 GeV/c keep their spin vectors aligned with their momentum vectors in any transverse electric field for the duration of the storage time. However, not all protons within the acceptance of the ring, are exactly at the magic momentum. An RF-cavity is used to eliminate the first order effects responsible for the spread of angles between the spin and momentum vectors. We have used particle simulations and fourth-order Runge-Kutta integration to determine the second order effects. The spin coherence time, a measure of the angle spreads, are found to be adequate for a total storage time of 10^3 s. Further improvements, using sextupole magnets and/or stochastic cooling, could bring another order of magnitude improvement in the statistical sensitivity.

1 Introduction

High intensity (of order 10^{11}) polarized proton and deuteron beams have been used successfully for several decades in storage rings around the world. The experience gained using those beams provides a unique opportunity in the study of the electric dipole moment (EDM) of the proton and deuteron. The storage ring EDM collaboration has recently completed the proposal for a proton EDM experiment with a sensitivity of $10^{-29}e\cdot$cm. The method is using protons at their magic momentum of 0.7 GeV/c in an all-electric storage ring [1]. Together with the neutron [2], the proton and deuteron EDM experiments will be able to decipher the CP-violating source should one particle is found to have a non-zero EDM. At $10^{-29}e\cdot$cm, they are testing the electroweak Baryogenesis model, and will constrain it severely in case they are found to be consistent with zero. For the experimental method and the physics reach of the proton, deuteron, and muon EDM experiments, see [1, 3, 4, 5, 6, 7].

Three discrete symmetries play an important role in the standard model (SM): Parity (P), time (T) reversal and charge (C) symmetry. The interaction energy of a particle under an electric field is given by: $H = -d(\vec{s}/s)\cdot\vec{E}$ where d is the magnitude of EDM, $\vec{E}$ is the electric field vector and $\vec{s}$ is the spin vector. This equation implies that the EDM vector is aligned with the spin vector, which has its origins from the requirement of non-degeneracy in quantum mechanics. Since $\vec{E}$ is a polar vector and $\vec{s}$ is an axial vector, the Hamiltonian is odd under P and T transformations. Therefore, a nonzero EDM means that both P and T symmetries are violated. Assuming CPT conservation, T-violation means CP-violation as well. Andrei Sakharov [8] first pointed out that CP-violation is one of the essential ingredients needed to make the baryon-antibaryon asymmetric universe we observe today from an initially symmetric one. Even though CP-violation has been observed in weak decays it is not nearly enough, by some eight orders of magnitude, to explain this baryon-antibaryon asymmetry. Therefore, a much stronger source of CP-violation is needed and if a proton EDM is observed it could provide it. Physics beyond the SM predicts new CP-violating phases well within the sensitivity level of the proton EDM experiment. The CP-violating phase in the weak interactions predicts a much smaller EDM for the proton, in the range of $10^{-31} - 10^{-33} e \cdot \mathrm{cm}$.

2 Frozen Spin Method

A charged particle in an electric field region will accelerate and get lost unless the electric field is compensated by another force. For particles in storage rings the electric field is compensated by the centripetal force. The storage ring EDM method can at the same time store a large number of polarized particles and arrange for maximum EDM sensitivity under certain conditions [1, 3, 4, 5, 7]. If the spin and the momentum of the particle precess at the same rate, then there will be a radial E-field in the particle rest frame acting on its EDM vector precessing it out of plane. The method, using high intensity proton beams (4×10^{10} per cycle) with small momentum spread $(dp/p)_{\mathrm{rms}} = 2 \times 10^{-4}$, is found to hold the most promise [1].

The general T-BMT equations give the spin and velocity precession of a relativistic particle in the presence of both E and B fields:

$$\frac{d\vec{s}}{dt} = \frac{e}{m}\vec{s} \times \left[\left(\frac{g}{2} - \frac{1-\gamma}{\gamma} \right) \vec{B} - \left(\frac{g}{2} - 1 \right) \frac{\gamma}{\gamma+1} \left(\vec{\beta} \cdot \vec{B} \right) \vec{\beta} - \left(\frac{g}{2} - \frac{\gamma}{\gamma+1} \right) \frac{\vec{\beta} \times \vec{E}}{c} \right] \tag{1}$$

$$\frac{d\vec{\beta}}{dt} = \frac{e}{\gamma m} \left[\vec{\beta} \times \vec{B} + \frac{\vec{E}}{c} - \vec{\beta} \frac{\left(\vec{\beta} \cdot \vec{E} \right)}{c} \right] \tag{2}$$

