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On the Status of α_s

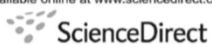
Insights and Open Issues

André Hoang,
Faculty for Physics
University of Vienna

Content

What this talk is NOT about:

World Average of α_s

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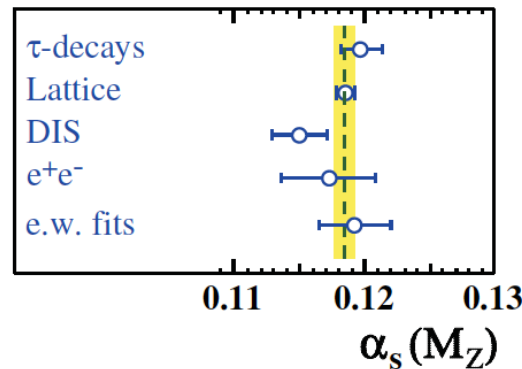
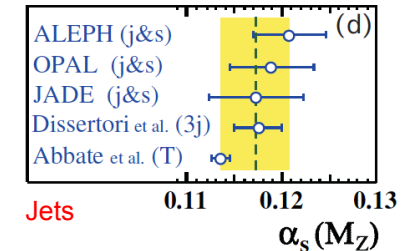
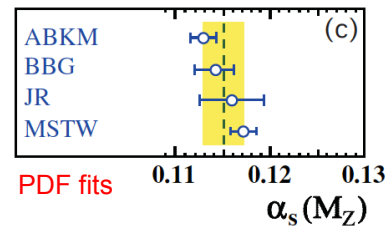
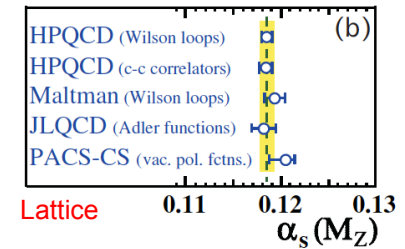
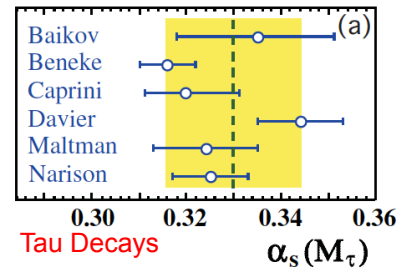
World Summary of α_s (2011)

Siegfried Bethke
 Max-Planck-Institute for Physics, Föhringer Ring 6, 80805 Munich, Germany

Abstract
 The most significant determinations of the strong coupling strength, α_s , are summarised and a new world average value of $\alpha_s(M_Z)$ is determined, using a new method of pre-averaging results within classes of measurements like hadronic τ decays, deep inelastic scattering processes, determinations on the lattice, electro-positron annihilation processes and electro-weak precision fits. The overall result is

$$\alpha_s(M_Z) = 0.1184 \pm 0.0007,$$

unchanged from its corresponding value obtained in 2009.



$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

Content

What this talk is NOT about:

World Average of α_s

Aims:

- Point out recent developments
- Provide some insight into some tricky points and open issues
- Motivate to push forward precise α_s determinations

This talk attempts to be objective, but certainly won't !

Apologies to those whose work is not mentioned and missing references !

Some material in this talk taken from presentations given at the Workshop
“Precision Measurements of α_s ” (MPI, Munich, February 9-11, 2012)



Content

- **Introduction**
- **Lattice Simulations**
- **τ -decays**
- **Eventshapes and Jet Rates**
- **DIS**
- **Conclusions**

Strong Coupling: Theory

SU(3) gauge theory:

$$\mathcal{L}_{\text{QCD}} = \mathcal{L}_{\text{classic}} + \mathcal{L}_{\text{gauge-fix}} + \mathcal{L}_{\text{ghost}}$$

$$\mathcal{L}_{\text{classic}} = -\frac{1}{4}F_{\alpha\beta}^A F_A^{\alpha\beta} + \sum_{\text{flavors } q} \bar{q}_\alpha (i\not{D} - m_q)_{\alpha\beta} q_\beta$$

$$D^\mu = \partial^\mu + ig_s A^\mu$$

$$\alpha_s \equiv \frac{g_s^2}{4\pi}$$

- SU(3) gauge symmetry does not fix strong coupling and quark masses
- QCD parameters are renormalized quantities and NOT physical
- Values are fixed by BSM short-distance physics

Strong Coupling: Theory

RG-evolution:

- Standard scheme: $\overline{\text{MS}}$ defined within dim reg
- 4-loop precision available

$$\frac{d\alpha_s(\mu)}{d \ln \mu} = \beta(\alpha_s(\mu)) = -2\alpha_s(\mu) \sum_{n=0}^{\infty} \beta_n \left(\frac{\alpha_s(\mu)}{4\pi} \right)^{n+1}$$

$$\begin{aligned}\beta_0 &= \frac{1}{4} \left[11 - \frac{2}{3} n_f \right] \\ \beta_1 &= \frac{1}{16} \left[102 - \frac{38}{3} n_f \right] \\ \beta_2 &= \frac{1}{64} \left[\frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2 \right] \\ \beta_3 &= \frac{1}{256} \left[\left(\frac{149753}{6} + 3564 \zeta_3 \right) - \left(\frac{1078361}{162} + \frac{6508}{27} \zeta_3 \right) n_f \right. \\ &\quad \left. + \left(\frac{50065}{162} + \frac{6472}{81} \zeta_3 \right) n_f^2 + \frac{1093}{729} n_f^3 \right]\end{aligned}$$

β_3 : Vermaseren, Larin, v.Ritbergen 1997
Czakon 2004

Decoupling:

