The statistical model for parton distributions

Claude Bourrely[1], *Franco Buccella*[2], *Jacques Soffer*[3]

[1] Aix-Marseille Université, Département de Physique, Faculté des Sciences de Luminy, 13288 Marseille, Cedex 09, France
[2] INFN, Sezione di Napoli, via Cintia, I-80126, Napoli
[3] Physics Department, Temple University Barton Hall, 1900 N, 13th Street Philadelphia, PA 19122-6082, USA

DOI: `http://dx.doi.org/10.3204/DESY-PROC-2012-02/95`

The phenomenological motivations, the expressions and the comparison with experiment of the parton distributions inspired by the quantum statistics are described. The Fermi-Dirac expressions for the quarks and their antiparticles automatically account for the correlation between the shape and the first moments of the valence partons, as well as the flavor and spin asymmetries of the sea. One is able to describe with a small number of parameters both unpolarized and polarized structure functions.

Let us first recall some of the basic ingredients for building up the parton distribution functions (PDF) in the statistical approach, as oppose to the standard polynomial type parametrizations, based on Regge theory at low x and counting rules at large x. The fermion distributions are given by the sum of two terms [1], the first one, a quasi Fermi-Dirac function and the second one, a flavor and helicity independent diffractive contribution equal for light quarks. So we have, at the input energy scale $Q_0^2 = 4\mathrm{GeV}^2$,

$$xq^h(x, Q_0^2) = \frac{AX_{0q}^h x^b}{\exp[(x - X_{0q}^h)/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ , \tag{1}$$

$$x\bar{q}^h(x, Q_0^2) = \frac{\bar{A}(X_{0q}^{-h})^{-1} x^{2b}}{\exp[(x + X_{0q}^{-h})/\bar{x}] + 1} + \frac{\tilde{A}x^{\tilde{b}}}{\exp(x/\bar{x}) + 1} \ . \tag{2}$$

Notice the change of sign of the potentials and helicity for the antiquarks. The parameter $\bar{x}$ plays the role of a *universal temperature* and $X_{0q}^{\pm}$ are the two *thermodynamical potentials* of the quark q, with helicity $h = \pm$. It is important to remark that the diffractive contribution occurs only in the unpolarized distributions $q(x) = q_+(x) + q_-(x)$ and it is absent in the valence $q_v(x) = q(x) - \bar{q}(x)$ and in the helicity distributions $\Delta q(x) = q_+(x) - q_-(x)$ (similarly for antiquarks). The *eight* free parameters[1] in Eqs. (1,2) were determined at the input scale from the comparison with a selected set of very precise unpolarized and polarized Deep-Inelastic Scattering (DIS) data [1]. They have the following values

$$\bar{x} = 0.09907,\ b = 0.40962,\ \tilde{b} = -0.25347,\ \tilde{A} = 0.08318, \tag{3}$$

$$X_{0u}^+ = 0.46128,\ X_{0u}^- = 0.29766,\ X_{0d}^- = 0.30174,\ X_{0d}^+ = 0.22775\ . \tag{4}$$

[1] $A = 1.74938$ and $\bar{A} = 1.90801$ are fixed by the following normalization conditions $u - \bar{u} = 2$, $d - \bar{d} = 1$.

For the gluons we consider the black-body inspired expression

$$xG(x,Q_0^2) = \frac{A_G x^{b_G}}{\exp(x/\bar{x}) - 1} \ , \tag{5}$$

a quasi Bose-Einstein function, with $b_G = 0.90$, the only free parameter [2], since $A_G = 20.53$ is determined by the momentum sum rule. We also assume that, at the input energy scale, the polarized gluon distribution vanishes, so $x\Delta G(x, Q_0^2) = 0$. For the strange quark distributions, the simple choice made in Ref. [1] was greatly improved in Ref. [2]. More recently, new tests against experimental (unpolarized and polarized) data turned out to be very satisfactory, in particular in hadronic collisions, as reported in Refs. [3, 4]. For illustration, we will just give

