On the pair correlations of neutral K, D, B and B_s mesons with close momenta produced in inclusive multiparticle processes

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The phenomenological structure of inclusive cross-sections of the production of two neutral K mesons in collisions of hadrons and nuclei is investigated taking into account the strangeness conservation in strong and electromagnetic interactions. Relations describing the dependence of the correlations of two short-lived and two long-lived neutral kaons $K_S^0 K_S^0$, $K_L^0 K_L^0$ and the correlations of "mixed" pairs $K_S^0 K_L^0$ at small relative momenta upon the space-time parameters of the generation region of K^0 and $\bar{K}^0$ mesons have been obtained. It is shown that under the strangeness conservation the correlation functions of the pairs $K_S^0 K_S^0$ and $K_L^0 K_L^0$, produced in the same inclusive process, coincide, and the difference between the correlation functions of the pairs $K_S^0 K_S^0$ and $K_S^0 K_L^0$ is conditioned by the production of the pairs of non-identical neutral kaons $K^0 \bar{K}^0$. Analogous correlations for the pairs of neutral heavy mesons D^0, B^0 and B_s^0, generated in multiple processes with the charm (beauty) conservation, are analyzed, and differences from the case of neutral K mesons are discussed .

1 Consequences of the strangeness conservation for neutral kaons

In the work [1] the properties of the density matrix of two neutral K mesons, following from the strangeness conservation in strong and electromagnetic interactions, have been investigated. By definition, the diagonal elements of the non-normalized two-particle density matrix coincide with the two-particle structure functions, which are proportional to the double inclusive cross-sections.

Strangeness is the additive quantum number. Taking into account the strangeness conservation, the pairs of neutral kaons $K^0 K^0$ (strangeness $S = +2$), $\bar{K}^0 \bar{K}^0$ (strangeness $S = -2$) and $K^0 \bar{K}^0$ (strangeness $S = 0$) are produced incoherently. This means that in the K^0-$\bar{K}^0$- representation the non-diagonal elements of the density matrix between the states $K^0 K^0$ and $\bar{K}^0 \bar{K}^0$, $K^0 K^0$ and $K^0 \bar{K}^0$, $\bar{K}^0 \bar{K}^0$ and $K^0 \bar{K}^0$ are equal to zero. However, the non-diagonal elements of the two-kaon density matrix between the two states $|K^0\rangle^{(\mathbf{p}_1)} |\bar{K}^0\rangle^{(\mathbf{p}_2)}$ and $|\bar{K}^0\rangle^{(\mathbf{p}_1)} |K^0\rangle^{(\mathbf{p}_2)}$ with the zero strangeness are not equal to zero, in general. Here $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the first and second kaons.

The internal states of K^0 meson ($S = 1$) and $\bar{K}^0$ meson ($S = -1$) are the superpositions of

the states $|K_S^0\rangle$ and $|K_L^0\rangle$, where K_S^0 is the short-lived neutral kaon and K_L^0 is the long-lived one. Neglecting the small effect of CP non-invariance, the CP-parity of the state K_S^0 is equal to $(+1)$, and the CP-parity of the state K_L^0 is equal to (-1); in doing so,

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_S^0\rangle + |K_L^0\rangle), \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}(|K_S^0\rangle - |K_L^0\rangle).$$

It is clear that both the quasistationary states of the neutral kaon have no definite strangeness.

It follows from the Bose-symmetry of the wave function of two neutral kaons with respect to the total permutation of internal states and momenta that the CP-parity of the system $K^0\bar{K}^0$ is always positive [2] (the C-parity is $(-1)^L$, the space parity is $P = (-1)^L$, where L is the orbital momentum).

The system of two non-identical neutral kaons $K^0\bar{K}^0$ in the symmetric internal state, corresponding to even orbital momenta, is decomposed into the schemes $|K_S^0\rangle|K_S^0\rangle$ and $|K_L^0\rangle|K_L^0\rangle$ [2]:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} + |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$

$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)} - |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)}); \tag{1}$$

meantime, the system $K^0\bar{K}^0$ in the antisymmetric internal state, corresponding to odd orbital momenta, is decomposed into the scheme $|K_S^0\rangle|K_L^0\rangle$ [2]:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} - |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$

$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)} - |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)}). \tag{2}$$

The strangeness conservation leads to the fact that all the double inclusive cross-sections of production of pairs $K_S^0 K_S^0$, $K_L^0 K_L^0$ and $K_S^0 K_L^0$ (two-particle structure functions) prove to be symmetric with respect to the permutation of momenta $\mathbf{p}_1$ and $\mathbf{p}_2$.