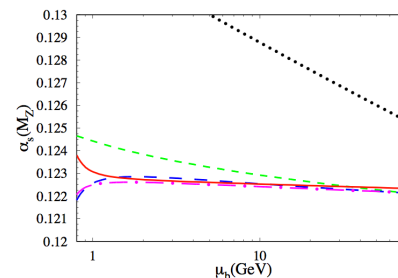
- Defined within an n_f -flavor effective theory
- Massive quarks integrated out
- 4-loop precision available

$$\alpha_s^{(n_f)}(m_h) = \alpha_s^{(n_f+1)}(m_h) \left(1 + \frac{11}{72} \left(\frac{\alpha_s^{(n_f+1)}(m_h)}{\pi} \right)^2 + \dots \right)$$

Schröder, Steinhauser 2005
Chetyrkin, Kühn, Sturm 2005

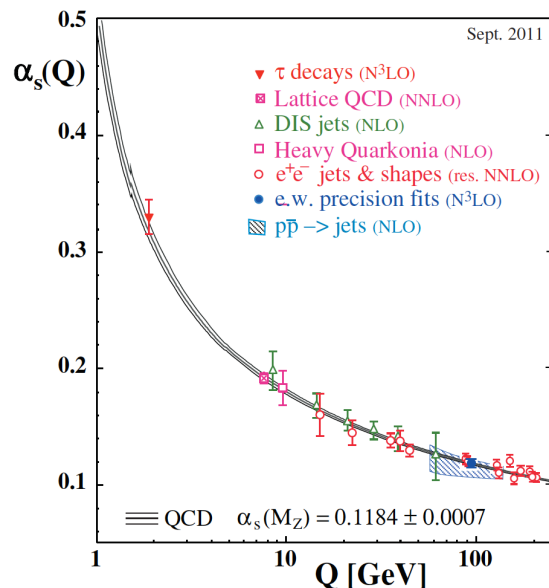
$$\alpha(m_\tau) \longrightarrow \alpha(m_Z) : \quad \delta\alpha(m_Z) = \pm 0.0004$$

Theory error smaller than most other uncertainties.



Strong Coupling: Theory

Asymptotic freedom:



- α_s perturbative only for scales larger than 1 GeV
- Currently we are only capable determining α_s from processes where perturbation theory dominates.

Λ_{QCD} :

$$\alpha_s^{(n_f)}(\mu) = \frac{4\pi}{\beta_0^n \ln(\frac{\mu^2}{\Lambda_{\text{QCD}}^2})} + \dots$$

$$\Lambda_{\overline{MS}}^{(5)} = (213 \pm 8) \text{ MeV} ,$$

$$\Lambda_{\overline{MS}}^{(4)} = (296 \pm 10) \text{ MeV}$$

$$\Lambda_{\overline{MS}}^{(3)} = (339 \pm 10) \text{ MeV}$$

- Parameter equivalent to α_s
- Frequently used to as a synonym for the scale of hadronization/confinement in dimensional counting

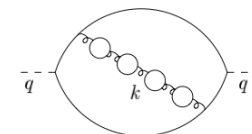
Strong Coupling: Theory

IR-Sensitivity:

- dim reg perturbation theory in gauge theories leads to enhanced sensitivity to scales $\mathcal{O}(\Lambda_{\text{QCD}})$
- „Renormalons“
- Conceptual origin of the OPE

$$\alpha_s(k) = \alpha_s(\mu) \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu) \beta_0}{4\pi} \right)^i \ln^i \left(\frac{\mu^2}{k^2} \right)$$

$$\int_0^1 dx x^{m-1} \ln^n(1/x) = \frac{n!}{m^{1+n}}$$

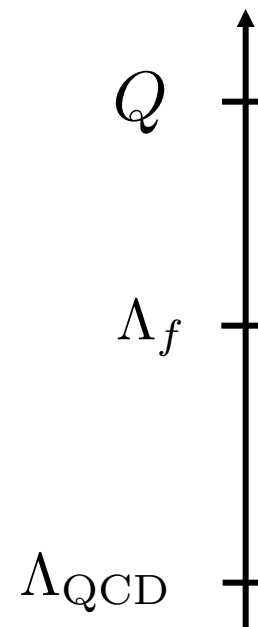


Wilson's OPE:

$$\Lambda_{\text{QCD}} < \Lambda_f < Q \quad (\text{cutoff regulator})$$

$$\sigma = C_0^W(Q, \Lambda_f) \theta_0^W(\Lambda_f) + C_1^W(Q, \Lambda_f) \frac{\theta_1^W(\Lambda_f)}{Q^p} + \dots$$

- Cleanest separation of short and long distance contributions
- renormalon-free
- EXTREMELY cumbersome for precision computations



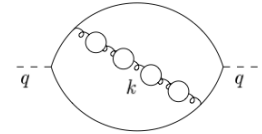
Strong Coupling: Theory

IR-Sensitivity:

- dim reg perturbation theory in gauge theories leads to enhanced sensitivity to scales $\mathcal{O}(\Lambda_{\text{QCD}})$
- „Renormalons“
- Perturbation theory only valid within OPE

$$\alpha_s(k) = \alpha_s(\mu) \sum_{i=0}^{\infty} \left(\frac{\alpha_s(\mu) \beta_0}{4\pi} \right)^i \ln^i \left(\frac{\mu^2}{k^2} \right)$$

$$\int_0^1 dx x^{m-1} \ln^n(1/x) = \frac{n!}{m^{1+n}}$$



OPE in $\overline{\text{MS}}$:

dim. reg. $d = 4 - 2\epsilon$

$$\begin{aligned} \sigma &\sim \mu^{2\epsilon} \int dk^{d-3} \frac{k^{p-1} f(k^2, \Lambda_{\text{QCD}}^2)}{(k^2 + Q^2)^{p/2}} \\ &= \mu^{2\epsilon} \int dk^{d-3} \frac{k^{p-1} f(k^2, 0) + \dots}{(k^2 + Q^2)^{p/2}} + \mu^{2\epsilon} \int dk^{d-3} k^{p-1} f(k^2, \Lambda_{\text{QCD}}) \left[\frac{1}{Q^p} + \dots \right] \\ &\quad \downarrow \qquad \qquad \qquad \downarrow \\ \sigma &= \bar{C}_0(Q, \mu) \bar{\theta}_0(\mu) + \bar{C}_1(Q, \mu) \frac{\bar{\theta}_1(\mu)}{Q^p} + \dots \end{aligned}$$