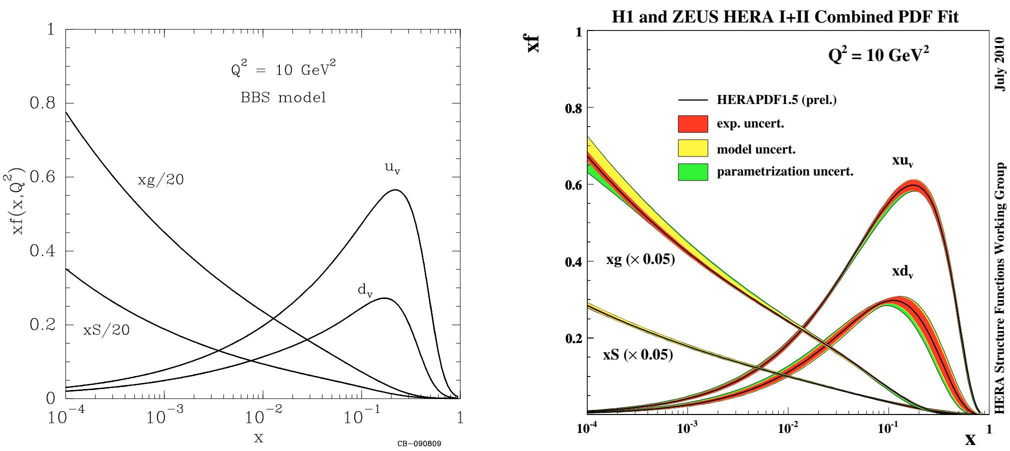

Figure 1: *Left* : BBS predictions for various statistical unpolarized parton distributions versus x at $Q^2 = 10\text{GeV}^2$. *Right* : Parton distributions at $Q^2 = 10\text{GeV}^2$, as determined by the HERAPDF fit, with different uncertainties (Taken from Ref. [5]).

one recent result, directly related to the determination of the quark distributions from unpolarized DIS. We display on Fig. 1 (*Left*), the resulting unpolarized statistical PDF versus x at Q^2=10 GeV^2, where xu_v is the u-quark valence, xd_v the d-quark valence, with their characteristic maximum around $x = 0.3$, xG the gluon and xS stands for twice the total antiquark contributions, $i.e.$ $xS(x) = 2x(\bar{u}(x) + \bar{d}(x) + \bar{s}(x)) + \bar{c}(x))$. Note that xG and xS are downscaled by a factor 0.05. They can be compared with the parton distributions as determined by the HERAPDF1.5 QCD NLO fit, shown also in Fig. 1 (*Right*), and there is a good agreement. The results are based on recent ep collider data from HERA, combined with previously published data and the accuracy is typically in the range of 1.3 - 2%. Another interesting point concerns the behavior of the ratio $d(x)/u(x)$, which depends on the mathematical properties of the ratio of two Fermi-Dirac factors, outside the region dominated by the diffractive contribution. So for $x > 0.1$, this ratio is expected to decrease faster for $X_{0d}^+ - \bar{x} < x < X_{0u}^+ + \bar{x}$ and then above, for $x > 0.6$ it flattens out. This change of slope is clearly visible in Fig. 2 (*Left*), with a very

[2] In Ref. [1] we were assuming that, for very small x, $xG(x, Q_0^2)$ has the same behavior as $x\bar{q}(x, Q_0^2)$, so we took $b_G = 1 + \tilde{b}$. However this choice leads to a too much rapid rise of the gluon distribution, compared to its recent determination from HERA data, which requires $b_G = 0.90$.

