



universität  
wien



# On the Status of !<sub>s</sub>

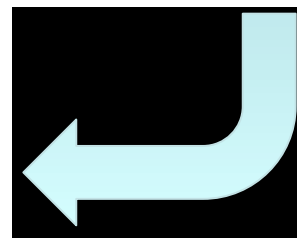
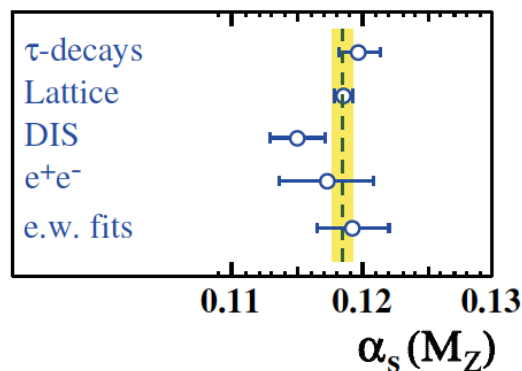
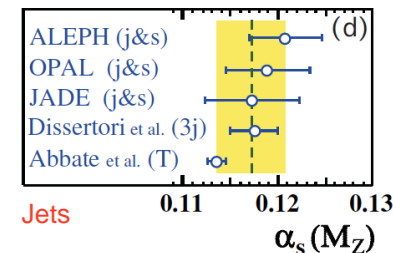
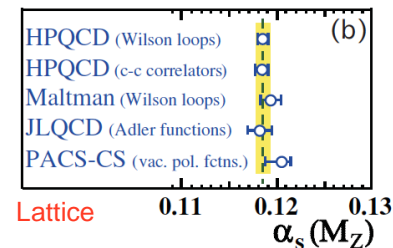
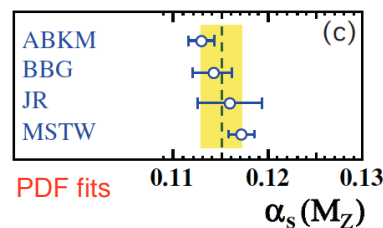
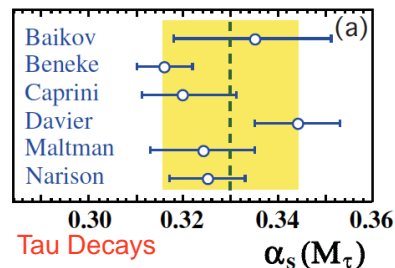
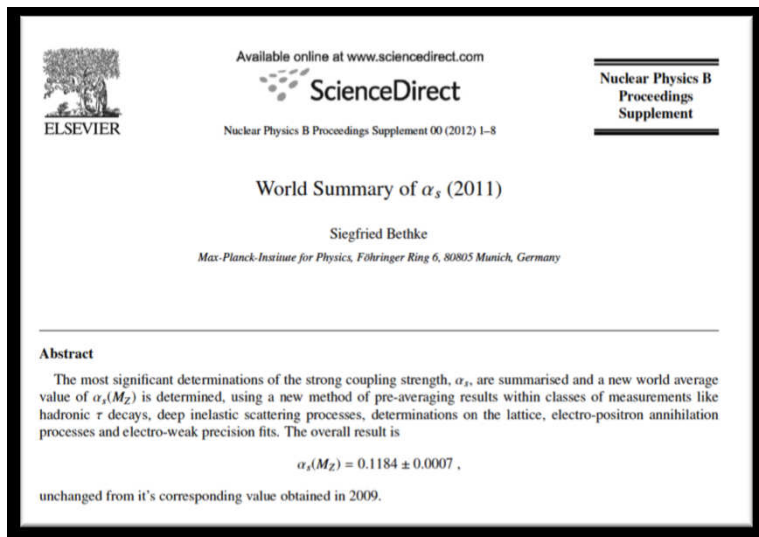
## Insights and Open Issues

André Hoang,  
Faculty for Physics  
University of Vienna

# Content

What this talk is NOT about:

World Average of  $\alpha_s$



$$\alpha_s(m_Z) = 0.1184 \pm 0.0007$$

# Content

What this talk is NOT about:

World Average of  $\alpha_s$

Aims:

- ! Point out recent developments
- ! Provide some insight into some tricky points and open issues
- ! Motivate to push forward precise  $\alpha_s$  determinations

This talk attempts to be objective, but certainly won't !

Apologies to those whose work is not mentioned and missing references !

Some material in this talk taken from presentations given at the Workshop

“Precision Measurements of  $\alpha_s$ ” (MPI, Munich, February 9-11, 2012)



# Content

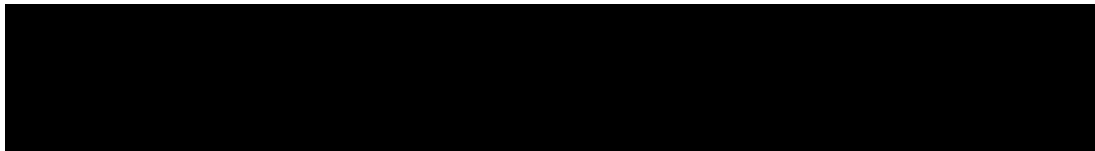
---

- ! Introduction
- ! Lattice Simulations
- !  $\Lambda$ -decays
- ! Eventshapes and Jet Rates
- ! DIS
- ! Conclusions

# Strong Coupling: Theory

---

SU(3) gauge theory:



$$D^\mu = \partial^\mu + ig_s A^\mu$$

$$\alpha_s \equiv \frac{g_s^2}{4\pi}$$

- ! SU(3) gauge symmetry does not fix strong coupling and quark masses
- ! QCD parameters are renormalized quantities and NOT physical
- ! Values are fixed by BSM short-distance physics