- High precision calculations manageable
- Renormalons can degrade perturbative behavior
- „Non-perturbative effects“ – and also „Duality Violation“

Lattice QCD

- QCD without gauge-fixing and ghosts
- Functional integrals formulated numerically on a finite Euklidean space-time lattice: size $N_s^3 \times N_4 \times a^4$

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp(-S) [\bullet]$$

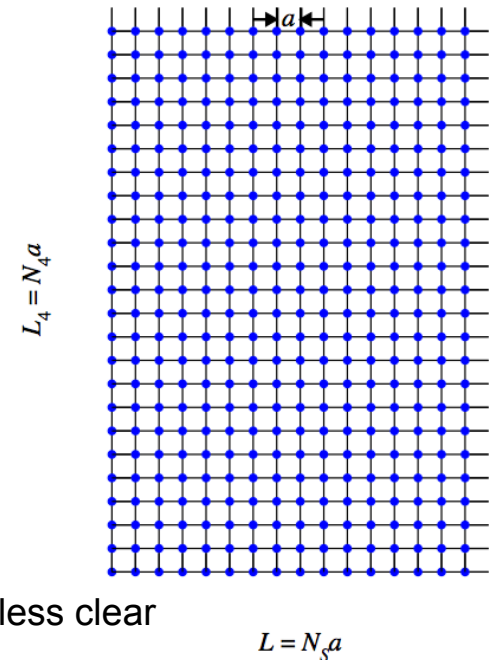
- Demanding: technical+conceptual implementation of virtual quarks (sea quarks): 4-fold replication on finite lattice

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \prod_{f=1}^{n_f} \det(\not{D} + m_f) \exp(-S_{\text{gauge}}) [\bullet]$$

- Staggered (4th root of det) fast numerically, conceptually less clear
 Wilson
 Domain-wall



clean conceptually, expensive

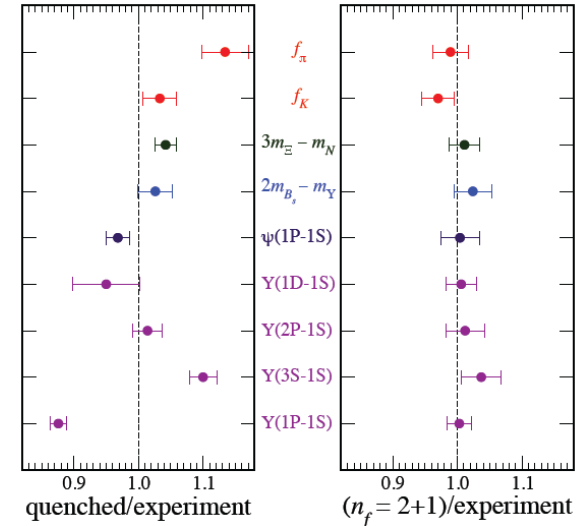


- Large scale separation vs. finite lattice: Effective Theories
 e.g. heavy quark masses: HQET, NRQCD
 Lattice spacing small: Symanzik improved action
- Chiral limit with staggered fermions: considered meaningful, but all results need to be checked with other fermion formulations
- 2(light)+1(strange) fermions: state-of-the-art

Lattice QCD

α_s -determinations:

- tune lattice to observed hadron spectrum
 - ➔ fix lattice parameters as fct of a
- Compute quantities on the lattice that can also be computed with perturbation theory („lattice replaces experiment“)
 - small Wilson loops (compare to lattice p.th.)
 - Current correlators (lattice + continuum p.th.)
 - Adler function (lattice + continuum p.th.)
 - QCD energy in cylinder (lattice p.th.)



$\alpha_{\overline{\text{MS}}}^{(n_f=5)}(M_Z)$	R	Q	range	$\mathcal{R}$	sea	Ref.
0.1170(12)			3			[9]
0.1183(8)	Wilson loops	a^{-1}	7	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[10]
0.1192(11)			7			[11]
0.1174(12)	quarkonium	$2m_c$	1–2	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[12]
0.1183(7)	correlators	$2m_Q$	3–6	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[13]
0.1181(3) $^{+14}_{-12}$	Adler function	Q	5	NNLO*	2+1 overlap	[14]
0.1205(8)(5) $^{+0}_{-17}$	“QCD in a can”		80		2+1 Wilson	[15]
0.1000(16) †	aka Schrödinger	L^{-1}	270	asymptotic	2 Wilson	[16]
0.1____(____)	functional		1000		2+1+1 Wilson	[17]

HPQCD

JLQCD

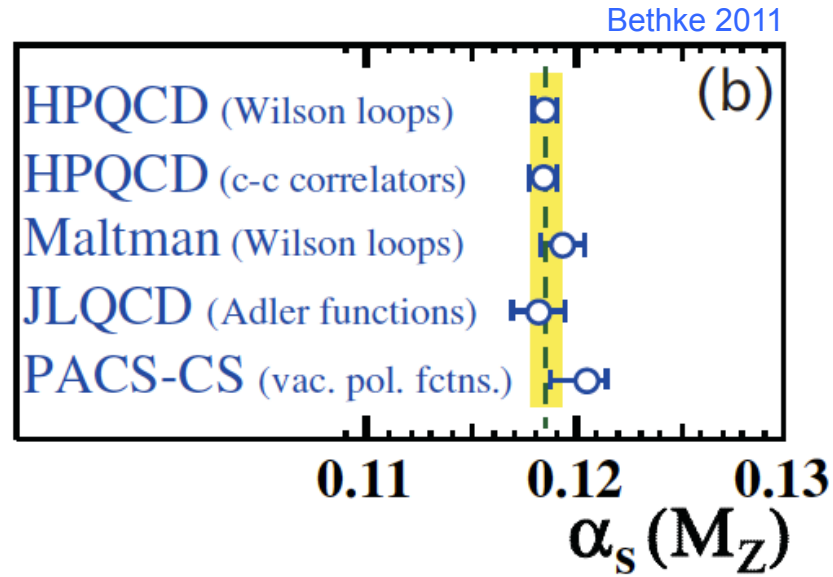
PACS-CS

ALPHA

α_s -determinations:

Results live from their consistency within well-developed methods.