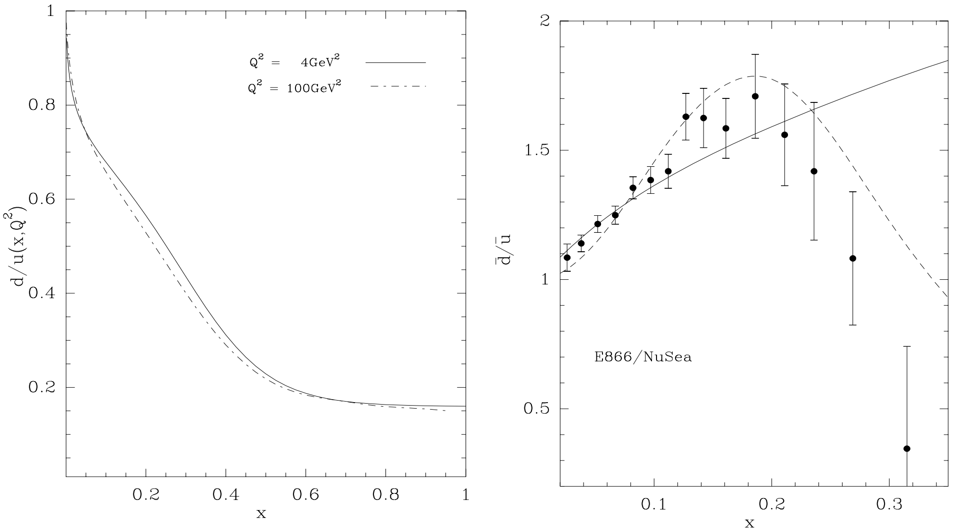

Figure 2: *Left* : The ratio $d(x)/u(x)$ as function of x for $Q^2 = 4\text{GeV}^2$ (solid line) and $Q^2 = 100\text{GeV}^2$ (dashed-dotted line). *Right* : Comparison of the data on $\bar{d}/\bar{u}$ versus x, from E866/NuSea at $Q^2 = 54\text{GeV}^2$ [7], with the prediction of the statistical model (solid curve) and the set 1 of the parametrization proposed in Ref. [8] (dashed curve).

little Q^2 dependence. Note that our prediction for the large x behavior, differs from most of the current literature, namely $d(x)/u(x) \to 0$ for $x \to 1$, but we find $d(x)/u(x) \to 0.16$ near the value 1/5, a prediction originally formulated in Ref. [6]. This is a very challenging question, since the very high-x region remains poorly known.

To continue our tests of the unpolarized parton distributions, we must come back to the important question of the flavor asymmetry of the light antiquarks. Our determination of $\bar{u}(x, Q^2)$ and $\bar{d}(x, Q^2)$ is perfectly consistent with the violation of the Gottfried sum rule, for which we found the value $I_G = 0.2493$ for $Q^2 = 4\text{GeV}^2$. Nevertheless there remains an open problem with the x distribution of the ratio $\bar{d}/\bar{u}$ for $x \geq 0.2$. According to the Pauli principle, this ratio is expected to remain above 1 for any value of x. However, the E866/NuSea Collaboration [7] has released the final results corresponding to the analysis of their full data set of Drell-Yan yields from an 800 GeV/c proton beam on hydrogen and deuterium targets and, for $Q^2 = 54\text{GeV}^2$, they obtain the ratio $\bar{d}/\bar{u}$ shown in Fig. 2 (*Right*). Although the errors are rather large in the high-x region, the statistical approach disagrees with the trend of the data. Clearly by increasing the number of free parameters, it is possible to build up a scenario which leads to the drop off of this ratio for $x \geq 0.2$. For example this was achieved in Ref. [8], as shown by the dashed curve in Fig. 2 (*Right*). There is no such freedom in the statistical approach, since quark and antiquark distributions are strongly related. On the experimental side, there are now new opportunities for extending the $\bar{d}/\bar{u}$ measurement to larger x up to $x = 0.7$, with the running E906 experiment at the 120 GeV Main Injector at Fermilab [9] and a proposed experiment at the new 30-50 GeV proton accelerator at J-PARC [10].

Analogous considerations can be made for the corresponding helicity distributions, whose most recent determinations are shown in Fig. 3 (*Left*). By using a similar argument as above, the ratio $\Delta u(x)/u(x)$ is predicted to have a rather fast increase in the x range $(X_{0u}^- - \bar{x}, X_{0u}^+ + \bar{x})$

and a smoother behaviour above, while $\Delta d(x)/d(x)$, which is negative, has a fast decrease in the x range $(X_{0d}^{+} - \bar{x}, X_{0d}^{-} + \bar{x})$ and a smooth one above. This is exactly the trends displayed in Fig. 3 (*Right*) and our predictions are in perfect agreement with the accurate high-x data. We note the behavior near $x = 1$, another typical property of the statistical approach, is also at variance with predictions of the current literature. The fact that $\Delta u(x)$ is more concentrated in the higher x region than $\Delta d(x)$, accounts for the change of sign of $g_1^n(x)$, which becomes positive for $x > 0.5$, as first observed at Jefferson Lab [13]. Concerning the light antiquark helicity