# Strong Coupling: Theory

## RG-evolution:

- Standard scheme:  $\overline{\text{MS}}$  defined within dim reg
- 4-loop precision available

$$\frac{d\alpha_s(\mu)}{d \ln \mu} = \beta(\alpha_s(\mu)) = -2\alpha_s(\mu) \sum_{n=0}^{\infty} \beta_n \left( \frac{\alpha_s(\mu)}{4\pi} \right)^{n+1}$$

$$\begin{aligned} \beta_0 &= \frac{1}{4} \left[ 11 - \frac{2}{3} n_f \right] \\ \beta_1 &= \frac{1}{16} \left[ 102 - \frac{38}{3} n_f \right] \\ \beta_2 &= \frac{1}{64} \left[ \frac{2857}{2} - \frac{5033}{18} n_f + \frac{325}{54} n_f^2 \right] \\ \beta_3 &= \frac{1}{256} \left[ \left( \frac{149753}{6} + 3564 \zeta_3 \right) - \left( \frac{1078361}{162} + \frac{6508}{27} \zeta_3 \right) n_f \right. \\ &\quad \left. + \left( \frac{50065}{162} + \frac{6472}{81} \zeta_3 \right) n_f^2 + \frac{1093}{729} n_f^3 \right] \end{aligned}$$

"<sub>3</sub> : Vermaseren, Larin, v.Ritbergen 1997  
Czakon 2004

## Decoupling:

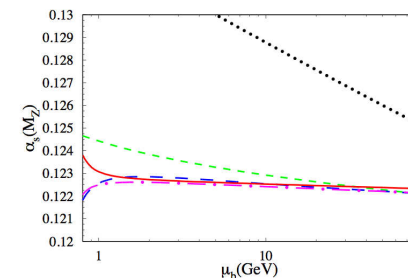
- Defined within an  $n_f$ -flavor effective theory
- Massive quarks integrated out
- 4-loop precision available

$$\alpha_s^{(n_f)}(m_h) = \alpha_s^{(n_f+1)}(m_h) \left( 1 + \frac{11}{72} \left( \frac{\alpha_s^{(n_f+1)}(m_h)}{\pi} \right)^2 + \dots \right)$$

Schröder, Steinhauser 2005  
Chetyrkin, Kühn, Sturm 2005

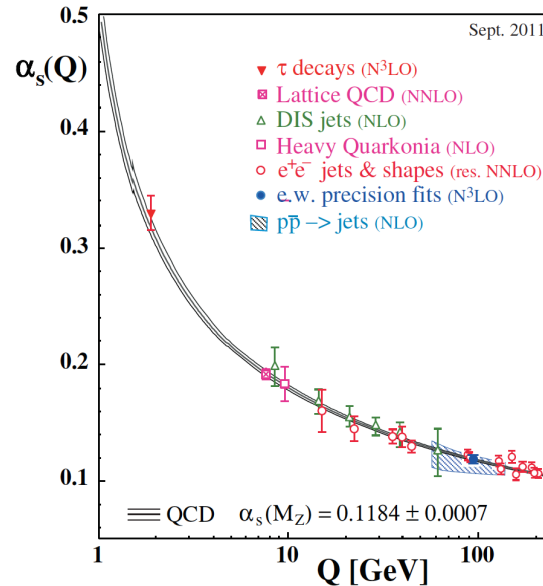
$$\alpha(m_\tau) \longrightarrow \alpha(m_Z) : \quad \delta\alpha(m_Z) = \pm 0.0004$$

Theory error smaller than most other uncertainties.



# Strong Coupling: Theory

## Asymptotic freedom:



- !  $\alpha_s$  perturbative only for scales larger than 1 GeV
- ! Currently we are only capable determining  $\alpha_s$  from processes where perturbation theory dominates.

$\Lambda_{\text{QCD}}$  :

$$\alpha_s^{(n_f)}(\mu) = \frac{4\pi}{\beta_0^n \ln(\frac{\mu^2}{\Lambda_{\text{QCD}}^2})} + \dots$$

$$\Lambda_{\overline{MS}}^{(5)} = (213 \pm 8) \text{ MeV} ,$$

$$\Lambda_{\overline{MS}}^{(4)} = (296 \pm 10) \text{ MeV}$$

$$\Lambda_{\overline{MS}}^{(3)} = (339 \pm 10) \text{ MeV}$$

- ! Parameter equivalent to  $\alpha_s$
- ! Frequently used to as a synonym for the scale of hadronization/confinement in dimensional counting

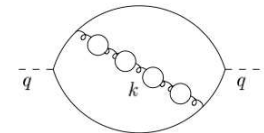
# Strong Coupling: Theory

## IR-Sensitivity:

- ! dim reg perturbation theory in gauge theories leads to enhanced sensitivity to scales  $\mathcal{O}(\Lambda_{\text{QCD}})$
- ! „Renormalons“
- ! Conceptual origin of the OPE

$$\alpha_s(k) = \alpha_s(\mu) \sum_{i=0}^{\infty} \left( \frac{\alpha_s(\mu) \beta_0}{4\pi} \right)^i \ln^i \left( \frac{\mu^2}{k^2} \right)$$

$$\int_0^1 dx x^{m-1} \ln^n(1/x) = \frac{n!}{m^{1+n}}$$

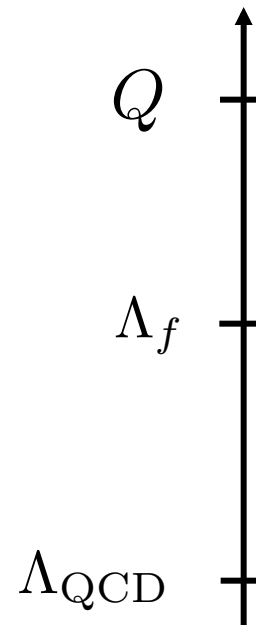


## Wilson's OPE:

$$\Lambda_{\text{QCD}} < \Lambda_f < Q \quad (\text{cutoff regulator})$$

$$\sigma = C_0^W(Q, \Lambda_f) \theta_0^W(\Lambda_f) + C_1^W(Q, \Lambda_f) \frac{\theta_1^W(\Lambda_f)}{Q^p} + \dots$$

- ! Cleanest separation of short and long distance contributions
- ! renormalon-free
- ! EXTREMELY cumbersome for precision computations