World average dominated by lattice results.



Is that it ?

Any reason to go on ? - YES !

- (1) Results should be cross checked (from lattice and other methods)
- (2) Finding the origin of disagreement to other methods motivates dedicated studies

Ability to make precision measurements reflects the true understanding of the method of theory.

τ Decay

Total hadronic τ width:
$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)}$$

Braaten, Narison, Pich 1992

→ Related to absorptive part of V and AV current correlators

$$R_{\tau,V+A} = N_C |V_{ud}|^2 S_{\text{EW}} \{1 + \delta_P + \delta_{\text{NP}}\}$$

electroweak corr.

$$S_{\text{EW}} = 1.0201 \pm 0.0003$$

non-perturb. corr. (OPE)

$$\delta_{\text{NP}} = -0.0059 \pm 0.0014$$

QCD corrections

$O(\alpha_s^4)$: Baikov, Chetyrkin, Kühn 2008

$$\delta_P = \sum_{n=1} K_n A^{(n)}(\alpha_s) = \sum_{n=1} (K_n + g_n) a_\tau^n \equiv \sum_{n=1} r_n a_\tau^n \quad a_\tau \equiv \frac{\alpha_s(m_\tau)}{\pi}$$

Contour-improved (CIPT)

Fixed-order (FOPT)

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left(\frac{\alpha_s(-s)}{\pi} \right)^n \left(1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right)$$

- Resums large β_n -logs to all orders
- Improved convergence

- Treats all corrections on equal footing
- Smaller scale-dependence

Contour-improved (CIPT)

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- Well behaved

Fixed-order (FOPT)

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Typical results at $O(\alpha_s^4)$ [lots of literature]

$$\alpha_s(m_\tau) = 0.344 \pm 0.015$$

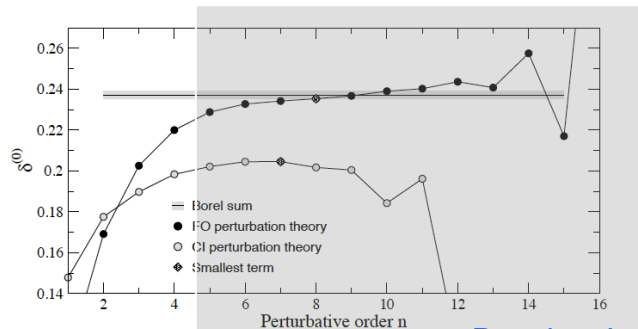
$$\alpha_s(m_Z) = 0.1210 \pm 0.0015$$

$$\alpha_s(m_\tau) = 0.321 \pm 0.015$$

$$\alpha_s(m_Z) = 0.1185 \pm 0.0015$$

→ Studies based on models of asymptotic behavior of the perturbative series do not favor any expansion

Beneke, Jamin 2008
Descots-Genon 2010



Beneke, Jamin

Contour-improved (CIPT)

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➔ Studies based on models of asymptotic behavior of the perturbative series do not favor any expansion

Beneke, Jamin 2008
Descots-Genon 2010

➔ Expansion look well, BUT ARE close to being unstable.

Common variation methods unreliable to estimate perturbative error.

What is the theoretical handle that explains the discrepancy ?

Duality Violation:

Boito, Cata, Golterman, Jamin, Maltman,
Osborne, Peris 2011

→ m_τ not sufficiently heavy such that OPE (p.th. + condensates) is justified
at very high precision

(resonances not sufficient “smeared” out)

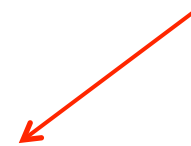
(recall: OPE = expansion in $\Lambda_{\text{QCD}}/m_\tau$)

$$\int_0^{s_0} ds w(s) \rho_{V,A}^{(1+0)}(s) = \frac{i}{2\pi} \oint_{|s|=s_0} ds w(s) \Pi_{V,A}^{(1+0)}(s)$$

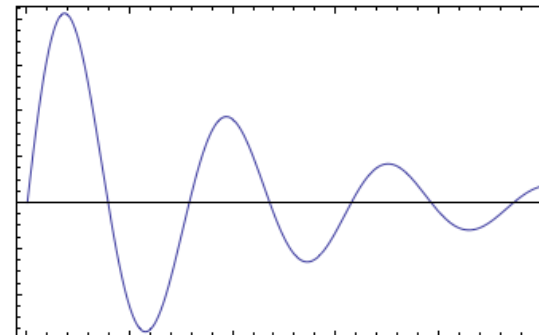
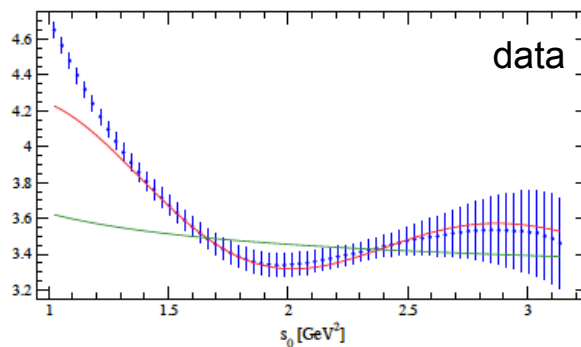
$$\Pi_{V,A}^{(1+0)}(s) = \Pi_{V,A}^{\text{OPE}}(s) + \Delta_{V,A}(s)$$

