Photon impact factor for BFKL pomeron at next-to-leading order

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DOI: `http://dx.doi.org/10.3204/DESY-PROC-2012-02/128`

An analytic expression in momentum space of the next-to-leading order photon impact factor for small-x deep inelastic scattering will be presented. The result is obtained using the operator product expansion in Wilson lines.

1 Introduction

In order to obtain an analytic expression of the NLO photon impact factor for BFKL pomeron, we use the high-energy Operator Product Expansion (OPE) in terms of Wilson lines. Our calculation mainly consist of three steps: we first obtain an analytic expression in coordinate space of the NLO impact factor, then, we obtain its Mellin representation, and finally we perform the Fourier transform in momentum space.

The logic of the high-energy OPE is the same as the one for the usual OPE. In order to find a certain asymptotical behavior of an amplitude using the OPE technique one introduces a factorization scale which factorize the amplitude into a product of coefficient functions and matrix elements of the relevant operators. Then, one has to find the evolution equations of the operators with respect to the factorization scale, solve the evolution equation and finally, convolute the solution with the initial conditions for the evolution and get the amplitude.

At high energy the scattering amplitude of the process is strongly ordered in rapidity space. For this reason it is natural to introduce a factorization parameter in rapidity space which factorize, order by order in perturbation theory, the scattering amplitude in coefficient functions and matrix elements of the relevant operator. The evolution of the matrix element with respect to the factorization scale is the non-linear BK equation [1, 2]. The linearization of this non-linear equation reproduce the BFKL equation [3]. Both, the BK equation and the BFKL equation are known at NLO accuracy in QCD and in $\mathcal{N}$=4 SYM theory [5, 6, 7, 8].

2 NLO Impact Factor for DIS

To better illustrate the logic of the OPE, let us consider the T-product of two electromagnetic currents which will be relevant for Deep Inelastic scattering (DIS) when evaluated in the target (nucleon or nucleus) state. The technique we are using is the background field technique: we consider the T-product of two electromagnetic currents in a background of gluon field. In the spectator frame the background field reduces to shock wave (for a review see [4]). In DIS, in the dipole model, the virtual photon which mediate the interaction between the lepton and the nucleon, splits in a quark anti-quark pair long before the interaction with the target. The propagation of the quark anti-quark pair in the background of a shock wave, reduces to two Wilson lines. If the quark fluctuate perturbatively in a quark and a gluon before interacting with the target, then the number of Wilson lines increases. Formally, we can write down the expansion of the T-product of two electromagnetic currents in the following way

$$
\begin{aligned}
T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} &= \int d^2z_1 d^2z_2\ I^{\rm LO}_{\mu\nu}(z_1,z_2,x,y)[\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm comp.}\\
&+ \int d^2z_1 d^2z_2 d^2z_3\ I^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}] + \cdots \qquad (1)
\end{aligned}
$$

where $U_x = \mathrm{Pexp}(\mathrm{ig}\int \mathrm{dx}^+ \mathrm{A}^-(\mathrm{x}^+ + \mathrm{x}_\perp)$ is the Wilson line. In Eq. (1), the coefficient $I^{\rm LO}_{\mu\nu}$ represents the leading order impact factor, while the NLO impact factor is given by the coefficient $I^{\rm NLO}_{\mu\nu}$. In QCD Feynmann diagrams at tree level are conformal invariant. The LO impact factor is indeed conformal invariant and it can be written in terms of conformal vectors [9] $\kappa = \frac{\sqrt{s}}{2x_*}(\frac{p_1}{s} - x^2p_2 + x_\perp) - \frac{\sqrt{s}}{2y_*}(\frac{p_1}{s} - y^2p_2 + y_\perp)$ and $\zeta_i = \left(\frac{p_1}{s} + z^2_{i\perp}p_2 + z_{i\perp}\right)$

$$
\begin{aligned}
\langle T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\}\rangle_A &= \frac{s^2}{2^9\pi^6 x_*^2 y_*^2}\int d^2z_{1\perp}d^2z_{2\perp}\frac{\mathrm{tr}\{U_{z_1}U^\dagger_{z_2}\}}{(\kappa\cdot\zeta_1)^3(\kappa\cdot\zeta_2)^3}\times\\
&\frac{\partial^2}{\partial x^\mu \partial y^\nu}\left[2(\kappa\cdot\zeta_1)(\kappa\cdot\zeta_2) - \kappa^2(\zeta_1\cdot\zeta_2)\right] + O(\alpha_s) \qquad (2)
\end{aligned}
$$