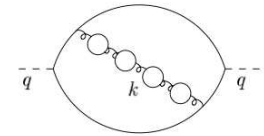


# Strong Coupling: Theory

## IR-Sensitivity:

- ! dim reg perturbation theory in gauge theories leads to enhanced sensitivity to scales  $\mathcal{O}(\Lambda_{\text{QCD}})$
- ! „Renormalons“
- ! Perturbation theory only valid within OPE

$$\alpha_s(k) = \alpha_s(\mu) \sum_{i=0}^{\infty} \left( \frac{\alpha_s(\mu) \beta_0}{4\pi} \right)^i \ln^i \left( \frac{\mu^2}{k^2} \right)$$



$$\int_0^1 dx x^{m-1} \ln^n(1/x) = \frac{n!}{m^{1+n}}$$

## OPE in $\overline{\text{MS}}$ :

dim. reg.  $d = 4 - 2\epsilon$

$$\begin{aligned} \sigma &\sim \mu^{2\epsilon} \int dk^{d-3} \frac{k^{p-1} f(k^2, \Lambda_{\text{QCD}}^2)}{(k^2 + Q^2)^{p/2}} \\ &= \mu^{2\epsilon} \int dk^{d-3} \frac{k^{p-1} f(k^2, 0) + \dots}{(k^2 + Q^2)^{p/2}} + \mu^{2\epsilon} \int dk^{d-3} k^{p-1} f(k^2, \Lambda_{\text{QCD}}) \left[ \frac{1}{Q^p} + \dots \right] \\ &\quad \downarrow \qquad \qquad \qquad \downarrow \\ \sigma &= \bar{C}_0(Q, \mu) \bar{\theta}_0(\mu) + \bar{C}_1(Q, \mu) \frac{\bar{\theta}_1(\mu)}{Q^p} + \dots \end{aligned}$$

- ! High precision calculations manageable
- ! Renormalons can degrade perturbative behavior
- ! „Non-perturbative effects“ – and also „Duality Violation“

# Lattice QCD

- ! QCD without gauge-fixing and ghosts
- ! Functional integrals formulated numerically on a finite Euklidean space-time lattice: size  $N_s^3 \times N_4 \times a^4$

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\Psi \mathcal{D}\bar{\Psi} \exp(-S) [\bullet]$$

- ! Demanding: technical+conceptual implementation of virtual quarks (sea quarks): 4-fold replication on finite lattice

$$\langle \bullet \rangle = \frac{1}{Z} \int \mathcal{D}U \prod_{f=1}^{n_f} \det(\not{D} + m_f) \exp(-S_{\text{gauge}}) [\bullet]$$

- ! Staggered (4<sup>th</sup> root of det) fast numerically, conceptually less clear

Wilson

Domain-wall



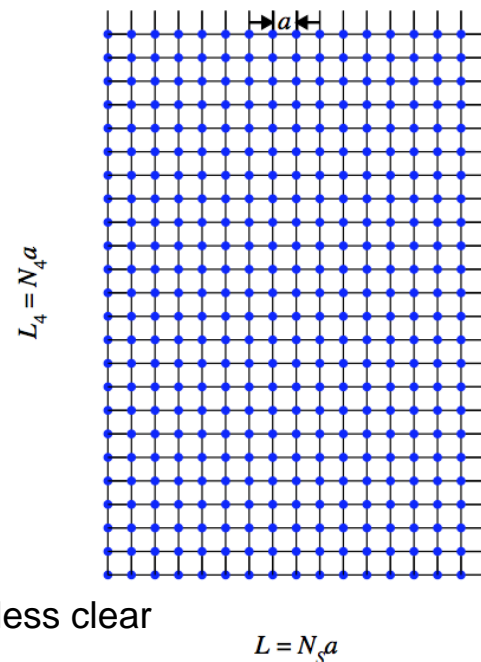
clean conceptually, expensive

- ! Large scale separation vs. finite lattice: Effective Theories

e.g. heavy quark masses: HQET, NRQCD

Lattice spacing small: Symanzik improved action

- ! Chiral limit with staggered fermions: considered meaningful, but all results need to be checked with other fermion formulations
- ! 2(light)+1(strange) fermions: state-of-the-art

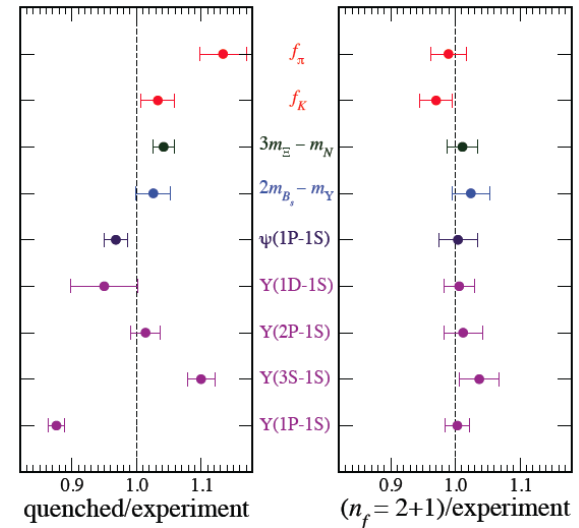


# Lattice QCD

## !<sub>s</sub>-determinations:

- ! tune lattice to observed hadron spectrum
  - ➡ fix lattice parameters as fct of a
- ! Compute quantities on the lattice that can also be computed with perturbation theory („lattice replaces experiment“)

- !! small Wilson loops (compare to lattice p.th.)
- !! Current correlators (lattice + continuum p.th.)
- !! Adler function (lattice + continuum p.th.)
- !! QCD energy in cylinder (lattice p.th.)



$\alpha_{\overline{\text{MS}}}^{(n_f=5)}(M_Z)$	$R$	$Q$	range	$\mathcal{R}$	sea	Ref.
0.1170(12)			3			[9]
0.1183(8)	Wilson loops	$a^{-1}$	7	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[10]
0.1192(11)			7			[11]
0.1174(12)	quarkonium	$2m_c$	1–2	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[12]
0.1183(7)	correlators	$2m_Q$	3–6	NNLO	2+1 $\sqrt[4]{\text{staggered}}$	[13]
0.1181(3) $^{+14}_{-12}$	Adler function	$Q$	5	NNLO*	2+1 overlap	[14]
0.1205(8)(5) $^{+0}_{-17}$	“QCD in a can”		80		2+1 Wilson	[15]
0.1000(16) $^\dagger$	aka Schrödinger	$L^{-1}$	270	asymptotic	2 Wilson	[16]
0.1____(____)	functional		1000		2+1+1 Wilson	[17]