$$\rho_{V,A}^{\text{DV}}(s) = \kappa_{V,A} e^{-\gamma_{V,A}s} \sin(\alpha_{V,A} + \beta_{V,A}s)$$

model (no first principle
computation possible)
parameters must be fitted



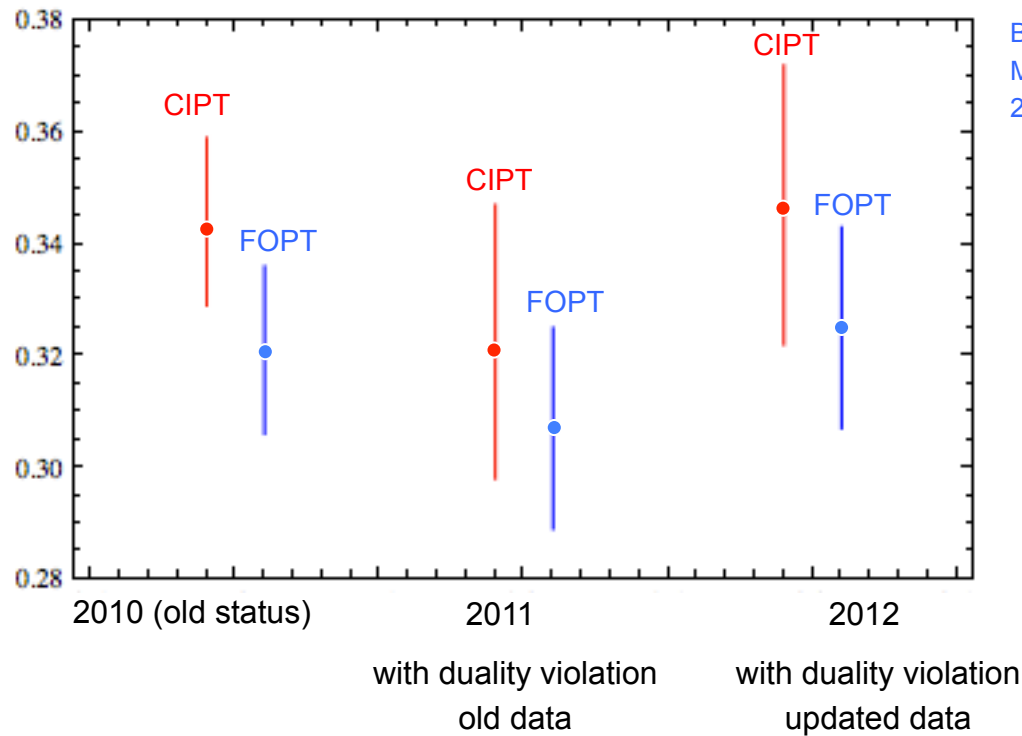
Behavior of non-perturbative effects in
the Minkowskian region



τ Decay

Duality Violation:

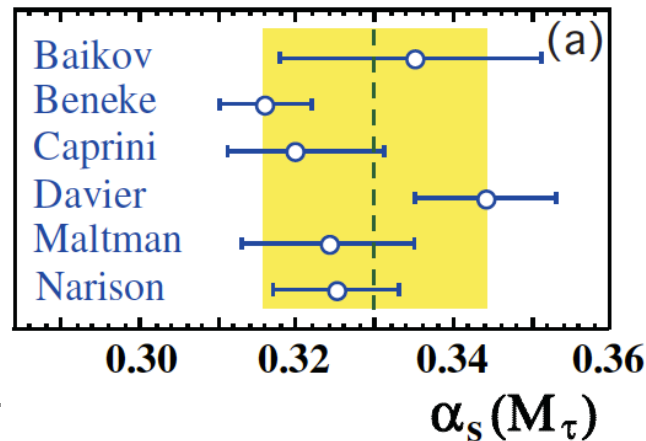
Discrepancy reconciled at the cost of larger errors.



