On the pair correlations of neutral K, D, B and B_s mesons with close momenta produced in inclusive multiparticle processes

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DOI: `http://dx.doi.org/10.3204/DESY-PROC-2012-02/17`

The phenomenological structure of inclusive cross-sections of the production of two neutral K mesons in collisions of hadrons and nuclei is investigated taking into account the strangeness conservation in strong and electromagnetic interactions. Relations describing the dependence of the correlations of two short-lived and two long-lived neutral kaons $K^0_S K^0_S$, $K^0_L K^0_L$ and the correlations of "mixed" pairs $K^0_S K^0_L$ at small relative momenta upon the space-time parameters of the generation region of K^0 and $\bar{K}^0$ mesons have been obtained. It is shown that under the strangeness conservation the correlation functions of the pairs $K^0_S K^0_S$ and $K^0_L K^0_L$, produced in the same inclusive process, coincide, and the difference between the correlation functions of the pairs $K^0_S K^0_S$ and $K^0_S K^0_L$ is conditioned by the production of the pairs of non-identical neutral kaons $K^0 \bar{K}^0$. Analogous correlations for the pairs of neutral heavy mesons D^0, B^0 and B^0_s, generated in multiple processes with the charm (beauty) conservation, are analyzed, and differences from the case of neutral K mesons are discussed .

1 Consequences of the strangeness conservation for neutral kaons

In the work [1] the properties of the density matrix of two neutral K mesons, following from the strangeness conservation in strong and electromagnetic interactions, have been investigated. By definition, the diagonal elements of the non-normalized two-particle density matrix coincide with the two-particle structure functions, which are proportional to the double inclusive cross-sections.

Strangeness is the additive quantum number. Taking into account the strangeness conservation, the pairs of neutral kaons $K^0 K^0$ (strangeness $S = +2$), $\bar{K}^0 \bar{K}^0$ (strangeness $S = -2$) and $K^0 \bar{K}^0$ (strangeness $S = 0$) are produced incoherently. This means that in the K^0-$\bar{K}^0$- representation the non-diagonal elements of the density matrix between the states $K^0 K^0$ and $\bar{K}^0 \bar{K}^0$, $K^0 K^0$ and $K^0 \bar{K}^0$, $\bar{K}^0 \bar{K}^0$ and $K^0 \bar{K}^0$ are equal to zero. However, the non-diagonal elements of the two-kaon density matrix between the two states $|K^0\rangle^{(\mathbf{p}_1)} |\bar{K}^0\rangle^{(\mathbf{p}_2)}$ and $|\bar{K}^0\rangle^{(\mathbf{p}_1)} |K^0\rangle^{(\mathbf{p}_2)}$ with the zero strangeness are not equal to zero, in general. Here $\mathbf{p}_1$ and $\mathbf{p}_2$ are the momenta of the first and second kaons.

The internal states of K^0 meson ($S = 1$) and $\bar{K}^0$ meson ($S = -1$) are the superpositions of the states $|K_S^0\rangle$ and $|K_L^0\rangle$, where K_S^0 is the short-lived neutral kaon and K_L^0 is the long-lived one. Neglecting the small effect of CP non-invariance, the CP-parity of the state K_S^0 is equal to $(+1)$, and the CP-parity of the state K_L^0 is equal to (-1); in doing so,

$$|K^0\rangle = \frac{1}{\sqrt{2}}(|K_S^0\rangle + |K_L^0\rangle), \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}(|K_S^0\rangle - |K_L^0\rangle).$$

It is clear that both the quasistationary states of the neutral kaon have no definite strangeness.

It follows from the Bose-symmetry of the wave function of two neutral kaons with respect to the total permutation of internal states and momenta that the CP-parity of the system $K^0\bar{K}^0$ is always positive [2] (the C-parity is $(-1)^L$, the space parity is $P = (-1)^L$, where L is the orbital momentum).

The system of two non-identical neutral kaons $K^0\bar{K}^0$ in the symmetric internal state, corresponding to even orbital momenta, is decomposed into the schemes $|K_S^0\rangle|K_S^0\rangle$ and $|K_L^0\rangle|K_L^0\rangle$ [2]:

$$|\psi^+\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} + |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$

$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)} - |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)}); \tag{1}$$

meantime, the system $K^0\bar{K}^0$ in the antisymmetric internal state, corresponding to odd orbital momenta, is decomposed into the scheme $|K_S^0\rangle|K_L^0\rangle$ [2]:

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle^{(\mathbf{p}_1)} \otimes |\bar{K}^0\rangle^{(\mathbf{p}_2)} - |\bar{K}^0\rangle^{(\mathbf{p}_1)} \otimes |K^0\rangle^{(\mathbf{p}_2)}) =$$

$$= \frac{1}{\sqrt{2}}(|K_S^0\rangle^{(\mathbf{p}_1)} \otimes |K_L^0\rangle^{(\mathbf{p}_2)} - |K_L^0\rangle^{(\mathbf{p}_1)} \otimes |K_S^0\rangle^{(\mathbf{p}_2)}). \tag{2}$$

The strangeness conservation leads to the fact that all the double inclusive cross-sections of production of pairs $K_S^0K_S^0$, $K_L^0K_L^0$ and $K_S^0K_L^0$ (two-particle structure functions) prove to be symmetric with respect to the permutation of momenta $\mathbf{p}_1$ and $\mathbf{p}_2$.

