Z' from GUTs, CPs, Axions, and the μ Problem

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We can think of a few interesting topics beyond the standard model(SM), “Are there a new U(1)$'$, axions, supersymmetry(SUSY), and string effects?” Here, I discuss my recent works related to U(1)$'$, the weak CP violation, and the μ related problems.

The most pending question beyond the standard model(BSM) is, “Is there a new U(1)$'$ at the TeV scale?” The SM is fitted to the vast electroweak data very successfully without a need for an extra neutral gauge boson(s). There already exist a vast references on the extra Z' [1]. A strongly motivated framework for extra Z' bosons is the grand unified thoeies(GUTs). The ranks of the SO(10) and E$_6$ are greater than 4 and hence in those GUTs there can exist an extra Z' boson(s) at the TeV scale if the spontaneous symmetry breaking of the GUTs leaves them light. In Ref. [2], we have shown that it is improbable to have a TeV scale Z' if the GUT group is a subgroup of E$_6$. This is a very simple remark since any generator corresponding to the U(1)$_{Z'}$ from E$_6$ belongs to the Cartan subalgebra of E$_6$ and is a linear combination of the diagonal E$_6$ generators. It is equivalent to considering the Cartan subalgebra of a rank 6 subgroup of E$_6$. For this purpose, the SU(6)×SU(2)$_h$subgroup of E$_6$ is very convenient because the SU(6)×SU(2)$_h$ quantum numbers can be read by representing $(\mathbf{15},\mathbf{1})$ and $(\overline{\mathbf{6}},\mathbf{2})$ like matrices. There can be a “No-go theorem” for U(1)$_{\rm Baryon\ number}$. For U(1)$_X$ with $X\ne B$, there is a subtlety as shown below, but it is not likely that the mass of Z' is below 10 TeV. The LHC preliminary result with a light lightest SUSY particle(LSP) is consistent with this claim as reported at this Meeting [3]. On the contrary, if Z' is found below 10 TeV, our understanding of the SM from the subgroups of E$_6$ is not realized. In particular, the SU(5), SO(10), and SU(3)×SU(3)×SU(3), SU(6)×SU(2), and flipped SU(5) GUTs are not acceptable.

Let the baryon number generator be as commented in the subsequent paragraph.

$$B = a\,Y + b\,Y_6 + c\,X_3 + d\,R \tag{1}$$

where we included the global R charge also. But we will neglect R since it is broken by the supergravity effects. The notations in Eq. (1) are derived from SU(6)×SU(2). The generators F_3, F_8, T_3, Y, and Y_6 belong to the algebra of the vertical group SU(6) and X_3 belongs to the algebra of the horizontal group SU(2). The leptons and the Higgs doublets do not carry the baryon number. Their B charges according to Eqs. (1,3) are

$$e^c : a-\frac{b}{3}=0,\ \ (\nu_e,e): -\frac{a}{2}+\frac{b}{6}+\frac{c}{2}=0,\ \ H_d: -\frac{a}{2}+\frac{b}{6}-\frac{c}{2}=0,\ \ H_u: \frac{a}{2}+\frac{2b}{3}=0 \tag{2}$$

which cannot be satisfied simultaneously.

The SU(6) GUT model discussed in connection with the F-theory [4] has been known since early 1980s [5]. For the diagonal subgroups of E_6, any U(1) generator can be a linear combination of the Cartan subgroup of E_6. Therefore, we can prove a no-go possibility in terms of the Cartan subgroup of SU(6)×SU(2)⊂E_6, where SU(6) will be called vertical and SU(2) will be called horizontal as shown below for $(\mathbf{15},\mathbf{1})$ and $(\overline{\mathbf{6}},\mathbf{2})$ of SU(6)×SU(2)$_h$ for the first family,

$$\mathbf{15}_L \equiv (\mathbf{15},\mathbf{1}) = \begin{pmatrix} 0 & u^c & -u^c & u & d & D \\ -u^c & 0 & u^c & u & d & D \\ u^c & -u^c & 0 & u & d & D \\ -u & -u & -u & 0 & e^c & H_u^+ \\ -d & -d & -d & -e^c & 0 & H_u^0 \\ -D & -D & -D & -H_u^+ & -H_u^0 & 0 \end{pmatrix},$$

$$\overline{\mathbf{6}}_{\mathbf{2},1} \equiv (\overline{\mathbf{6}},\mathbf{2}^{\uparrow}) = \begin{pmatrix} d^c \\ d^c \\ d^c \\ -\nu_e \\ e \\ N \end{pmatrix},\ \overline{\mathbf{6}}_{\mathbf{2},2} \equiv (\overline{\mathbf{6}},\mathbf{2}^{\downarrow}) = \begin{pmatrix} D^c \\ D^c \\ D^c \\ -H_d^0 \\ H_d^- \\ N' \end{pmatrix}. \tag{3}$$