Boito, Cata, Golterman, Jamin, Maltman, Osborne, Peris 2011+2012

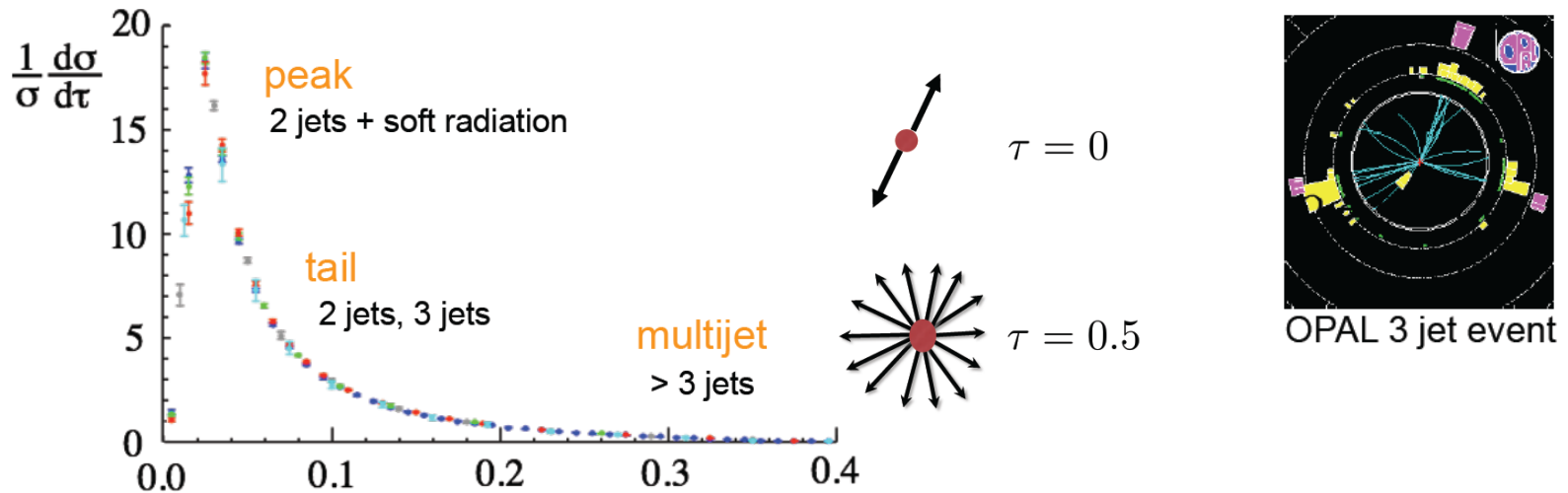
Bethke 2011

Uncertainties from spread of central values.



Eventshape Distributions (e+e-)

- Single variable measure of global topology properties of hadronic events (primary $q\bar{q}$)
Variable zero for $q\bar{q}$ final state (2 back-to-back zero mass jets)



Thrust:

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \quad \tau = 1 - T$$

Heavy jet mass $p_{H\pm}$:

Heavier of the invariant masses of all particle in the 2 hemispheres w.r. to the thrust axis

C-parameter:

$$C = \frac{3}{2} \frac{\sum_{ij} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$

Jet broadening:

$$B_k = \left(\frac{\sum_{i \in H_k} |\vec{p}_i \times \vec{n}_T|}{2 \sum_i |\vec{p}_i|} \right) \quad B_T = B_1 + B_2$$

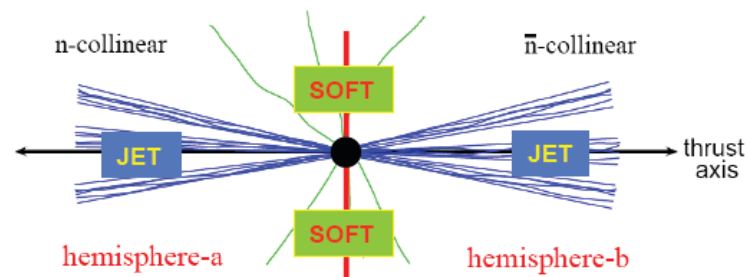
y_{23} : Transition value between 2 and 3 jets (Durham jet algorithm)

Eventshape Distributions (e+e-)

→ Perturbation theory: $\frac{d\sigma}{d\tau} \sim \delta(\tau) + \mathcal{O}(\alpha_s)$

$\mathcal{O}(\alpha_s)$ is LO for $\tau > 0$

→ IR-safe & perturbatively inclusive (e.g. thrust)



$$\frac{d\sigma}{d\tau} = \sigma_0 H(Q, \mu) Q \int d\ell J_T(Q^2 \tau - Q\ell, \mu) S_T(\ell, \mu)$$

Korshemski, Sterman

$$\mu_h \sim Q$$

perturbative

$$\mu_{\text{jet}} \sim Q\sqrt{\tau}$$

perturbative

$$\mu_{\text{soft}} \sim Q\tau \sim \Lambda$$

ρ_H, C : similar to thrust

B: $\mu_{\text{jet}} \sim \mu_{\text{soft}}$

→ High precision era: NNLO, $\mathcal{O}(\alpha_s^3)$ perturbative corrections
partonic Monte-Carlo program

Gehrmann-DeRidder, Gehrmann,
Glover, Heinrich 2007

Weinzierl 2009

Eventshape Distributions (e+e-)

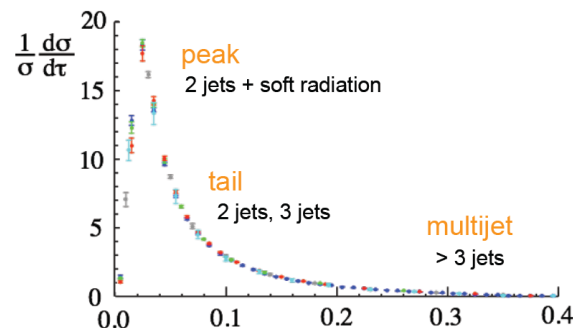
→ Fixed-Order and log summation:

- NNLO (fixed-order)
- NNLO + NLL (classic resummation) Catani ea; Dokshitzer ea
- NNLO + N³LL (SCET factorization)

} all eventshapes
 $\tau, \rho_H(B)$

→ Non-perturbative Effects (tail):

- MC: hadron mode – partonic mode
- Power corrections from data

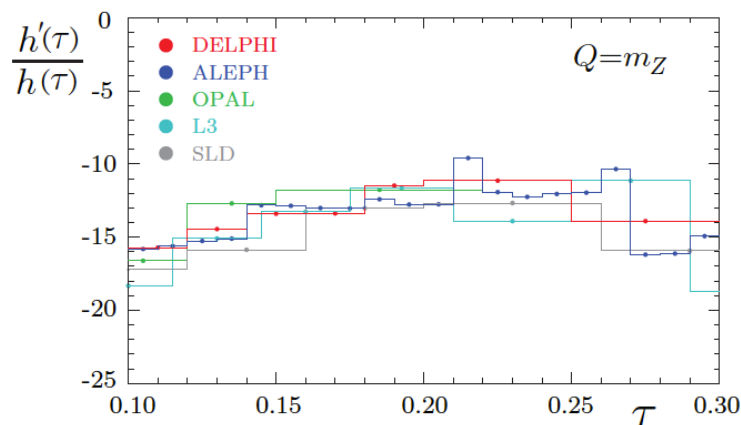


Problem of MC-estimate:

- Hadronization corrections in MC are not in the scheme needed for calculations based on NNLO corrections (MSbar+dim reg)
- MC estimates much too small (tiny ! Few % for α_s)

$$\frac{d\sigma}{d\tau} \sim h(\tau) \rightarrow \left(\tau - \frac{2\Lambda}{Q} \right)$$

$$\frac{\delta\alpha_s}{\alpha_s} \sim \frac{2\Lambda}{Q} \frac{h'(\tau)}{h(\tau)} \sim 9\%$$

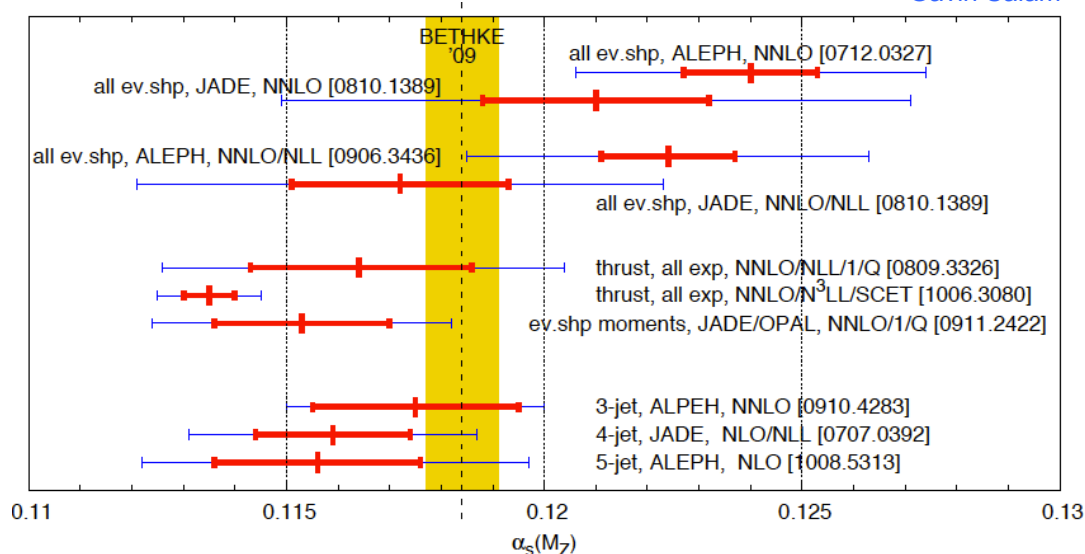


Abbate, Fickinger, Hoang, Mateu, Stewart 2010

Eventshape Distributions (e+e-)

Only 3 NNLO-based analyses left :

Gavin Salam



Davison, Webber 2009 (all event shapes)
 Abbate, Fickinger, Hoang, Mateu, Stewart 2010 (thrust)
 Gehrmann, Jaquier, Luisoni 2010 (all event shapes)

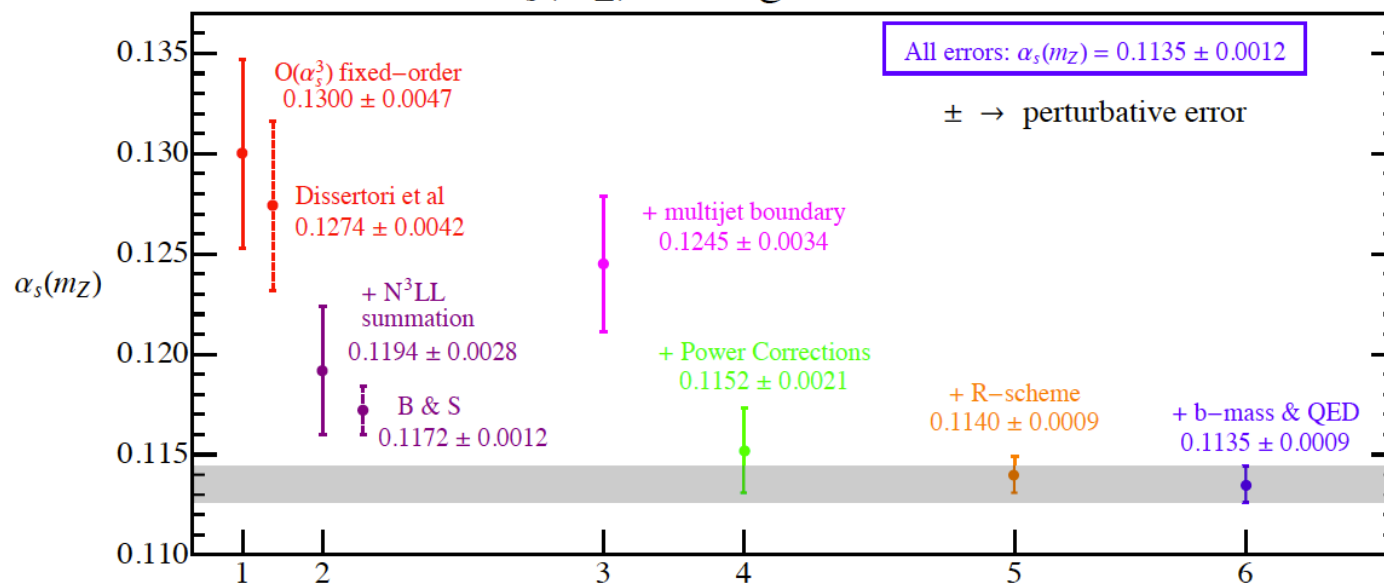
Jet rates

All results for $\alpha_s(m_Z)$ are low !!