HPQCD

JLQCD

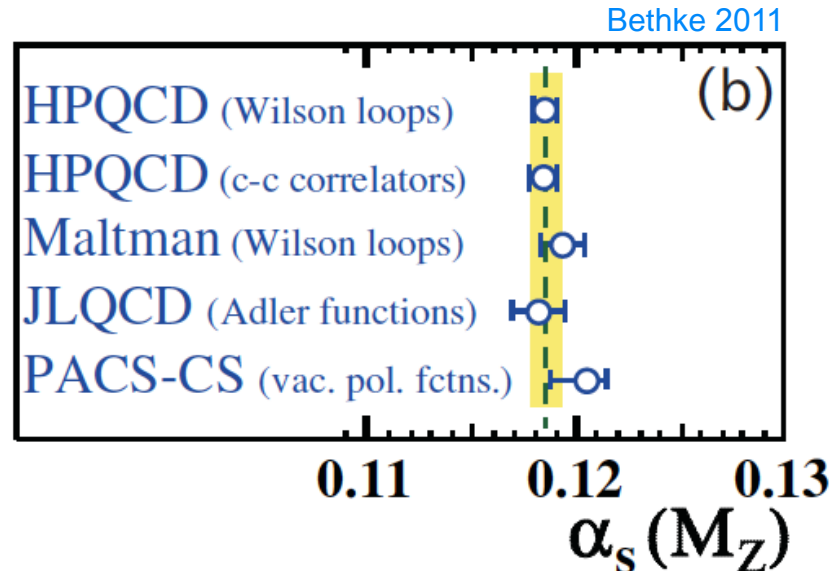
PACS-CS

ALPHA

## $\alpha_s$ -determinations:

Results live from their consistency within well-developed methods.

World average dominated by lattice results.



Is that it ?

Any reason to go on ? - YES !

- (1)! Results should be cross checked (from lattice and other methods)
- (2)! Finding the origin of disagreement to other methods motivates dedicated studies

Ability to make precision measurements reflects the true understanding of the method of theory.

# " Decay

Total hadronic " width: 
$$R_\tau = \frac{\Gamma(\tau \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau \rightarrow \nu_\tau e \bar{\nu}_e)}$$

Braaten, Narison, Pich 1992

→ Related to absorptive part of V and AV current correlators

$$R_{\tau,V+A} = N_C |V_{ud}|^2 S_{\text{EW}} \{1 + \delta_P + \delta_{\text{NP}}\}$$

electroweak corr.

$$S_{\text{EW}} = 1.0201 \pm 0.0003$$

QCD corrections

$O(\alpha_s^4)$  : Baikov, Chetyrkin, Kühn 2008

non-perturb. corr. (OPE)

$$\delta_{\text{NP}} = -0.0059 \pm 0.0014$$

$$\delta_P = \sum_{n=1} K_n A^{(n)}(\alpha_s) = \sum_{n=1} (K_n + g_n) a_\tau^n \equiv \sum_{n=1} r_n a_\tau^n \quad a_\tau \equiv \frac{\alpha_s(m_\tau)}{\pi}$$

Contour-improved (CIPT)

Fixed-order (FOPT)

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left( \frac{\alpha_s(-s)}{\pi} \right)^n \left( 1 - 2 \frac{s}{m_\tau^2} + 2 \frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right)$$

- ! Resums large "  $n$ -logs to all orders
- ! Improved convergence

- ! Treats all corrections on equal footing
- ! Smaller scale-dependence

# " Decay

## Contour-improved (CIPT)

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left( \frac{\alpha_s(-s)}{\pi} \right)^n \left( 1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right)$$

- ! Resums large "  $n$ -logs to all orders
- ! Improved convergence
- ! Well behaved

## Fixed-order (FOPT)

- ! Treats all corrections on equal footing
- ! Smaller scale-dependence
- ! Well behaved

Typical results at  $O(\alpha_s^4)$  [lots of literature]

$$\alpha_s(m_\tau) = 0.344 \pm 0.015$$

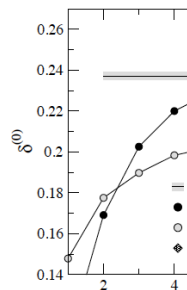
$$\alpha_s(m_Z) = 0.1210 \pm 0.0015$$

$$\alpha_s(m_\tau) = 0.321 \pm 0.015$$

$$\alpha_s(m_Z) = 0.1185 \pm 0.0015$$

→ Studies based on models of asymptotic behavior of the perturbative series do not favor any expansion

Beneke, Jamin 2008  
Descots-Genon 2010



Beneke, Jamin

# " Decay

## Contour-improved (CIPT)

$$A^{(n)}(\alpha_s) = \frac{1}{2\pi i} \oint_{|s|=m_\tau^2} \frac{ds}{s} \left( \frac{\alpha_s(-s)}{\pi} \right)^n \left( 1 - 2\frac{s}{m_\tau^2} + 2\frac{s^3}{m_\tau^6} - \frac{s^4}{m_\tau^8} \right)$$

- ! Resums large "  $n$ -logs to all orders
- ! Improved convergence
- ! Well behaved

## Fixed-order (FOPT)

- ! Treats all corrections on equal footing
- ! Smaller scale-dependence
- ! Well behaved

Typical results at  $O(\alpha_s^4)$  [lots of literature]

$$\alpha_s(m_\tau) = 0.344 \pm 0.015$$

$$\alpha_s(m_Z) = 0.1210 \pm 0.0015$$

$$\alpha_s(m_\tau) = 0.321 \pm 0.015$$

$$\alpha_s(m_Z) = 0.1185 \pm 0.0015$$

→ Studies based on models of asymptotic behavior of the perturbative series do not favor any expansion

Beneke, Jamin 2008  
Descots-Genon 2010

→ Expansion look well, BUT ARE close to being unstable.

Common variation methods unreliable to estimate perturbative error.

What is the theoretical handle that explains the discrepancy ?