Besides, due to the strangeness conservation, the structure functions of neutral K mesons produced in inclusive processes are invariant with respect to the replacement of the short-lived state K_S^0 by the long-lived state K_L^0, and *vice versa* [1]:

$$f_{SS}(\mathbf{p}_1, \mathbf{p}_2) = f_{LL}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{4}\,[f_{K^0K^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0\bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) +$$

$$+ f_{K^0\bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0K^0}(\mathbf{p}_1, \mathbf{p}_2)] + \frac{1}{2}\,\mathrm{Re}\,\rho_{K^0\bar{K}^0 \to \bar{K}^0K^0}(\mathbf{p}_1, \mathbf{p}_2), \tag{3}$$

$$f_{SL}(\mathbf{p}_1, \mathbf{p}_2) = f_{LS}(\mathbf{p}_1, \mathbf{p}_2) = \frac{1}{4}\,[f_{K^0K^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0\bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) +$$

$$+ f_{K^0\bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2) + f_{\bar{K}^0K^0}(\mathbf{p}_1, \mathbf{p}_2)] - \frac{1}{2}\,\mathrm{Re}\,\rho_{K^0\bar{K}^0 \to \bar{K}^0K^0}(\mathbf{p}_1, \mathbf{p}_2), \tag{4}$$

where $\rho_{K^0\bar{K}^0 \to \bar{K}^0K^0}(\mathbf{p}_1, \mathbf{p}_2) = (\rho_{\bar{K}^0K^0 \to K^0\bar{K}^0}(\mathbf{p}_1, \mathbf{p}_2))^*$ are the non-diagonal elements of the two-kaon density matrix. The difference between the two-particle structure functions f_{SS} and f_{SL} is connected just with the contribution of these non-diagonal elements.

2 Structure of pair correlations of identical and non-identical neutral kaons with close momenta

Now let us consider, within the model of one-particle sources [2-7], the correlations of pairs of neutral K mesons with close momenta (see also [8-10]). In the case of the identical states $K_S^0 K_S^0$ and $K_L^0 K_L^0$ we obtain the following expressions for the correlation functions R_{SS}, R_{LL} (proportional to the structure functions), normalized to unity at large relative momenta:

$$R_{SS}(\mathbf{k}) = R_{LL}(\mathbf{k}) = \lambda_{K^0K^0}\left[1 + F_{K^0}(2\mathbf{k}) + 2\,b_{\rm int}(\mathbf{k})\right] +$$

$$+\,\lambda_{\bar{K}^0\bar{K}^0}\left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\,\tilde{b}_{\rm int}(\mathbf{k})\right] + \lambda_{K^0\bar{K}^0}\left[1 + F_{K^0\bar{K}^0}(2\mathbf{k}) + 2\,B_{\rm int}(\mathbf{k})\right]. \qquad (5)$$

Here $\mathbf{k}$ is the momentum of one of the kaons in the c.m. frame of the pair, and the quantities $\lambda_{K^0K^0}$, $\lambda_{\bar{K}^0\bar{K}^0}$ and $\lambda_{K^0\bar{K}^0}$ are the relative fractions of the average numbers of produced pairs K^0K^0, $\bar{K}^0\bar{K}^0$ and $K^0\bar{K}^0$, respectively $(\lambda_{K^0K^0} + \lambda_{\bar{K}^0\bar{K}^0} + \lambda_{K^0\bar{K}^0} = 1)$. The "formfactors" $F_{K^0}(2\mathbf{k})$, $F_{\bar{K}^0}(2\mathbf{k})$ and $F_{K^0\bar{K}^0}(2\mathbf{k})$ appear due to the contribution of Bose-statistics:

$$F_{K^0}(2\mathbf{k}) = \int W_{K^0}(\mathbf{r})\,\cos(2\mathbf{kr})\,d^3\mathbf{r}, \qquad F_{\bar{K}^0}(2\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r})\,\cos(2\mathbf{kr})\,d^3\mathbf{r},$$

$$F_{K^0\bar{K}^0}(2\mathbf{k}) = \int W_{K^0\bar{K}^0}(\mathbf{r})\,\cos(2\mathbf{kr})\,d^3\mathbf{r}. \qquad (6)$$