Besides, due to the strangeness conservation, the structure functions of neutral K mesons produced in inclusive processes are invariant with respect to the replacement of the short-lived state K_S^0 by the long-lived state K_L^0, and *vice versa* [1]:

$$f_{SS}(\mathbf{p}_1, \mathbf{p}_2) = f_{LL}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{4} \left[f_{K^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + \right.$$

$$\left. + f_{K^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) \right] + \frac{1}{2} \operatorname{Re} \rho_{K^0 \bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2), \tag{3}$$

$$f_{SL}(\mathbf{p}_1, \mathbf{p}_2) = f_{LS}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{4} \left[f_{K^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + \right.$$

$$\left. + f_{K^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) \right] - \frac{1}{2} \operatorname{Re} \rho_{K^0 \bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2), \tag{4}$$

where $\rho_{K^0 \bar{K}^0 \to \bar{K}^0 K^0}(\mathbf{p}_1, \mathbf{p}_2) = (\rho_{\bar{K}^0 K^0 \to K^0 \bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2))^*$ are the non-diagonal elements of the two-kaon density matrix. The difference between the two-particle structure functions f_{SS} and f_{SL} is connected just with the contribution of these non-diagonal elements.

2 Structure of pair correlations of identical and non-identical neutral kaons with close momenta

Now let us consider, within the model of one-particle sources [2-7], the correlations of pairs of neutral K mesons with close momenta (see also [8-10]). In the case of the identical states $K_S^0 K_S^0$ and $K_L^0 K_L^0$ we obtain the following expressions for the correlation functions R_{SS}, R_{LL} (proportional to the structure functions), normalized to unity at large relative momenta:

$$R_{SS}(\mathbf{k}) = R_{LL}(\mathbf{k}) = \lambda_{K^0 K^0} \left[1 + F_{K^0}(2\mathbf{k}) + 2\, b_{\text{int}}(\mathbf{k}) \right] +$$

$$+ \lambda_{\bar{K}^0 \bar{K}^0} \left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\, \tilde{b}_{\text{int}}(\mathbf{k}) \right] + \lambda_{K^0 \bar{K}^0} \left[1 + F_{K^0 \bar{K}^0}(2\mathbf{k}) + 2\, B_{\text{int}}(\mathbf{k}) \right]. \tag{5}$$

Here $\mathbf{k}$ is the momentum of one of the kaons in the c.m. frame of the pair, and the quantities $\lambda_{K^0 K^0}$, $\lambda_{\bar{K}^0 \bar{K}^0}$ and $\lambda_{K^0 \bar{K}^0}$ are the relative fractions of the average numbers of produced pairs $K^0 K^0$, $\bar{K}^0 \bar{K}^0$ and $K^0 \bar{K}^0$, respectively ($\lambda_{K^0 K^0} + \lambda_{\bar{K}^0 \bar{K}^0} + \lambda_{K^0 \bar{K}^0} = 1$) . The "formfactors" $F_{K^0}(2\mathbf{k})$, $F_{\bar{K}^0}(2\mathbf{k})$ and $F_{K^0 \bar{K}^0}(2\mathbf{k})$ appear due to the contribution of Bose-statistics:

$$F_{K^0}(2\mathbf{k}) = \int W_{K^0}(\mathbf{r})\, \cos(2\mathbf{k}\mathbf{r})\, d^3\mathbf{r}, \qquad F_{\bar{K}^0}(2\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r})\, \cos(2\mathbf{k}\mathbf{r})\, d^3\mathbf{r},$$

$$F_{K^0 \bar{K}^0}(2\mathbf{k}) = \int W_{K^0 \bar{K}^0}(\mathbf{r})\, \cos(2\mathbf{k}\mathbf{r})\, d^3\mathbf{r}. \tag{6}$$