where β, e and m are the velocity, charge and mass of the particle respectively, $\gamma = \sqrt{1-\beta^2}$, c is the speed of light and g is the g-factor for spin, which is $g = 2.792847356(23)$ for the proton. Setting $B = 0$, and assuming $(\vec{\beta} \cdot \vec{E} = 0)$, the so-called $g-2$ precession rate, i.e., the precession rate of the angle between the spin and momentum vectors, is:

$$\vec{\omega}_a = \frac{e}{m} \left[\frac{1}{\gamma^2 - 1} - a \right] \frac{\vec{\beta} \times \vec{E}}{c} \tag{3}$$

where $a = (g-2)/2$ is the anomalous magnetic moment. Setting $\gamma = \sqrt{1/a + 1}$ locks the angle between the spin and momentum vectors as a function of time. This method is called the "frozen spin" method. This specific case leads to a "magic" momentum value for the proton: $p_0 = m/\sqrt{a} = 0.707$ GeV$/c$.

For a particle at rest, E-fields only couple to EDMs (d) and magnetic fields only to magnetic dipole moments (μ), with the spin precession given by: $d\vec{s}/dt = \vec{d} \times \vec{E} + \vec{\mu} \times \vec{B}$. When the magnetic field is zero and the particle is at its magic momentum, the spin precession is affected only by EDM: $d\vec{s}/dt = \vec{d} \times \vec{E}$ causing a *vertical* spin precession for the duration of the storage

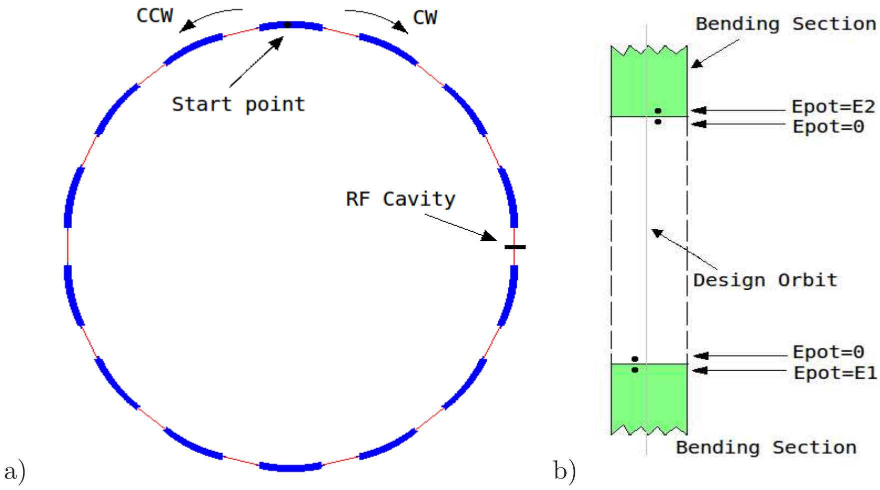

Figure 1: a) The ring includes 14 bending and straight sections, with one RF cavity. b) At the interface of bending and straight sections, kinetic and potential energy changes are made to keep the total energy conserved.

time. Since not all particles will be exactly at the "magic" momentum there is going to be a spread in the spin angles relative to their momentum vectors. The linear spread in angles is cancelled by using an RF-cavity in a straight section. Next we present a simulation that is sensitive to the second order effects.

The simulated ring is composed of 14 bending and straight sections (see Fig. 1 (a)). The assumed ring radius is $r = 40$ m and the radial electric field is 10.5 MV/m for a 3 cm plate separation. The electric field is provided by cylindrical plates, slightly modified to provide vertical focusing. The vertical tune was 0.2, while the E-field gradients comply with Maxwell's equations up to fourth order. One of the straight sections includes an RF cavity. In the bending section, the position and spin of the particle are estimated by integrating Eqs. 1, 2 using the fourth order Runge-Kutta method [9]. At the boundaries of the straight sections, the fringe field is approximated with a sharp transition from a field region to no-field region (see Fig. 1 (b)). The potential energy of the particle is converted into kinetic energy at the entrance of the straight section. This is done by changing the longitudinal velocity of the particle, which is a good approximation. At the other end, when the particle reaches the bending section, its kinetic energy is again changed and shared again by kinetic and potential energy, according to its position. For example a particle at 1.5 cm away from the design orbit has a potential energy of about 150 keV. This corresponds to $dp/p = 3.5 \times 10^{-4}$.