where $W_{K^0}(\mathbf{r})$, $W_{\bar{K}^0}(\mathbf{r})$ and $W_{K^0\bar{K}^0}(\mathbf{r})$ are the probability distributions of distances between the sources of emission of two K^0 mesons, between the sources of emission of two $\bar{K}^0$ mesons and between the sources of emission of the K^0 meson and $\bar{K}^0$ meson, respectively, in the c.m. frame of the kaon pair. Meantime, the quantity $b_{\rm int}(\mathbf{k})$ describes the contribution of the S-wave interaction of two K^0 mesons, the quantity $\tilde{b}_{\rm int}(\mathbf{k})$ describes the contribution of the S-wave interaction of two $\bar{K}^0$ mesons and the quantity $B_{\rm int}(\mathbf{k})$ describes the contribution of the S-wave interaction of the K^0 meson with the $\bar{K}^0$ meson. Due to the CP invariance, the quantities $b_{\rm int}(\mathbf{k})$ and $\tilde{b}_{\rm int}(\mathbf{k})$ can be expressed by means of averaging the same function $b(\mathbf{k},\mathbf{r})$ over the different distributions:

$$b_{\rm int}(\mathbf{k}) = \int W_{K^0}(\mathbf{r})b(\mathbf{k},\mathbf{r})d^3\mathbf{r}, \quad \tilde{b}_{\rm int}(\mathbf{k}) = \int W_{\bar{K}^0}(\mathbf{r})b(\mathbf{k},\mathbf{r})d^3\mathbf{r}.$$

The quantity $B_{\rm int}(\mathbf{k})$ has the structure : $B_{\rm int}(\mathbf{k}) = \int W_{K^0\bar{K}^0}(\mathbf{r})B(\mathbf{k},\mathbf{r})d^3\mathbf{r}$, where $B(\mathbf{k},\mathbf{r}) \neq b(\mathbf{k},\mathbf{r})$.

Let us emphasize that when the pair of non-identical neutral kaons $K^0\bar{K}^0$ is produced but the pair of identical quasistationary states $K_S^0K_S^0$ (or $K_L^0K_L^0$) is registered over decays, the two-particle correlations at small relative momenta have the same character as in the case of usual identical bosons with zero spin [2].

For the pairs of non-identical kaon states $K_S^0K_L^0$ the correlation functions at small relative momenta have the form:

$$R_{SL}(\mathbf{k}) = R_{LS}(\mathbf{k}) = \lambda_{K^0K^0}\left[1 + F_{K^0}(2\mathbf{k}) + 2\,b_{\rm int}(\mathbf{k})\right] +$$

$$+\,\lambda_{\bar{K}^0\bar{K}^0}\left[1 + F_{\bar{K}^0}(2\mathbf{k}) + 2\,\tilde{b}_{\rm int}(\mathbf{k})\right] + \lambda_{K^0\bar{K}^0}\left[1 - F_{K^0\bar{K}^0}(2\mathbf{k})\right]. \qquad (7)$$

It follows from Eqs.(5) and (7) that the correlation functions of pairs of neutral K mesons with close momenta, which are created in inclusive processes, satisfy the relation

$$R_{SS}(\mathbf{k}) + R_{LL}(\mathbf{k}) - R_{SL}(\mathbf{k}) - R_{LS}(\mathbf{k}) = 2\left[R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k})\right] = \\ = 4\lambda_{K^0\bar{K}^0}\left[F_{K^0\bar{K}^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k})\right]. \quad (8)$$

We see that the difference between the correlation functions of the pairs of identical neutral kaons $K_S^0 K_S^0$ and pairs of non-identical neutral kaons $K_S^0 K_L^0$ is conditioned exclusively by the generation of $K^0\bar{K}^0$-pairs.

The relations connecting the contribution of the S-wave strong interaction into the pair correlations of particles at small relative momenta with the parameters of low-energy scattering were obtained earlier in the papers [4-7]. It is essential that the "formfactors" (6) and the functions $b_{\text{int}}(\mathbf{k})$, $\tilde{b}_{\text{int}}(\mathbf{k})$ and $B_{\text{int}}(\mathbf{k})$ depend on the space-time parameters of the generation region of neutral kaons and tend to zero at high values of the relative momentum $q = 2|\mathbf{k}|$ of two neutral kaons. Concretely, the expression for the function $B(\mathbf{k}, \mathbf{r})$ in the case of the $K^0\bar{K}^0$ system has been obtained in the paper [10]. In the same paper, the estimate of contribution of the transition $K^+K^- \to K^0\bar{K}^0$ has also been presented .