The NLO impact factor is also a tree level diagram, but it is not conformal invariant due to the rapidity divergence present at this order. Since we regularize such divergence by rigid cut-off, we introduce terms which violate the conformal invariance. In order to restore the symmetry we introduce counterterms which form the composite operator. The procedure of restoring the loss of conformal symmetry due to the regularization of the rapidity divergence by rigid cut-off, is analog to the procedure of restoring gauge invariance by adding counterterms to local operator when the rigid cut-off is used instead of dimensional regularization, which automatically preserve gauge symmetry, to regulate ultraviolet divergence at one loop order. In Eq. (1), the composite operator is

$$
\begin{aligned}
[\mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]^{\rm conf} &= \mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}\\
&+ \frac{\alpha_s}{4\pi}\int d^2z_3\ \frac{z^2_{12}}{z^2_{13}z^2_{23}}[\frac{1}{N_c}\mathrm{tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}\mathrm{tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - \mathrm{Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]\ln\frac{az^2_{12}}{z^2_{13}z^2_{23}} + O(\alpha_s^2) \qquad (3)
\end{aligned}
$$

The parameter a is analog to μ_F in the usual OPE. Note also that at this order the operator proportional to the NLO impact factor does not need to be modified. It would get a counterterm at NNLO accuracy. Using, then, the composite operator the NLO impact factor is conformal invariant and it can be written entirely in terms of the conformal vectors we defined above. See Ref. [10] for its explicit expression. Such result is an analytic expression of the photon impact factor in coordinate space which is relevant for DIS off a large nucleus where the non linear operator appearing at NLO level is relevant at high parton density regime. What we are interested in is the NLO impact factor for BFKL pomeron. Thus, our next step, before proceeding to the calculation of the Mellin representation, is to obtain the linearization of result in coordinate space in the non-linear case. It turns out that the coordinate representation of the NLO impact factor in the linearized case can be written as a linear combination of five conformal tensor structures [10]. In Ref. [9] it was indeed predicted that any impact factor can be written as a linear combination of the same conformal tensor structures. The projection of the impact factor on the Lipatov eigenfunctions with conformal spin 0 is related to the unpolarized structure function for DIS. While the projection on the Lipatov eigenfunction with conformal spin 2 is related to the polarized structure function. The result of the Mellin representation can be found in Ref. [11]. Once we have performed the Mellin representation, we are ready to perform the Fourier transform in momentum space. The result is

$$\begin{aligned}I^{\mu\nu}(q,k_\perp) &= \frac{N_c}{32}\int\frac{d\nu}{\pi\nu}\frac{\sinh\pi\nu}{(1+\nu^2)\cosh^2\pi\nu}\Big(\frac{k_\perp^2}{Q^2}\Big)^{\frac{1}{2}-i\nu}\Big\{\Big[\Big(\frac{9}{4}+\nu^2\Big)\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\mathcal{F}_1(\nu)\Big)P_1^{\mu\nu}\\ &+\Big(\frac{11}{4}+3\nu^2\Big)\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\mathcal{F}_2(\nu)\Big)P_2^{\mu\nu}\Big]\\ &+\frac{\frac{1}{4}+\nu^2}{2k_\perp^2}\Big(1+\frac{\alpha_s}{\pi}+\frac{\alpha_sN_c}{2\pi}\mathcal{F}_3(\nu)\Big)\big[\tilde{P}^{\mu\nu}\bar{k}^2+\bar{P}^{\mu\nu}\tilde{k}^2\big]\Big\}\end{aligned} \tag{4}$$

where

$$\begin{aligned}P_1^{\mu\nu} &= g^{\mu\nu}-\frac{q_\mu q_\nu}{q^2}; \qquad P_2^{\mu\nu} = \frac{1}{q^2}\Big(q^\mu-\frac{p_2^\mu q^2}{q\cdot p_2}\Big)\Big(q^\nu-\frac{p_2^\nu q^2}{q\cdot p_2}\Big)\\ \bar{P}^{\mu\nu} &= \big(g^{\mu1}-ig^{\mu2}-p_2^\mu\frac{\bar{q}}{q\cdot p_2}\big)\big(g^{\nu1}-ig^{\nu2}-p_2^\nu\frac{\bar{q}}{q\cdot p_2}\big)\\ \tilde{P}^{\mu\nu} &= \big(g^{\mu1}+ig^{\mu2}-p_2^\mu\frac{\tilde{q}}{q\cdot p_2}\big)\big(g^{\nu1}+ig^{\nu2}-p_2^\nu\frac{\tilde{q}}{q\cdot p_2}\big)\end{aligned}$$