where $W_{K^0}(\mathbf{r})$, $W_{\bar{K}^0}(\mathbf{r})$ and $W_{K^0 \bar{K}^0}(\mathbf{r})$ are the probability distributions of distances between the sources of emission of two K^0 mesons, between the sources of emission of two $\bar{K}^0$ mesons and between the sources of emission of the K^0 meson and $\bar{K}^0$ meson, respectively, in the c.m. frame of the kaon pair. Meantime, the quantity $b_{\text{int}}(\mathbf{k})$ describes the contribution of the S-wave interaction of two K^0 mesons, the quantity $\tilde{b}_{\text{int}}(\mathbf{k})$ describes the contribution of the S-wave interaction of two $\bar{K}^0$ mesons and the quantity $B_{\text{int}}(\mathbf{k})$ describes the contribution of the S-wave interaction of the K^0 meson with the $\bar{K}^0$ meson. Due to the CP invariance, the quantities $b_{\text{int}}(\mathbf{k})$ and $\tilde{b}_{\text{int}}(\mathbf{k})$ can be expressed by means of averaging the same function $b(\mathbf{k}, \mathbf{r})$ over the different distributions:

$$b_{\text{int}}(\mathbf{k}) = \int W_{K^0}(\mathbf{r}) b(\mathbf{k}, \mathbf{r}) d^3\mathbf{r}, \quad \tilde{b}_{\text{int}}(\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r}) b(\mathbf{k}, \mathbf{r}) d^3\mathbf{r}.$$

The quantity $B_{\text{int}}(\mathbf{k})$ has the structure : $B_{\text{int}}(\mathbf{k}) = \int W_{K^0 \bar{K}^0}(\mathbf{r}) B(\mathbf{k}, \mathbf{r}) d^3\mathbf{r}$, where $B(\mathbf{k}, \mathbf{r}) \neq b(\mathbf{k}, \mathbf{r})$.

Let us emphasize that when the pair of non-identical neutral kaons $K^0 \bar{K}^0$ is produced but the pair of identical quasistationary states $K_S^0 K_S^0$ (or $K_L^0 K_L^0$) is registered over decays, the two-particle correlations at small relative momenta have the same character as in the case of usual identical bosons with zero spin [2].

For the pairs of non-identical kaon states $K_S^0 K_L^0$ the correlation functions at small relative momenta have the form:

$$R_{SL}(\mathbf{k}) = R_{LS}(\mathbf{k}) = \lambda_{K^0 K^0} \left[1 + F_{K^0}(2\mathbf{k}) + 2\, b_{\text{int}}(\mathbf{k}) \right] +$$

$$+ \lambda_{\bar{K}^0 \bar{K}^0} \left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\, \tilde{b}_{\text{int}}(\mathbf{k}) \right] + \lambda_{K^0 \bar{K}^0} \left[1 - F_{K^0 \bar{K}^0}(2\mathbf{k}) \right]. \tag{7}$$

It follows from Eqs.(5) and (7) that the correlation functions of pairs of neutral K mesons with close momenta, which are created in inclusive processes, satisfy the relation

$$R_{SS}(\mathbf{k}) + R_{LL}(\mathbf{k}) - R_{SL}(\mathbf{k}) - R_{LS}(\mathbf{k}) = 2\left[R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k})\right] =$$
$$= 4\lambda_{K^0\bar{K}^0}\left[F_{K^0\bar{K}^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k})\right]. \tag{8}$$

We see that the difference between the correlation functions of the pairs of identical neutral kaons $K_S^0 K_S^0$ and pairs of non-identical neutral kaons $K_S^0 K_L^0$ is conditioned exclusively by the generation of $K^0\bar{K}^0$-pairs.

The relations connecting the contribution of the S-wave strong interaction into the pair correlations of particles at small relative momenta with the parameters of low-energy scattering were obtained earlier in the papers [4-7]. It is essential that the "formfactors" (6) and the functions $b_{\text{int}}(\mathbf{k})$, $\tilde{b}_{\text{int}}(\mathbf{k})$ and $B_{\text{int}}(\mathbf{k})$ depend on the space-time parameters of the generation region of neutral kaons and tend to zero at high values of the relative momentum $q = 2|\mathbf{k}|$ of two neutral kaons. Concretely, the expression for the function $B(\mathbf{k}, \mathbf{r})$ in the case of the $K^0\bar{K}^0$ system has been obtained in the paper [10]. In the same paper, the estimate of contribution of the transition $K^+K^- \to K^0\bar{K}^0$ has also been presented.