# " Decay

## Duality Violation:

Boito, Cata, Golterman, Jamin, Maltman,  
Osborne, Peris 2011

→  $m_\tau$  not sufficiently heavy such that OPE (p.th. + condensates) is justified  
at very high precision

(resonances not sufficient “smeared” out)

(recall: OPE = expansion in  $\Lambda_{\text{QCD}}/m_\tau$ )

$$\int_0^{s_0} ds w(s) \rho_{V,A}^{(1+0)}(s) = \frac{i}{2\pi} \oint_{|s|=s_0} ds w(s) \Pi_{V,A}^{(1+0)}(s)$$

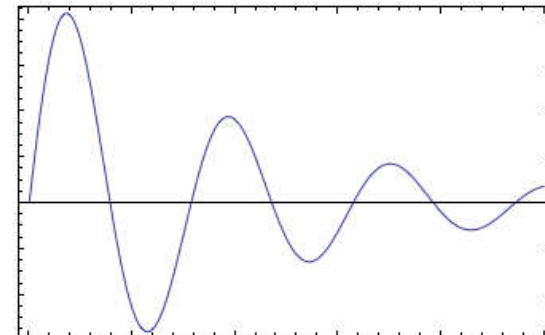
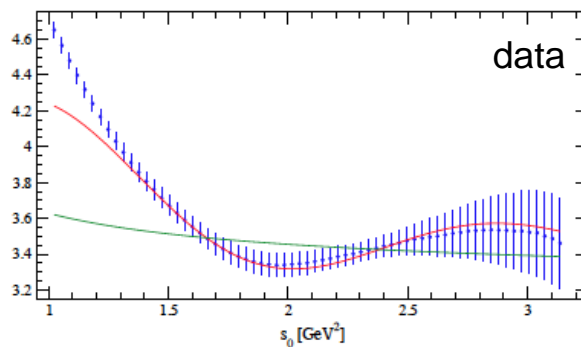
$$\Pi_{V,A}^{(1+0)}(s) = \Pi_{V,A}^{\text{OPE}}(s) + \Delta_{V,A}(s)$$

$$\rho_{V,A}^{\text{DV}}(s) = \kappa_{V,A} e^{-\gamma_{V,A}s} \sin(\alpha_{V,A} + \beta_{V,A}s)$$

model (no first principle  
computation possible)  
parameters must be fitted



Behavior of non-perturbative effects in  
the Minkowskian region

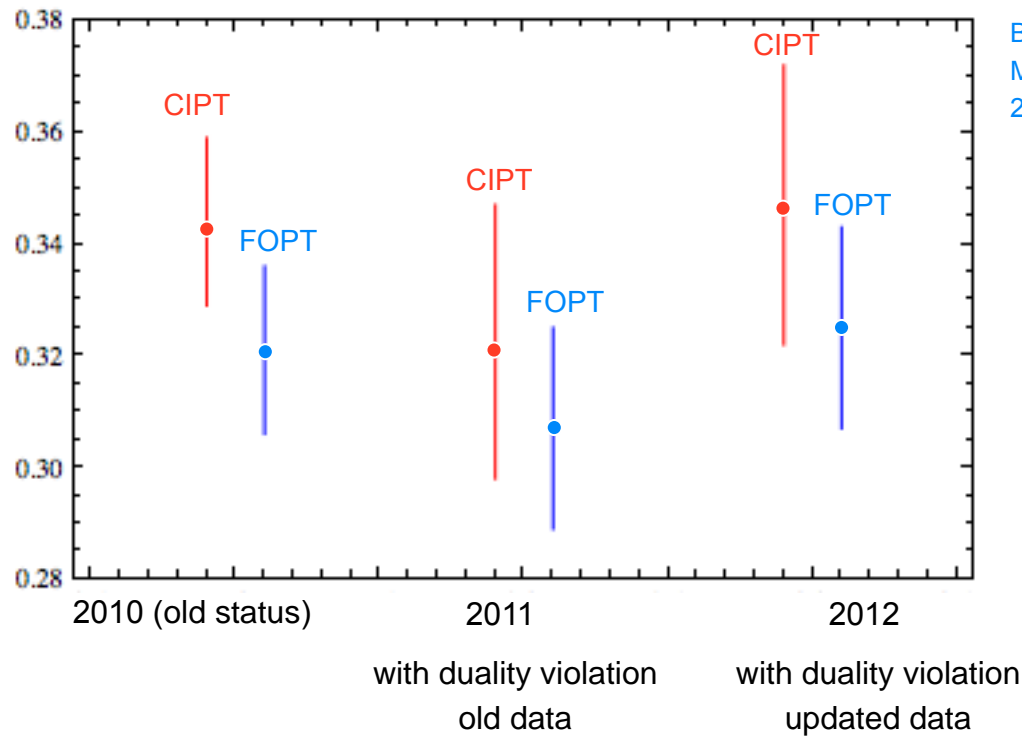




# " Decay

## Duality Violation:

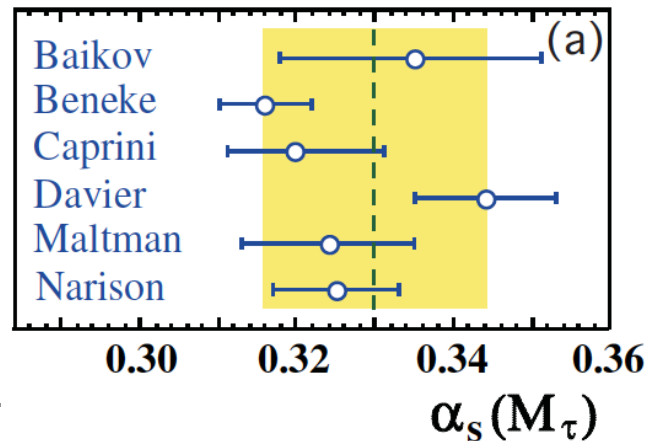
Discrepancy reconciled at the cost of larger errors.



