Distribution of linearly polarized gluons inside a large nucleus

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The distribution of linearly polarized gluons inside a large nucleus is studied in the framework of the color glass condensate. We find that the Weizsäcker-Williams distribution saturates the positivity bound at large transverse momenta and is suppressed at small transverse momenta, whereas the dipole distribution saturates the bound for any value of the transverse momentum. We also discuss processes in which both distributions of linearly polarized gluons can be probed.

1 Introduction

Recently, transverse momentum dependent parton distributions (TMDs) inside a nucleon have attracted a lot of interest. So far, the main focus of the field has been on quark TMDs. On the other hand, the available studies of (polarized) gluon TMDs are still rather sparse. Among them the distribution of linearly polarized gluons inside an unpolarized nucleon ($h_1^{\perp g}$ in the notation of Ref. [1]) is of particular interest. It is the only polarization dependent gluon TMD for an unpolarized nucleon, and is a time-reversal TMD implying that initial/final state interactions are not needed for its existence. This distribution, in principle, can be accessed through measuring azimuthal asymmetries in processes such as jet or heavy quark pair production in electron-nucleon scattering as well as nucleon-nucleon scattering, and photon pair production in hadronic collisions [2, 3, 4]. Moreover, it has been found that the linearly polarized gluon distribution may affect the transverse momentum distribution of Higgs bosons produced from gluon fusion [5, 6, 7]

It has long been recognized that the $k_\perp$ dependent unpolarized gluon distribution f_1^g plays a central role in small x saturation phenomena. Due to the presence of a semi-hard scale (the so-called saturation scale), $f_1^g(x, k_\perp)$ at small x can be computed using an effective theory which is also known as the color glass condensate (CGC) framework. There are two widely used $k_\perp$ dependent unpolarized gluon distributions with different gauge link structures: (1) the Weizsäcker-Williams (WW) distribution [8, 9, 10] describing the gluon number density, and (2) the so-called dipole distribution which appears, for instance, in the description of inclusive particle production in pA collisions [11, 12]. Moreover, it has shown that both types of $k_\perp$ dependent gluon distributions can be directly probed through two-particle correlations in various high energy scattering reactions [13, 14, 7].

In a recent paper [15], we extended the calculation of $f_1(x, k_\perp)$ to the case of $h_1^{\perp g}(x, k_\perp)$.

By following the procedure outlined in [13, 14] we further demonstrated that the WW distribution and the dipole distribution can be accessed by measuring a $\cos 2\phi$ asymmetry for dijet production in lepton nucleus scattering and for production of a virtual photon plus a jet in nucleon nucleus scattering, respectively.

2 Distribution of linearly polarized gluons at small x

We first introduce the operator definition of the Weizsäcker-Williams gluon distribution inside a large nucleus [16, 1],

$$\begin{aligned} M^{ij}_{WW} &= \int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_\perp\cdot\vec{\xi}_\perp} \langle A|F^{+i}(\xi^- + y^-, \xi_\perp + y_\perp)\, L^\dagger_{\xi+y}\, L_y\, F^{+j}(y^-, y_\perp)|A\rangle \\ &= \frac{\delta^{ij}_\perp}{2}\, x f^g_{1,WW}(x,k_\perp) + \left(\frac{1}{2}\hat{k}^i_\perp \hat{k}^j_\perp - \frac{1}{4}\delta^{ij}_\perp\right) x h^{\perp g}_{1,WW}(x,k_\perp)\,, \end{aligned} \tag{1}$$

where $\hat{k}^i_\perp = k^i_\perp/|\vec{k}_\perp|$. Color gauge invariance is ensured by two (future-pointing) gauge links in the adjoint representation. We use

$$L_\xi = \mathcal{P}\, e^{-ig\int_{\xi^-}^{\infty^-} d\zeta^- A^+(\zeta^-,\xi_\perp)}\, \mathcal{P}\, e^{-ig\int_{\xi_\perp}^{\infty} d\vec{\zeta}_\perp\cdot\vec{A}_\perp(\zeta_\perp,\xi^-=\infty^-)}\,, \tag{2}$$

where $A^\mu = A^\mu_a t_a$ with $(t_a)_{bc} = -if_{abc}$, and f_{abc} denoting the structure constants of the $SU(3)$ group. We performed the calculation of the WW gluon distributions in the CGC framework in the light-cone gauge by following the standard procedure, and we obtained [15],