3 Correlations of neutral heavy mesons

Formally, analogous relations are valid also for the neutral heavy mesons D^0, B^0 and B_s^0. In doing so, the role of strangeness conservation is played, respectively, by the conservation of charm and beauty in inclusive multiple processes with production of these mesons. In these cases the quasistationary states are also states with definite CP parity, neglecting the effects of CP nonconservation. For example,

$$|B_S^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle + |\bar{B}^0\rangle), \ CP \text{ parity } (+1); \quad |B_L^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle - |\bar{B}^0\rangle), \ CP \text{ parity } (-1).$$

The difference of masses between the respective CP-odd and CP-even states is very insignificant in all the cases, ranging from 10^{-12} MeV for K^0 mesons up to 10^{-8} MeV for B_s^0 mesons. Concerning the lifetimes of these states, in the case of K^0 mesons they differ by 600 times, but for D^0, B^0 and B_s^0 mesons the respective lifetimes are almost the same. In connection with this, it is practically impossible to distinguish the states of D^0, B^0 and B_s^0 mesons with definite CP parity by the difference in their lifetimes. These states, in principle, can be identified through the purely CP-even and purely CP-odd decay channels ; however, in fact the branching ratio for such decays is very small. For example,

$$Br\left(D^0 \to \pi^+\pi^-\right) = 1.62 \cdot 10^{-3} \ (\ CP = +1\ ); \quad Br\left(D^0 \to K^+K^-\right) = 4.25 \cdot 10^{-3} \ (\ CP = +1\ ) ;$$

$$Br\left(B_s^0 \to J/\Psi \ \pi^0\right) < 1.2 \cdot 10^{-3} \ (\ CP = +1\ ); \quad Br\left(B^0 \to J/\Psi \ K_S^0\right) = 9 \cdot 10^{-4} \ (\ CP = -1\ ) .$$

Just as in the case of neutral K mesons, the correlation functions for the pairs of states of neutral D, B and B_s mesons with the same CP parity ($R_{SS} = R_{LL}$) and for the pairs of states with different CP parity (R_{SL}) do not coincide, and the difference between them is conditioned exclusively by the production of pairs $D^0\bar{D}^0$, $B^0\bar{B}^0$ and $B_s^0\bar{B}_s^0$, respectively. In particular, for B_s^0 mesons the following relation holds:

$$R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k}) = 2\lambda_{B_s^0\bar{B}_s^0}\left[F_{B_s^0\bar{B}_s^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k})\right]; \tag{9}$$

here $\lambda_{B_s^0 \bar{B}_s^0}$ is the relative fraction of generated pairs $B_s^0 \bar{B}_s^0$,

$$F_{B_s^0 \bar{B}_s^0}(2\mathbf{k}) = \int W_{B_s^0 \bar{B}_s^0}(\mathbf{r}) \cos(2\mathbf{kr}) \, d^3\mathbf{r}, \qquad B_{\text{int}}(\mathbf{k}) = \int W_{B_s^0 \bar{B}_s^0}(\mathbf{r}) \, B(\mathbf{k},\mathbf{r}) \, d^3\mathbf{r},$$

$$B(\mathbf{k},\mathbf{r}) = |A_{B_s^0 \bar{B}_s^0}(k)|^2 \frac{1}{r^2} + 2\,\mathrm{Re}\left(A_{B_s^0 \bar{B}_s^0}(k) \frac{\exp(ikr)\cos \mathbf{kr}}{r}\right),$$

where $A_{B_s^0 \bar{B}_s^0}(k) \equiv A_{B_s^0 \bar{B}_s^0 \to B_s^0 \bar{B}_s^0}(k)$ is the amplitude of S-wave $B_s^0 \bar{B}_s^0$ - scattering, $k = |\mathbf{k}|$, $r = |\mathbf{r}|$. Let us remark that the B_s^0 and $\bar{B}_s^0$ mesons do not have charged partners (the isotopic spin equals zero) and, on account of that, in the given case the transition similar to $K^+ K^- \to K^0 \bar{K}^0$ is absent .

4   Summary

1. It is shown that, taking into account the strangeness conservation, the correlation functions for two short-lived neutral K mesons (R_{SS}) and two long-lived neutral K mesons (R_{LL}) are equal to each other. This result is the direct consequence of the strangeness conservation.

2. It is shown that the production of $K^0 \bar{K}^0$-pairs with the zero strangeness leads to the difference between the correlation functions R_{SS} and R_{SL} of two neutral kaons.

3. The character of analogous correlations for neutral heavy mesons D^0 , B^0, B_s^0 with nonzero charm and beauty is discussed . Contrary to the case of K^0 mesons, here the distinction of respective CP-even and CP-odd states encounters difficulties, which are connected with the insignificant difference of their lifetimes and the relatively small probability of purely CP -even and purely CP-odd decay channels .

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