Equation (1) for B is equivalent to discussing a U(1) subgroup of E_6. Thus, B cannot be a generator belonging to E_6. On the other hand, three conditions except the H_u condtion in Eq. (2) can be satisfied, which is called the *leptophobic* case. H_u carries a nonvanishing Y' and also N of Eq. (3) carries a nonvanishing Y'. Therefore, the singlet neutrino mass scale is the Z' mass scale. In this case, we consider Z' couplings both to B and L. We considered SU(6)×SU(2) for the Z_6 hexality and the $Z-Z'$ mass (with fine tuned coupling constants). In the latter case, of course we assumed the lepton coupling to Z' as phrased above as *Z' couplings both to B and L*. Then, the LEP2 precision experiment bound on the ρ parameter is crucial to constrain the model [2], but the leptophobic case in terms of kinetic mixing softens this condition.

Another issue going beyond the SM is to understand how the weak CP violation is realized at the high energy scale. CP violation observed in the K-meson and the B-meson systems is given by the Cabibbo-Kobayashi-Maskawa(CKM) matrix [6]. Recently, we presented an exact CKM matrix [7], replacing the approximate Wolfenstein form. It is worthwhile to write any convenient form if it is useful for obtaining some information on the high energy scale physics. The well-known facts about the CKM matrix are: (1) Det. $V_{\rm CKM}$ is better to be real, (2) the 3×3 $V_{\rm CKM}$ is complex to describe the weak CP violation, (3) any among the nine elements is zero, then there is no weak CP violation, (4) there is a good expansion parameter λ, (5) the product of the elements $V_{\rm CKM(31)}\cdot V_{\rm CKM(22)}\cdot V_{\rm CKM(13)}$ is the barometer of weak CP violation, and (6) eventually, $V_{\rm CKM}$ will be derivable from the Yukawa texture. The fact (1) is related to the issue of Arg.Det.M_q which hints a relation to the strong CP problem [8]. If Arg.Det.M_q is not zero, then we remove this to define a good quark mass basis, using the PQ symmetry [9] or by some other mechanism [8]. But this reality condition is not absolutely necessary, but only a very convenient choice. As done with the expansion parameter λ, the expansion can be achieved in terms of the $V_{\rm CKM(12)}$ angle θ_1 since θ_2 and θ_3 are known to be of order θ_1^2. If the CKM matrix is expanded in terms of θ_1 instead of λ, it is easy to write an exact form. Now we can write an exact CKM matrix, satisfying all the above requirements [7],

$$V_{\rm KS} = \begin{pmatrix} c_1 & s_1c_3 & s_1s_3 \\ -c_2s_1 & e^{-i\delta}s_2s_3 + c_1c_2c_3 & -e^{-i\delta}s_2c_3 + c_1c_2s_3 \\ -e^{i\delta}s_1s_2 & -c_2s_3 + c_1s_2c_3e^{i\delta} & c_2c_3 + c_1s_2s_3e^{i\delta} \end{pmatrix} \tag{4}$$

where $s_i = \sin\theta_i$ and $c_i = \cos\theta_i$, and the parameters are determined as $\theta_1 = 13.0305^{\rm o} \pm 0.0123^{\rm o} = 0.227426 \pm (2.14\times 10^{-4})$, $\theta_2 = 2.42338^{\rm o} \pm 0.1705^{\rm o} = 0.042296 \pm (2.976\times 10^{-3})$, $\theta_3 = 1.54295^{\rm o} \pm 0.1327^{\rm o} = 0.027567 \pm (2.315\times 10^{-3})$, and $\delta = 89.0^{\rm o} \pm 4.4^{\rm o}$. The determinant is real, but its six elements are complex with the following δ dependent parts, $V_{11}V_{22}V_{33} \to 2c_1c_2c_3s_2s_3\cos\delta - c_1c_2c_3s_1^2s_2s_3e^{i\delta}$, $V_{11}V_{23}V_{32} \to 2c_1c_2c_3s_2s_3\cos\delta - c_1c_2c_3s_1^2s_2s_3e^{i\delta}$, $V_{12}V_{23}V_{31} \to -c_1c_2c_3s_1^2s_2s_3e^{i\delta}$, $V_{12}V_{21}V_{33} \to -c_1c_2c_3s_1^2s_2s_3e^{i\delta}$, $V_{13}V_{21}V_{32} = -c_1c_2c_3s_1^2s_2s_3e^{i\delta}$, $V_{13}V_{22}V_{31} \to -c_1c_2c_3s_1^2s_2s_3e^{i\delta}$. Each of the six products has the same imaginary part. Therefore, the weak CP violation is unambiguously signaled by the product of the CKM matrix elements, e.g. the imaginary part of $V_{\rm CKM(31)}\cdot V_{\rm CKM(22)}\cdot V_{\rm CKM(13)}$ is $-ic_1c_2c_3s_1^2s_2s_3\sin\delta$. One more merit is that one can read the Jarlskog triangles directly from Eq. (4). For a Jarlskog triangle, three lines of the Jarlskog triangle are given by three elements obtained from two columns(or two rows) of Eq. (4). Three Jarlskog triangles for columns are presented in Ref. [7].