$$\begin{aligned} x h^{\perp g}_{1,WW}(x,k_\perp) &= \left(4\hat{k}^i_\perp\hat{k}^j_\perp - 2\delta^{ij}_\perp\right) M^{ij}_{WW} \\ &= \frac{N_c^2-1}{4\pi^3}\, S_\perp \int d|\xi_\perp| \frac{K_2(|k_\perp||\xi_\perp|)}{\frac{1}{4\mu_A}|\xi_\perp| Q_s^2}\left(1 - e^{-\frac{\xi_\perp^2 Q_s^2}{4}}\right). \end{aligned} \tag{3}$$

where K_2 is the second order Bessel function. Let us now discuss the above expression in the limit of high and low transverse momenta,

$$x h^{\perp g}_{1,WW}(x,k_\perp) \simeq 2S_\perp \frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{k_\perp^2} \qquad (k_\perp \gg Q_s)\,, \tag{4}$$

$$x h^{\perp g}_{1,WW}(x,k_\perp) \simeq 2S_\perp \frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{Q_s^2} \qquad (\Lambda_{QCD} \ll k_\perp \ll Q_s)\,. \tag{5}$$

On the other hand, in these limits the unpolarized gluon distribution takes the form [9, 10]

$$x f^g_{1,WW}(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{\mu_A}{k_\perp^2} \qquad (k_\perp \gg Q_s)\,, \tag{6}$$

$$x f^g_{1,WW}(x,k_\perp) \simeq S_\perp \frac{N_c^2-1}{4\pi^3}\frac{1}{\alpha_s N_c}\ln\frac{Q_s^2}{k_\perp^2} \qquad (\Lambda_{QCD} \ll k_\perp \ll Q_s)\,. \tag{7}$$

From those results one immediately finds that for large $k_\perp$ the distribution of linearly polarized gluons saturates the positivity limit, which in our notation reads $h_1^{\perp g} \leq 2 f_1^g$ [16]. In contrast, the ratio $h^{\perp g}_{1,WW}/f^g_{1,WW}$ is suppressed in the region of small $k_\perp$.

We now present the expression for the dipole distribution. In that case the operator definition reads [17, 13, 14]

$$
\begin{aligned}
M_{DP}^{ij} &= 2\int \frac{d\xi^- d^2\xi_\perp}{(2\pi)^3 P^+} e^{ixP^+\xi^- - i\vec{k}_\perp\cdot\vec{\xi}_\perp} \langle A|\mathrm{Tr}\, F^{+i}(\xi^- + y^-, \xi_\perp + y_\perp) U_{\xi+y}^{[-]\dagger} F^{+j}(y^-, y_\perp) U_{\xi+y}^{[+]}|A\rangle \\
&= \frac{\delta_\perp^{ij}}{2} x f_{1,DP}^g(x,k_\perp) + \left(\frac{1}{2}\hat{k}_\perp^i \hat{k}_\perp^j - \frac{1}{4}\delta_\perp^{ij}\right) x h_{1,DP}^{\perp g}(x,k_\perp)\,, \qquad (8)
\end{aligned}
$$

where $U_\xi^{[-]} = U^n(0,-\infty;0)U^n(-\infty,\xi^-;\xi_\perp)$ and $U_\xi^{[+]} = U^n(0,+\infty;0)U^n(+\infty,\xi^-;\xi_\perp)$ are gauge links in the fundamental representation. In covariant gauge, the only nontrivial component of the field strength tensor is $F^{+i}(y_\perp) = -\partial_\perp^i \alpha(y_\perp)$, which can be viewed as the realization of the eikonal approximation in the McLerran-Venugopalan model. By noticing this fact, one may easily obtain,

$$
x h_{1,DP}^{\perp g}(x,k_\perp) = 2x f_{1,DP}^g(x,k_\perp) = \frac{k_\perp^2 N_c}{\pi^2 \alpha_s} S_\perp \int \frac{d^2\xi_\perp}{(2\pi)^2} e^{-ik_\perp\cdot\xi_\perp} e^{-\frac{Q_{sq}^2 \xi_\perp^2}{4}}\,, \qquad (9)
$$

which means that the positivity bound is saturated for any value of $k_\perp$.

3 Observables

For the unpolarized case it has been shown that the results from the TMD factorization are in agreement with the results obtained by extrapolating the CGC calculation to the correlation limit [13, 14]. By applying a corresponding power counting in the correlation limit where the transverse momentum imbalance between two final state particles (or jets) is much smaller than the individual transverse momenta, we found a complete matching between the effective TMD factorization and the CGC calculation in the polarized case as well [15].