Boito, Cata, Golterman, Jamin, Maltman, Osborne, Peris 2011+2012

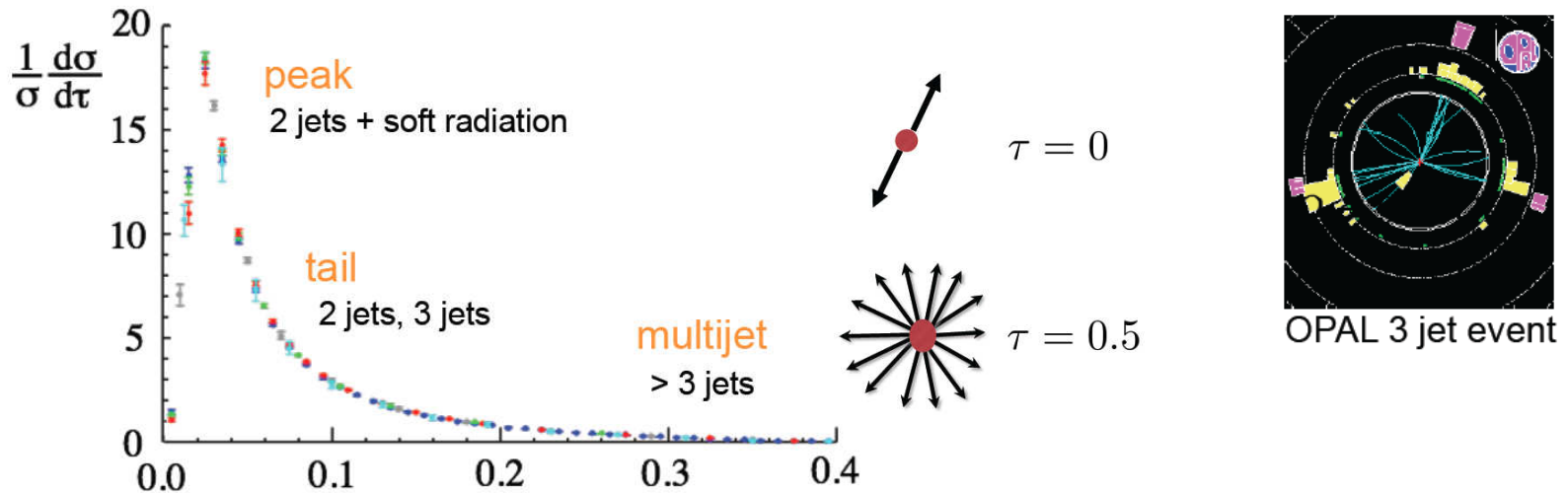
Bethke 2011

Uncertainties from spread of central values.



# Eventshape Distributions (e+e-)

- Single variable measure of global topology properties of hadronic events (primary  $q\bar{q}$ )  
Variable zero for  $q\bar{q}$  final state (2 back-to-back zero mass jets)



Thrust:

$$T = \max_{\hat{t}} \frac{\sum_i |\hat{t} \cdot \vec{p}_i|}{\sum_i |\vec{p}_i|} \quad \tau = 1 - T$$

Heavy jet mass  $\#_{H\pm}$ :

Heavier of the invariant masses of all particle in the 2 hemispheres w.r. to the thrust axis

C-parameter:

$$C = \frac{3}{2} \frac{\sum_{ij} |\vec{p}_i| |\vec{p}_j| \sin^2 \theta_{ij}}{(\sum_i |\vec{p}_i|)^2}$$

Jet broadening:

$$B_k = \left( \frac{\sum_{i \in H_k} |\vec{p}_i \times \vec{n}_T|}{2 \sum_i |\vec{p}_i|} \right) \quad B_T = B_1 + B_2$$

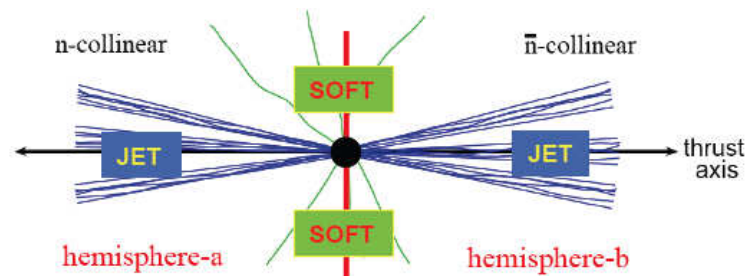
$\gamma_{23}$ : Transition value between 2 and 3 jets (Durham jet algorithm)

# Eventshape Distributions (e+e-)

→ Perturbation theory:  $\frac{d\sigma}{d\tau} \sim \delta(\tau) + \mathcal{O}(\alpha_s)$

$\mathcal{O}(\alpha_s)$  is LO for  $\tau > 0$

→ IR-safe & perturbatively inclusive (e.g. thrust)



Korshemski, Serman

$$\mu_h \sim Q$$

perturbative

$$\mu_{\text{jet}} \sim Q\sqrt{\tau}$$

perturbative

$$\mu_{\text{soft}} \sim Q\tau \sim \Lambda$$

#<sub>H</sub>, C: similar to thrust

B:  $\mu_{\text{jet}} \sim \mu_{\text{soft}}$

→ High precision era: NNLO,  $\mathcal{O}(\alpha_s^3)$  perturbative corrections  
partonic Monte-Carlo program

Gehrmann-DeRidder, Gehrmann,  
Glover, Heinrich 2007

Weinzierl 2009

# Eventshape Distributions (e+e-)

## → Fixed-Order and log summation:

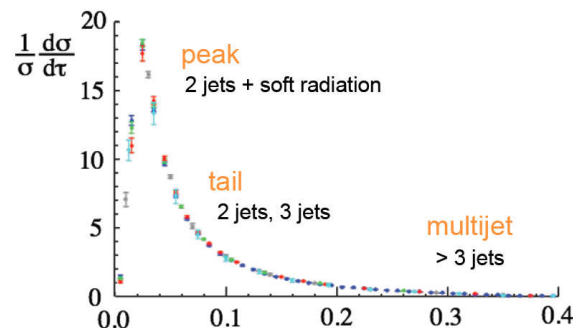
- ! NNLO (fixed-order)
- ! NNLO + NLL (classic resummation)
- ! NNLO + N<sup>3</sup>LL (SCET factorization)

Catani ea; Dokshitzer ea

} all eventshapes  
", #<sub>H</sub> (B)