3 Correlations of neutral heavy mesons

Formally, analogous relations are valid also for the neutral heavy mesons D^0, B^0 and B_s^0 . In doing so, the role of strangeness conservation is played, respectively, by the conservation of charm and beauty in inclusive multiple processes with production of these mesons . In these cases the quasistationary states are also states with definite CP parity, neglecting the effects of CP nonconservation. For example,

$$|B_S^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle + |\bar{B}^0\rangle),\ CP \text{ parity } (+1); \quad |B_L^0\rangle = \frac{1}{\sqrt{2}}(|B^0\rangle - |\bar{B}^0\rangle),\ CP \text{ parity } (-1).$$

The difference of masses between the respective CP-odd and CP-even states is very insignificant in all the cases, ranging from 10^{-12} MeV for K^0 mesons up to 10^{-8} MeV for B_s^0 mesons . Concerning the lifetimes of these states, in the case of K^0 mesons they differ by 600 times, but for D^0, B^0 and B_s^0 mesons the respective lifetimes are almost the same. In connection with this, it is practically impossible to distinguish the states of D^0, B^0 and B_s^0 mesons with definite CP parity by the difference in their lifetimes. These states, in principle, can be identified through the purely CP-even and purely CP-odd decay channels ; however, in fact the branching ratio for such decays is very small . For example,

$$Br\,(D^0 \to \pi^+\pi^-) = 1.62 \cdot 10^{-3}\ (\ CP = +1\ ); \quad Br\,(D^0 \to K^+K^-) = 4.25 \cdot 10^{-3}\ (\ CP = +1\ )\ ;$$

$$Br\,(B_s^0 \to J/\Psi\ \pi^0) < 1.2 \cdot 10^{-3}\ (\ CP = +1\ ); \quad Br\,(B^0 \to J/\Psi\ K_S^0) = 9 \cdot 10^{-4}\ (\ CP = -1\ )\ .$$

Just as in the case of neutral K mesons, the correlation functions for the pairs of states of neutral D, B and B_s mesons with the same CP parity ($R_{SS} = R_{LL}$) and for the pairs of states with different CP parity (R_{SL}) do not coincide, and the difference between them is conditioned exclusively by the production of pairs $D^0\bar{D}^0$, $B^0\bar{B}^0$ and $B_s^0\bar{B}_s^0$, respectively. In particular, for B_s^0 mesons the following relation holds:

$$R_{SS}(\mathbf{k}) - R_{SL}(\mathbf{k}) = 2\lambda_{B_s^0\bar{B}_s^0}\left[F_{B_s^0\bar{B}_s^0}(2\mathbf{k}) + B_{\text{int}}(\mathbf{k})\right]; \quad (9)$$

here $\lambda_{B_s^0 \bar{B}_s^0}$ is the relative fraction of generated pairs $B_s^0 \bar{B}_s^0$,

$$F_{B_s^0 \bar{B}_s^0}(2\mathbf{k}) = \int W_{B_s^0 \bar{B}_s^0}(\mathbf{r}) \cos(2\mathbf{kr})\, d^3\mathbf{r}, \qquad B_{\rm int}(\mathbf{k}) = \int W_{B_s^0 \bar{B}_s^0}(\mathbf{r})\, B(\mathbf{k},\mathbf{r})\, d^3\mathbf{r},$$

$$B(\mathbf{k},\mathbf{r}) = |A_{B_s^0 \bar{B}_s^0}(k)|^2 \frac{1}{r^2} + 2\,{\rm Re}\left(A_{B_s^0 \bar{B}_s^0}(k) \frac{\exp(ikr)\cos\mathbf{kr}}{r} \right),$$

where $A_{B_s^0 \bar{B}_s^0}(k) \equiv A_{B_s^0 \bar{B}_s^0 \to B_s^0 \bar{B}_s^0}(k)$ is the amplitude of S-wave $B_s^0 \bar{B}_s^0$ - scattering, $k = |\,\mathbf{k}\,|$, $r = |\,\mathbf{r}\,|$. Let us remark that the B_s^0 and $\bar{B}_s^0$ mesons do not have charged partners (the isotopic spin equals zero) and, on account of that, in the given case the transition similar to $K^+K^- \to K^0\bar{K}^0$ is absent .

4 Summary

1. It is shown that, taking into account the strangeness conservation, the correlation functions for two short-lived neutral K mesons (R_{SS}) and two long-lived neutral K mesons (R_{LL}) are equal to each other. This result is the direct consequence of the strangeness conservation.
2. It is shown that the production of $K^0\bar{K}^0$-pairs with the zero strangeness leads to the difference between the correlation functions R_{SS} and R_{SL} of two neutral kaons.
3. The character of analogous correlations for neutral heavy mesons D^0 , B^0, B_s^0 with nonzero charm and beauty is discussed . Contrary to the case of K^0 mesons, here the distinction of respective CP-even and CP-odd states encounters difficulties, which are connected with the insignificant difference of their lifetimes and the relatively small probability of purely CP -even and purely CP-odd decay channels .

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