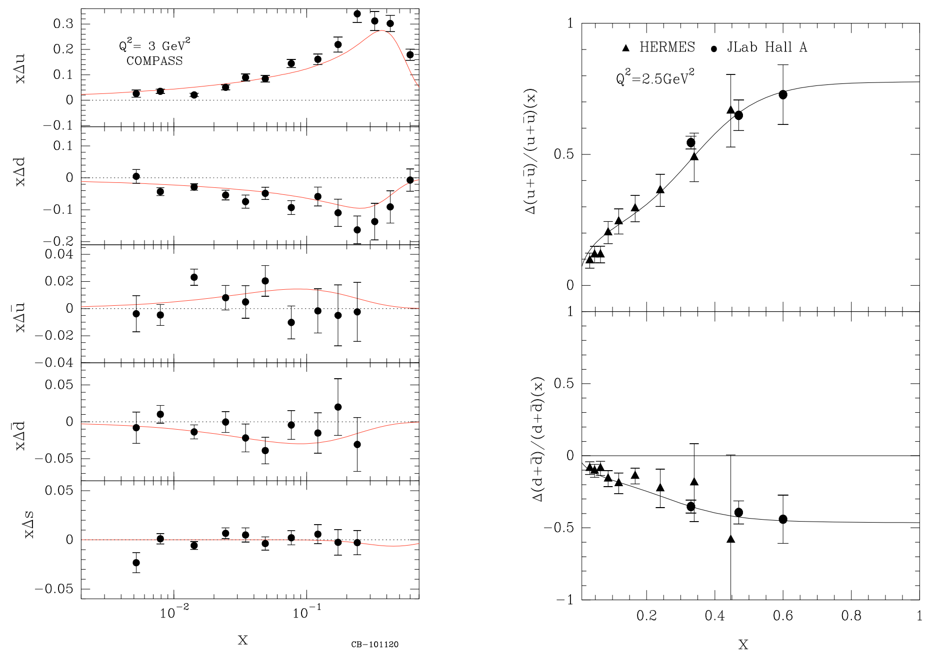

Figure 3: *Left* : Quark and antiquark helicity distributions as a function of x for $Q^2 = 3\text{GeV}^2$. Data from COMPASS [11]. The curves are predictions from the statistical approach. *Right* : Ratios $(\Delta u + \Delta\bar{u})/(u + \bar{u})$ and $(\Delta d + \Delta\bar{d})/(d + \bar{d})$ as a function of x. Data from Hermes for $Q^2 = 2.5\text{GeV}^2$ [12] and a JLab Hall A experiment [13]. The curves are predictions from the statistical approach.

distributions, the statistical approach imposes a strong relationship to the corresponding quark helicity distributions. In particular, it predicts $\Delta\bar{u}(x) > 0$ and $\Delta\bar{d}(x) < 0$, with almost the same magnitude, in contrast with the simplifying assumption $\Delta\bar{u}(x) = \Delta\bar{d}(x)$, often adopted in the literature. The COMPASS data [14] give $\Delta\bar{u}(x) + \Delta\bar{d}(x) \simeq 0$, which implies either small or opposite values for $\Delta\bar{u}(x)$ and $\Delta\bar{d}(x)$. Indeed $\Delta\bar{u}(x) > 0$ and $\Delta\bar{d}(x) < 0$, predicted by the statistical approach [1] (see Fig. 3 (*Left*), lead to a non negligible positive contribution of the sea to the Bjorken sum rule, an interesting consequence. For lack of space we only mention the extension to the transverse momentum dependence (TMD), an important aspect of the statistical PDF and we refer the reader to Ref. [15].

A new set of PDF was constructed in the framework of a statistical approach of the nucleon. All unpolarized and polarized distributions depend upon *nine* free parameters for light quarks and gluon, with some physical meaning. New tests against experimental (unpolarized and polarized) data on DIS, Semi-inclusive DIS and also hadronic processes, are very satisfactory. It has a good predictive power, but some special features remain to be verified, specially in the high-x region, a serious challenge for the future.

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