## → Non-perturbative Effects (tail):

- ! MC: hadron mode – partonic mode
- ! Power corrections from data

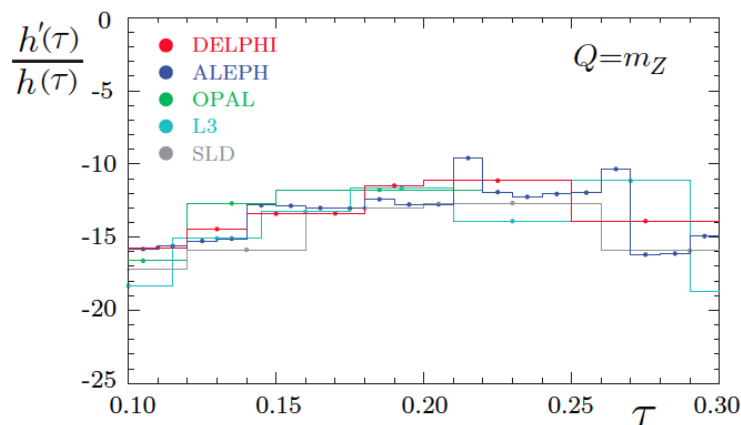


## Problem of MC-estimate:

- ! Hadronization corrections in MC are not in the scheme needed for calculations based on NNLO corrections (MSbar+dim reg)
- ! MC estimates much too small (tiny ! Few % for !<sub>s</sub>)

$$\frac{d\sigma}{d\tau} \sim h(\tau) \rightarrow \left( \tau - \frac{2\Lambda}{Q} \right)$$

$$\frac{\delta\alpha_s}{\alpha_s} \sim \frac{2\Lambda}{Q} \frac{h'(\tau)}{h(\tau)} \sim 9\%$$

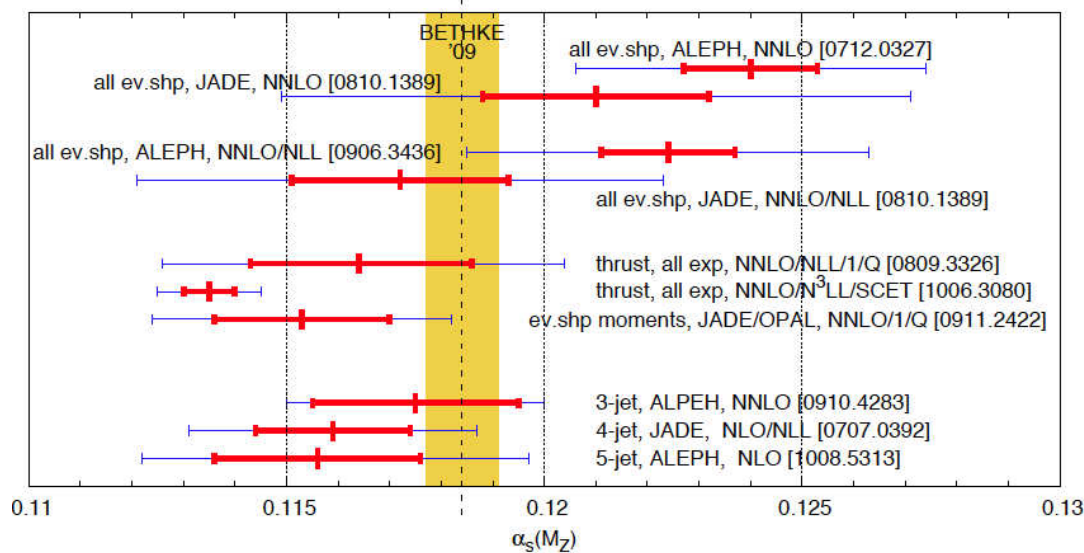


Abbate, Fickinger, Hoang, Mateu, Stewart 2010

# Eventshape Distributions (e+e-)

Only 3 NNLO-based analyses left :

Gavin Salam



Davison, Webber 2009 (all event shapes)  
 Abbate, Fickinger, Hoang, Mateu, Stewart 2010 (thrust)  
 Gehrmann, Jaquier, Luisoni 2010 (all event shapes)

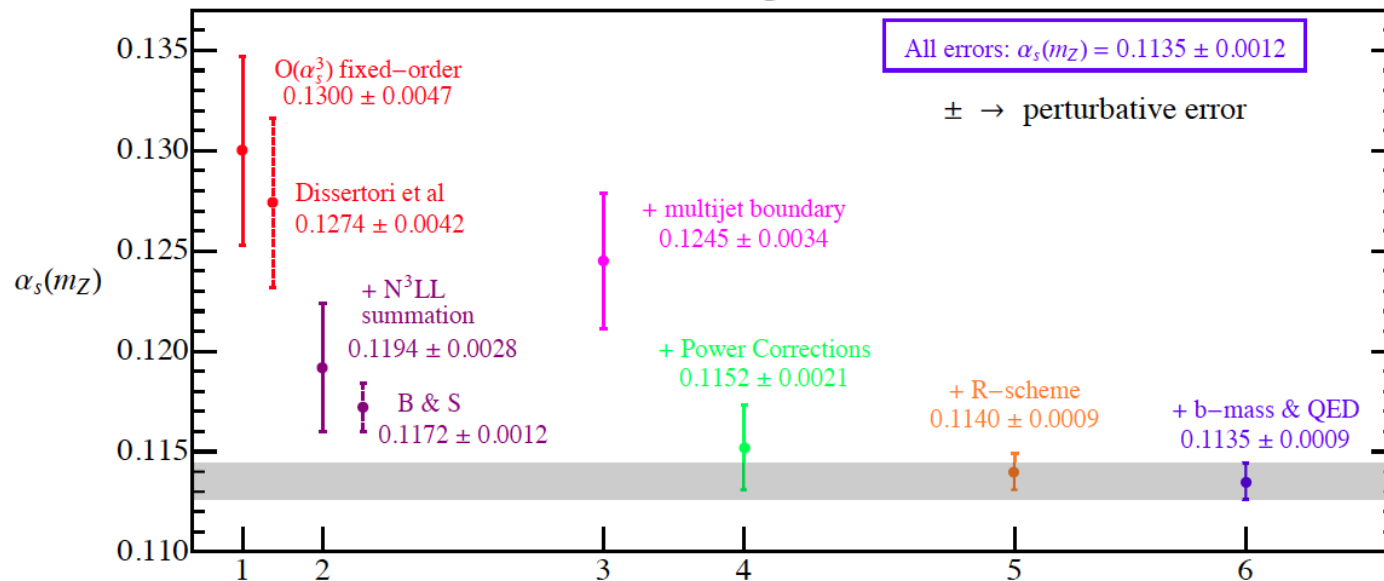
Jet rates

All results for  $\alpha_s(m_Z)$  are low !!