The CKM matrix does not fix a BSM. The chief reason is that the unitary matrices of the right-handed fields, R, are not completely fixed. One interesting choice is $R = L$. In Ref. [10], $R = L$ is used to determine the maximal CP phase through the Froggatt-Nielsen mechanism.

The good choice of the phases such that Det.$V_{\rm CKM}$ = real is related to the PQ symmetry. The PQ symmetry needs heavy quarks [11] or two Higgs doublets [12]. Supersymmetry(SUSY) needs two Higgs doublets also, H_u and H_d. But, the PQ symmetry forbids the μH_uH_d term in the superpotential W, which is the so-called μ problem [13]. This is a serious problem challenging a TeV scale electroweak symmetry breaking. We must achieve the SU(2)×U(1) breaking at a TeV scale. This μ problem has been expressed in several different objectives: (1) The doublet-triplet splitting problem in SUSY GUTs, (2) "Is there the PQ symmetry?", (3) "How large is the μ term?", (4) The B_μ problem in the GMSB, (5) "Why is there only one pair of Higgs doublets?", etc. To forbid a GUT scale μ, the PQ or R symmetries have been used, e.g. $W = \mu H_uH_d$ is forbidden if $X_{\rm PQ}(H_u) = 1$ and $X_{\rm PQ}(H_d) = 1$. Generating a TeV scale μ is another problem. There are two well-known method generating a TeV scale μ [13, 14]. At this PATRAS Workshop, the axion solution is of common interest, employing a nonrenormalizable term using singlet fields $S_{1,2}$, $W \sim (1/M_P)S_1S_2H_uH_d$, with $\langle S_{1,2}\rangle \approx 10^{10-12}$ GeV. This leads to the very light axion and axion cosmology. The axion cosmology restricts the axion decay constant below 10^{12} GeV, but it also depends on the initial misalignment angle θ_1 as recently calculated in [15], taking into account the overshoot factor and the anharmonic correction.

String compactification has been used to study the axion, the μ problem and R-parity. Since there is no exact global symmetry in string, the U(1)$_{\rm PQ}$ and U(1)$_R$ symmetries (if needed) must be approximate. For the PQ symmetry, the coupling $c_{a\gamma\gamma}$ turns out to be small [16].

Finally, we comment on the realization of one Higgs doublet pair and an exact R-parity, by enlarging the electroweak group to SU(3)×U(1) at the GUT scale. It has been worked out in a $\mathbb{Z}_{12-I}$ orbifold compactification [17]. Under SU(3)$_c$×SU(3)$_W$, the left-handed three quark families appear as $3\,(\mathbf{3}_c, \mathbf{3}_W)$. To cancel SU(3)$_W$ anomaly, we must have nine $\overline{\mathbf{3}}_W$. Nine $\overline{\mathbf{3}}_W$'s split into three $\overline{\mathbf{3}}_W$'s, each constituting like (N, ν_e, e). The remaining six contain H_u and H_d: $T_u = (S^+, H_u^+, H_u^0)$ and $T_d = (S^0, H_d^0, H_d^-)$. Now, we note that H_u and H_d coupling must come from a product of $\overline{\mathbf{3}}$'s. Note that it is not possible to write a term with two $\overline{\mathbf{3}}$'s. A

SU(3)$_W$ invariant coupling is possible by multiplying three $\overline{\mathbf{3}}$'s with antisymmetric combinations, $\epsilon_{IJK}\overline{\mathbf{3}}^I\overline{\mathbf{3}}^J\overline{\mathbf{3}}^K$. Therefore, in the flavor space the $H_u - H_d$ mass matrix is antisymmetric and hence its determinant is zero. It presents a bosonic flavor symmetry of Higgs doublets, and in effect one Higgs doublet pair must be light [17]. In fact, we realize this type of realization in an F-theory compactification [4]. We had this in the orbifold compactification. Of course, SU(5) GUT cannot house SU(3)$_c\times$SU(3)$_W\times$U(1). To have a hexality, we must extend to an SU(6) GUT since the center of SU(6) is $\mathbb{Z}_6$. A hexality model, housing an exact R-parity, has been constructed in [4].

In conclusion, I presented three topics beyond the SM paying attention to my recent papers: There is no Z' below 10 TeV, otherwise our wisdom to the SM is in trouble, a useful suggestion for the CKM matrix, and a realization of one pair of Higgs doublets and an exact R-parity in the MSSM from a string compactification.

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