$$\begin{aligned}\mathcal{F}_{1(2)}(\nu) &= \Phi_{1(2)}(\nu)+\chi_\gamma\Psi(\nu), \qquad \mathcal{F}_3(\nu) = F_6(\nu)+\Big(\chi_\gamma-\frac{1}{\bar{\gamma}\gamma}\Big)\Psi(\nu),\\ \Psi(\nu) &\equiv \psi(\bar{\gamma})+2\psi(2-\gamma)-2\psi(4-2\gamma)-\psi(2+\gamma),\\ F_6(\gamma) &= F(\gamma)-\frac{2C}{\bar{\gamma}\gamma}-1-\frac{2}{\gamma^2}-\frac{2}{\bar{\gamma}^2}-3\frac{1+\chi_\gamma-\frac{1}{\gamma\bar{\gamma}}}{2+\bar{\gamma}\gamma},\\ \Phi_1(\nu) &= F(\gamma)+\frac{3\chi_\gamma}{2+\bar{\gamma}\gamma}+1+\frac{25}{18(2-\gamma)}+\frac{1}{2\bar{\gamma}}-\frac{1}{2\gamma}-\frac{7}{18(1+\gamma)}+\frac{10}{3(1+\gamma)^2}\\ \Phi_2(\nu) &= F(\gamma)+\frac{3\chi_\gamma}{2+\bar{\gamma}\gamma}+1+\frac{1}{2\bar{\gamma}\gamma}-\frac{7}{2(2+3\bar{\gamma}\gamma)}+\frac{\chi_\gamma}{1+\gamma}+\frac{\chi_\gamma(1+3\gamma)}{2+3\bar{\gamma}\gamma},\\ F(\gamma) &= \frac{2\pi^2}{3}-\frac{2\pi^2}{\sin^2\pi\gamma}-2C\chi_\gamma+\frac{\chi_\gamma-2}{\bar{\gamma}\gamma}\end{aligned}$$

In order to obtain the full expression in momentum space of the NLO DIS amplitude, we need to perform also the Fourier transform of the NLO linearized BK equation for the dipole form of the unitegrated gluon distribution $\mathcal{V}(z)=z^{-2}\mathcal{U}(z)$ where $\mathcal{U}(x,y)=1-N_c^{-1}\mathrm{tr}\{U(x_\perp U^\dagger(y_\perp)\}$ [11]. The $k_{\rm T}$ factorized form of the NLO amplitude for DIS can be found in Ref. [11].

3 Conclusions

An analytic expression of the NLO photon impact factor in momentum space for the pomeron contribution, given in formula (4), and the $k_{\rm T}$ factorization formula, up to NLO accuracy, of the NLO DIS amplitude, given in [11], has been presented. So far the NLO impact factor was available only as a combination of numerical and analytical expressions [12]. The $k_{\rm T}$ factorization formula given in [11] is intimately related to the two-gluon (pomeron) exchange. In order, then, to obtain a factorized form of the DIS amplitude in $k_{\rm T}$ space, we considered the linearized case. We then obtained first a Mellin representation of the NLO impact factor and finally performed the Fourier transform in momentum space and obtained the first main result presented here, formula (4). A $k_{\rm T}$ factorization formula can be obtained also for the NLO amplitude of the γ^*-γ^* scattering (see Ref. [13] for the $\mathcal{N}$=4 SYM theory case). One simply needs to perform the Fourier transform of the NLO evolution equation for the $\mathcal{U}(x)$ operator. This would represent a proof, for the first time, of the validity of the factorization in $k_{\rm T}$ space at NLO accuracy also for γ^*-γ^* scattering, since an analytic expression of the NLO photon impact factor in momentum space was not known before. The $k_{\rm T}$ factorization form of DIS amplitude [11] as well as the one for γ^*-γ^* scattering are simply a consequence of the success of the high-energy OPE.

The author is grateful to the organizers of DIS 2012 conference for financial support.

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