Figure 2 shows the angle for $x_0 = z_0 = 0$ and $dp/p = 2 \times 10^{-4}$ for a ring with 28 m of total straight section length. It shows that the average spin precession rate is less than 0.004 rad/s for particles at the edge of the ring acceptance. Defining spin coherence time (SCT) as the time needed for the angle between spin and momentum to become 1 rad, Fig. 2 gives more than 250 seconds of SCT for particles off the "magic" momentum. Since those particles can be taken out at the polarimeter detector at early times, this finding supports a storage time of 10^3 s for each cycle without the need of compensating sextuples. Using sextupoles and/or stochastic cooling the obtained SCT can be two orders of magnitude larger, further increasing the sensitivity of the

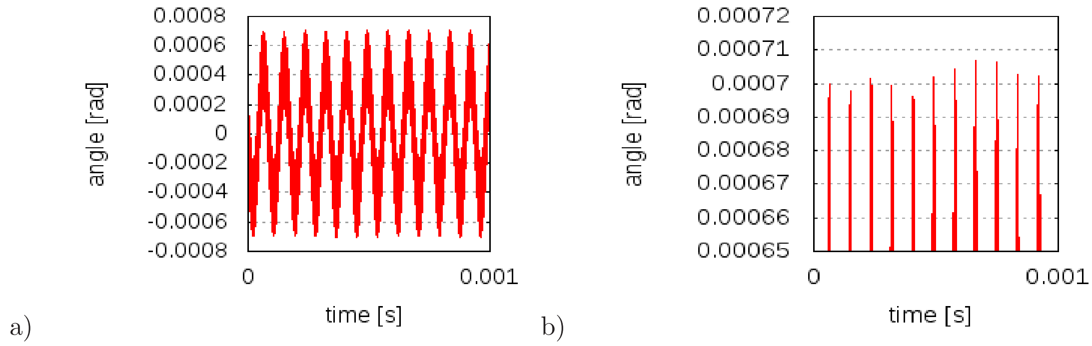

Figure 2: a) The angle between the spin and momentum vectors for $dp/p = 2 \times 10^{-4}$ and 28 m of total straight section length. b) Zoom-in the top region of (a) shows that the angle fluctuates but remains constant to about 0.25×10^{-5} radians over 1ms.

experiment by another order of magnitude. Finally, using the same tracking simulation, we have determined the acceptance of the ring to be: $\epsilon = x'^2 \frac{R}{Q_h} = (0.43 \times 10^{-3})^2 \times \frac{40}{1.3} = 5.6\,\mathrm{mm\,mrad}$.

3 Conclusion

The simulation results show that the proposed frozen spin method, using "magic" momentum protons in an all-electric ring, provides a SCT that is adequate for a storage time of 10^3 seconds without the need of compensating sextupoles. In addition, for an all-electric ring with 3 cm aperture, the acceptance of the ring is found to be 5.6 mm mrad. In a future upgrade, applying compensating sextupoles and/or stochastic cooling could further improve the statistical sensitivity of the method by another order of magnitude.

References

[1] V. Anastassopoulos *et al.* [Storage Ring EDM Collaboration], "A Proposal to Measure the Proton EDM with $10^{-29} e \cdot \mathrm{cm}$ Sensitivity", available from the collaboration web site: http://www.bnl.gov/edm.

[2] There are several planned neutron EDM experiments: at the spallation neutron source (SNS) at Oak Ridge, Tennessee; at ILL at Grenoble, in France; at PSI, Villigen, Switzerland; and at more places under planning. Their sensitivity goal is between $10^{-27} e \cdot \mathrm{cm}$ and $10^{-28} e \cdot \mathrm{cm}$, limited mostly by statistics.

[3] Y.K. Semertzidis *et al.*, "Sensitive Search for a Permanent Muon EDM", proceedings of HIMUS99 workshop, Tsukuba, Japan, (1999), [arXiv:0012087 [hep-ph]].

[4] Y.K. Semertzidis, "A New Method for a Sensitive deuteron EDM Experiment", [arXiv:0308063 [hep-ex]].

[5] F.J.M. Farley *et al.*, Phys. Rev. Lett. **93**, 052001 (2004), [arXiv:0307006 [hep-ex]].

[6] Y.F. Orlov, W.M. Morse, and Y.K. Semertzidis, Phys. Rev. Lett. **96**, 214802 (2006).

[7] http://www.bnl.gov/edm where the muon EDM Letter of Intent to J-PARC, and the deuteron, and proton EDM proposals are available.

[8] A.D. Sakharov, JETP Lett. **5**, 24 (1967).

[9] W.H. Press, S.A. Teukolsky, W.T. Vetterling, Numerical Recipes, (2007).