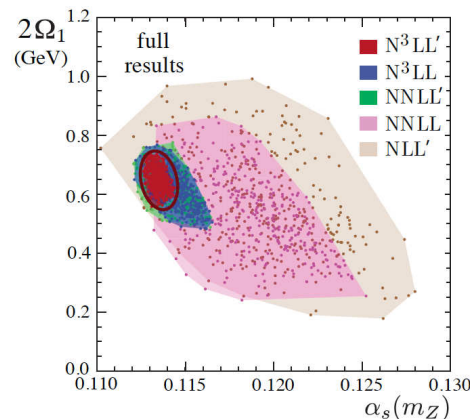
# Eventshape Distributions (e+e-)

$$\left(\frac{d\sigma}{d\tau}\right)_{\text{part}}^{\text{sing}} \sim \sigma_0 H(Q, \mu_Q) U_H(Q, \mu_Q, \mu_s) \int d\ell d\ell' U_J(Q\tau - \ell - \ell', \mu_Q, \mu_s) J_T(Q\ell', \mu_j) S_T(\ell - \Delta, \mu_s)$$

$\alpha_s(m_Z)$  from global thrust fits



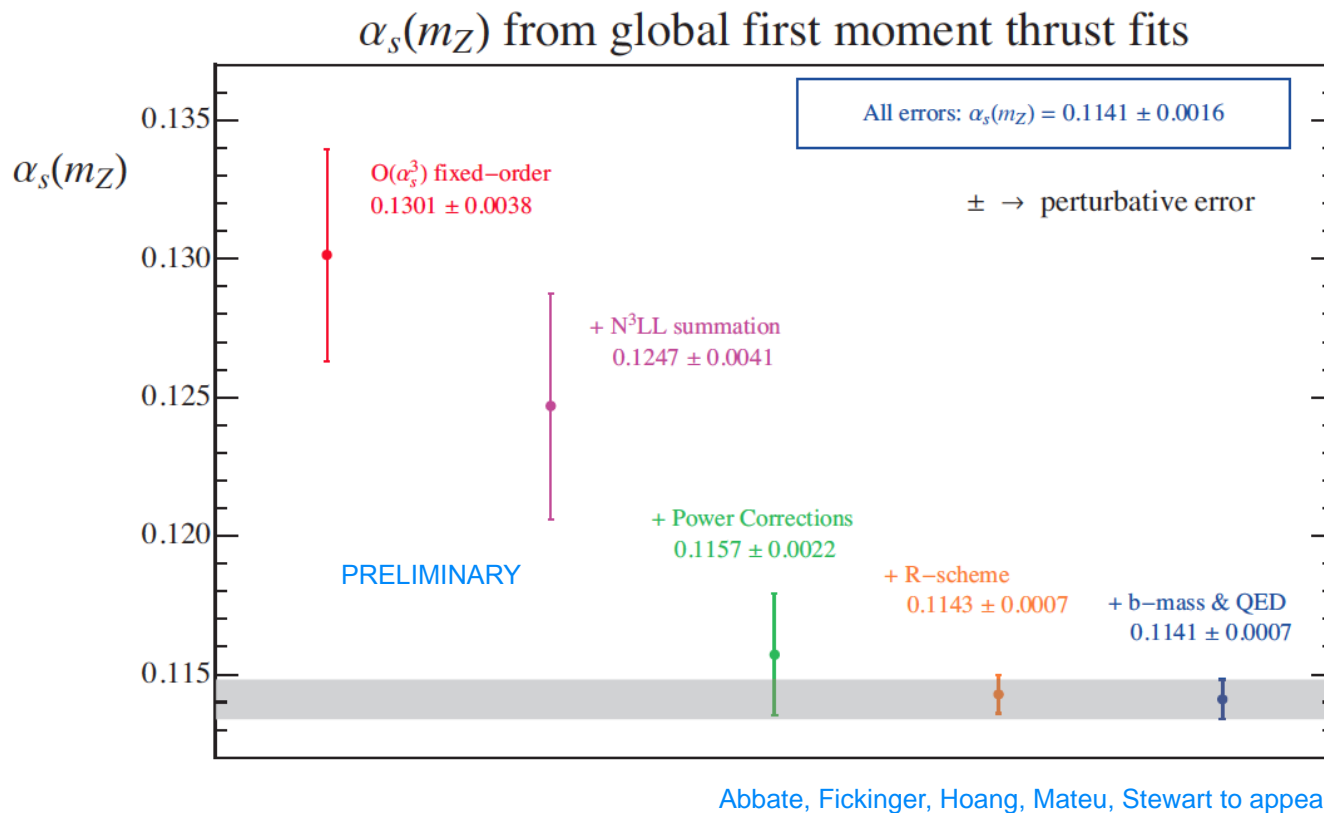
Abbate, Fickinger, Hoang, Mateu, Stewart 2010



- Central values for  $\alpha_s$  decrease with order
- Very good convergence

# Eventshape Distributions (e+e-)

NEW:



- ! SCET analyses for other event shapes needed !

	$\alpha_s(M_Z)$	
BBG	$0.1134^{+0.0019}_{-0.0021}$	valence analysis, NNLO [60]
BB	$0.1132 \pm 0.0022$	valence analysis, NNLO [110]
GRS	0.112	valence analysis, NNLO [160]
ABKM	$0.1135 \pm 0.0014$	HQ: FFNS $n_f = 3$ [16]
ABKM	$0.1129 \pm 0.0014$	HQ: BSMN-approach [16]
JR	$0.1124 \pm 0.0020$	dynamical approach [20]
JR	$0.1158 \pm 0.0035$	standard fit [20]
ABM11	$0.1134 \pm 0.0011$	
MSTW	$0.1171 \pm 0.0014$	[158]
NN21	$0.1173 \pm 0.0007$	[124]
CT10	$0.118 \pm 0.005$	[161]

Table 4.10: Summary of recent NNLO QCD analyses of the DIS world data, supplemented by related measurements using other processes.

- ! Central values for  $\alpha_s$  decrease NLO to NNLO
- ! All below the world average.



# Conclusions

---

- ! Ability to make precise measurements of the strong coupling reflects our understanding and control of QCD in various areas.
- ! High precision measurements and methods claiming very small theoretical uncertainties HAVE TO BE challenged and questioned. Even new and difficult-to-be-answered questions should be asked.
- ! Why do jet observables all seem to give low  $\alpha_s$  with NNLO included ?  
What does this mean ?  
Maybe NNNLO helps ?
- ! Eventually: measurements of  $\alpha_s$  at the LHC