First, we discuss dijet production in lepton nucleus scattering. In fact, we consider the process $\gamma^* + A \to q(p_1) + \bar{q}(p_2) + X$ for both transversely and longitudinally polarized photons. The correct gluon TMD entering the factorization formula is the WW distribution. The calculation provides

$$
\begin{aligned}
\frac{d\sigma^{\gamma_T^* A\to q\bar{q}+X}}{dP.S.} &= \delta(x_{\gamma^*}-1) H_{\gamma_T^* g\to q\bar{q}} \Big\{ x f_{1,WW}^g(x,k_\perp) \\
&\quad - \frac{[z_q^2 + (1-z_q)^2]\epsilon_f^2 P_\perp^2 - m_q^2 P_\perp^2}{[z_q^2 + (1-z_q)^2](\epsilon_f^4 + P_\perp^4) + 2m_q^2 P_\perp^2} \cos(2\phi) x h_{1,WW}^{\perp g}(x,k_\perp) \Big\}\,, \qquad (10) \\
\frac{d\sigma^{\gamma_L^* A\to q\bar{q}+X}}{dP.S.} &= \delta(x_{\gamma^*}-1) H_{\gamma_L^* g\to q\bar{q}} \Big\{ x f_{1,WW}^g(x,k_\perp) + \frac{1}{2}\cos(2\phi) x h_{1,WW}^{\perp g}(x,k_\perp) \Big\}\,, \qquad (11)
\end{aligned}
$$

where $x_{\gamma^*} = z_q + z_{\bar{q}}$, with z_q, $z_{\bar{q}}$ being the momentum fractions of the virtual photon carried by the quark and antiquark, respectively. The phase space factor is defined as $dP.S. = dy_1 dy_2 d^2P_\perp d^2k_\perp$, where y_1, y_2 are rapidities of the two outgoing quarks in the lab frame. Moreover, $\vec{P}_\perp = (\vec{p}_{1\perp} - \vec{p}_{2\perp})/2$, and $\epsilon_f^2 = z_q(1-z_q)Q^2 + m_q^2$. The transverse momenta are defined in the $\gamma^* A$ cm frame. In the correlation limit, one has $|P_\perp| \simeq |p_{1\perp}| \simeq |p_{2\perp}| \gg |k_\perp| = |p_{1\perp} + p_{2\perp}|$. The (azimuthal) angle between $\vec{k}_\perp$ and $\vec{P}_\perp$ is denoted by ϕ. The hard partonic cross sections $H_{\gamma_{T,L}^* g\to q\bar{q}}$ can be found in Ref. [14]. The $\cos(2\phi)$-modulation of the cross section allows one to address the distribution of linearly polarized gluons. For intermediate values of x this was already pointed out in Ref. [3].

Let us now turn to the dipole distribution at small x. From a theoretical point of view, the simplest process to address $h_{1,DP}^{\perp g}$ seems to be back-to-back virtual photon plus jet production in pA collisions, i.e., $p + A \to \gamma^*(p_1) + q(p_2) + X$. The differential cross section, obtained in the effective TMD factorization, reads

$$\frac{d\sigma^{pA\to\gamma^* q+X}}{dP.S.} = \sum_q x_p f_1^q(x_p)\left\{ H_{qg\to\gamma^* q}^{UU}\, x f_{1,DP}^g(x,k_\perp) + \cos(2\phi) H_{qg\to\gamma^* q}^{\cos(2\phi)}\, x h_{1,DP}^{\perp g}(x,k_\perp) \right\}, \tag{12}$$

where the partonic cross sections are given by

$$H_{qg\to\gamma^* q}^{UU} = \frac{\alpha_s \alpha_{em} e_q^2}{N_c \hat{s}^2}\left(-\frac{\hat{s}}{\hat{u}} - \frac{\hat{u}}{\hat{s}} - \frac{2Q^2\hat{t}}{\hat{s}\hat{u}} \right), \qquad H_{qg\to\gamma^* q}^{\cos(2\phi)} = \frac{\alpha_s \alpha_{em} e_q^2}{N_c \hat{s}^2}\left(\frac{-Q^2\hat{t}}{\hat{s}\hat{u}} \right). \tag{13}$$

Here we used the partonic Mandelstam variables $\hat{s} = (p_1 + p_2)^2$, $\hat{u} = (p_1 - p)^2$ and $\hat{t} = (p_2 - p)^2$, with p denoting the momentum carried by the incoming quark from the proton. Note that this effect drops out for prompt (real) photon production.

4 Summary

We derived both the WW distribution and the dipole distribution of linearly polarized gluons in a large nucleus by using the CGC formalism. We further demonstrated that the WW and the dipole gluon distribution can be probed by measuring a $\cos 2\phi$ asymmetry for dijet production in DIS, and for virtual photon-jet production in pA collisions, respectively. Such observables can, in principle, be measured at a future Electron Ion Collider, at RHIC and the LHC.

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