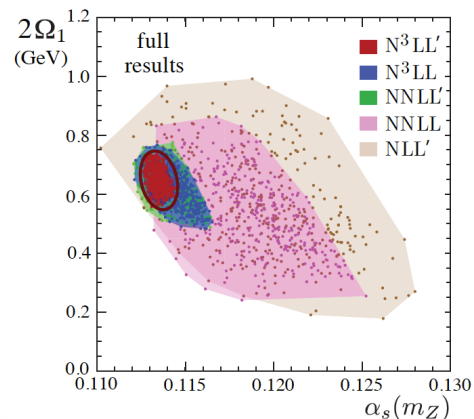
Eventshape Distributions (e+e-)

$$\left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{sing}} \sim \sigma_0 H(Q, \mu_Q) U_H(Q, \mu_Q, \mu_s) \int d\ell d\ell' U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s) J_T(Q\ell', \mu_j) S_T(\ell - \Delta, \mu_s)$$

$\alpha_s(m_Z)$ from global thrust fits



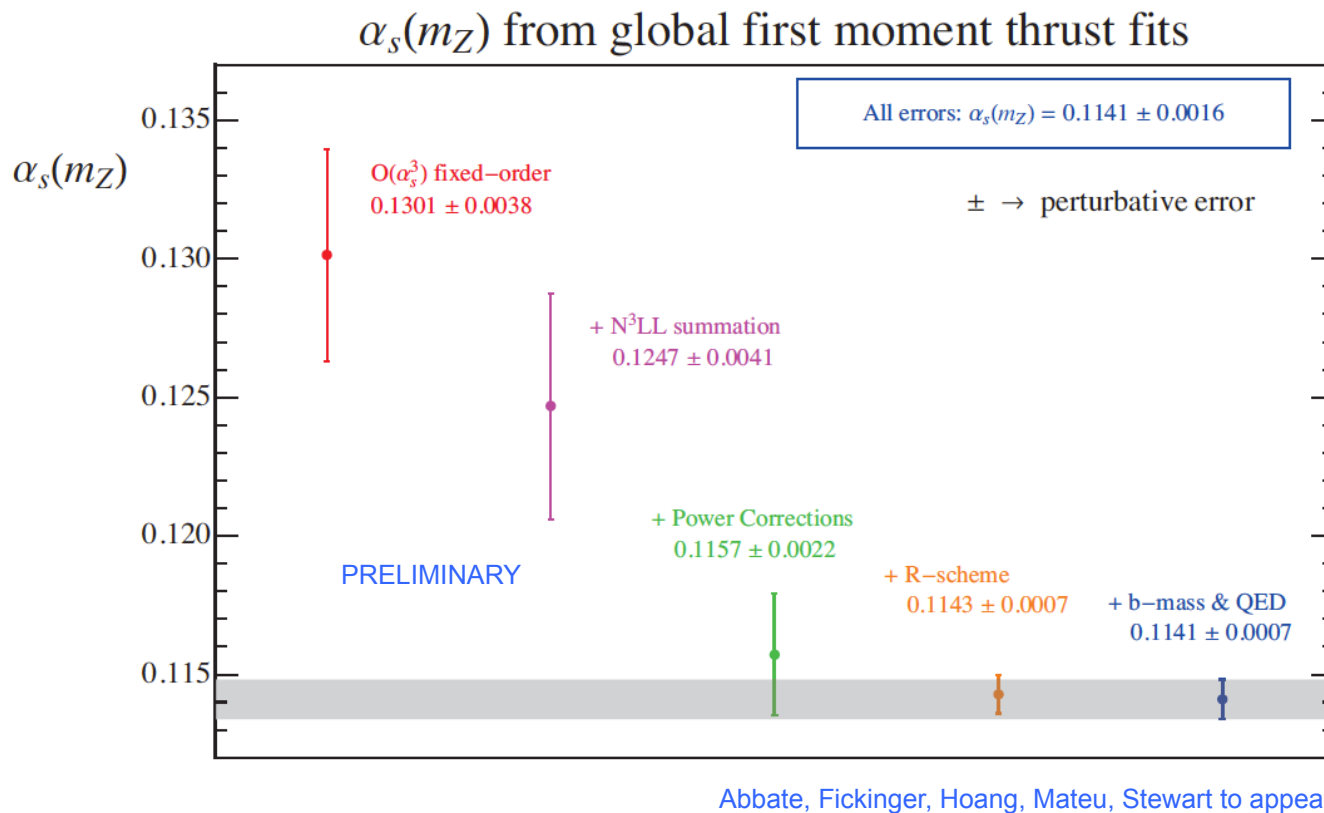
Abbate, Fickinger, Hoang, Mateu, Stewart 2010



- Central values for α_s decrease with order
- Very good convergence

Eventshape Distributions (e+e-)

NEW:



- SCET analyses for other event shapes needed !

	$\alpha_s(M_Z)$	
BBG	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO [60]
BB	0.1132 ± 0.0022	valence analysis, NNLO [110]
GRS	0.112	valence analysis, NNLO [160]
ABKM	0.1135 ± 0.0014	HQ: FFNS $n_f = 3$ [16]
ABKM	0.1129 ± 0.0014	HQ: BSMN-approach [16]
JR	0.1124 ± 0.0020	dynamical approach [20]
JR	0.1158 ± 0.0035	standard fit [20]
ABM11	0.1134 ± 0.0011	
MSTW	0.1171 ± 0.0014	[158]
NN21	0.1173 ± 0.0007	[124]
CT10	0.118 ± 0.005	[161]

Table 4.10: Summary of recent NNLO QCD analyses of the DIS world data, supplemented by related measurements using other processes.

- Central values for α_s decrease NLO to NNLO
- All below the world average.

Conclusions

- Ability to make precise measurements of the strong coupling reflects our understanding and control of QCD in various areas.
- High precision measurements and methods claiming very small theoretical uncertainties HAVE TO BE challenged and questioned. Even new and difficult-to-be-answered questions should be asked.
- Why do jet observables all seem to give low α_s with NNLO included ?
What does this mean ?
Maybe NNNLO helps ?
- Eventually: measurements of α_s at the LHC