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Relating quarks and leptons with the T_7 flavour group

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ABSTRACT

In this letter we present a model for quarks and leptons based on T_7 as flavour symmetry, predicting a canonical mass relation between charged leptons and down-type quarks proposed earlier. Neutrino masses are generated through a Type-I seesaw mechanism, with predicted correlations between the atmospheric mixing angle and neutrino masses. Compatibility with oscillation results leads to lower bounds for the lightest neutrino mass as well as for the neutrinoless double beta decay rates, even for normal neutrino mass hierarchy.

1. Introduction

Ever since the discovery of the muon in the thirties particle physicists have wondered about a possible simple understanding of fermion mass and mixing patterns. The experimental confirmation of neutrino oscillations [1–4] has brought again the issue into the spotlight. Yet despite many attempts, so far the origin of neutrino mass and its detailed flavour structure remains one of the most well-kept secrets of nature. In particular the observed values of neutrino oscillation parameters [5] pose the challenge to figure out why lepton mixing angles are so different from those of quarks. Indeed the sharp differences between the flavour mixing parameters characterising the quark and lepton sectors escalate the complexity of the flavour problem. Many extensions of the Standard Model (SM) have been proposed in order to induce nonzero neutrino masses [6] and to predict the oscillation parameters such as the neutrino mass ordering, the octant of the atmospheric mixing angle and the value of the CP-violating phase in the lepton sector.

A popular approach to tackle these issues is the use of discrete non-Abelian flavour symmetries which are known to be far more restrictive than Abelian ones [7]. In the literature there are many models based on, for instance, A_4 (the group of even permutations of a tetrahedron) whose simplest realisations predict zero reactor mixing angle and maximal atmospheric angle [8–10]. However, this nice prediction has now been experimentally ruled out [1–4] so that the corresponding models need to be revamped in order to account for observations [11].

A variety of possible predictions of flavour symmetry based models can be found, for instance [12]:

i) *neutrino mass sum rules* leading to restrictions on the effective mass parameter $|m_{ee}|$ characterising neutrinoless double beta decay ($0\nu\beta\beta$) processes [13–16];
ii) *neutrino mixing sum rules* [17].

Here we concentrate on the alternative possibility of having *mass relations* in the charged fermion sector. For definiteness we focus on the relation in Eq. (1),

$$\frac{m_b}{\sqrt{m_d m_s}} \approx \frac{m_\tau}{\sqrt{m_e m_\mu}}. \tag{1}$$

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Table 1
Matter assignments of the model where $a^7 = 1$.

	$\bar{L}$	ℓ_R	N_R	ν_R	$\overline{Q}$	d_R	u_{R_i}	H	φ_ν	φ_u	φ_d	ξ_ν
T_7	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_i$	$\mathbf{1}_0$	$\mathbf{3}$	$\bar{\mathbf{3}}$	$\mathbf{3}$	$\mathbf{1}_0$
$\mathbb{Z}_7$	a^3	a^3	a^5	a^2	a^3	a^3	a^2	1	a^4	a^2	a^1	a^3

Table 2
Vacuum expectation value alignments.

Flavon	VEV alignment in T_7	Model
φ_ν	$(1,1,0)$	$(1+\delta_{\nu_1}, 1, \delta_{\nu_2})$
φ_u	$(0,0,1)$	$(\delta_{u_1}, \delta_{u_2}, 1)$
φ_d	$(1,0,0)$	$(1, \delta_{d_1}, \delta_{d_2})$

This relation was suggested in [18–21] and can hold at the electroweak scale.[1] First we note that such a relation between down-type quark and charged lepton masses can be understood because of group structure, when there are three vacuum expectation values and only two invariant contractions (Yukawas) in the product, $\mathbf{3}\otimes\mathbf{3}\otimes\mathbf{3}$. For example, such relation can be obtained with other groups containing three-dimensional irreducible representations (irreps) such as, for example, $T_n \cong Z_n \rtimes Z_3$ (with $n = 7, 13, 19, 31, 43, 49$; [23]) as well as T'.

In what follows we build a flavour model for quarks and leptons based upon the smallest non-Abelian group after A_4, namely the flavour group T_7 [24–29] leading to the mass relation in Eq. (1). Neutrino masses are generated by implementing a Type-I seesaw [30] in contrast to the dimensional-five Weinberg operator approach used in previous Refs. [18–20]. We discuss the resulting phenomenological predictions, namely, a correlation between the lightest neutrino mass and the atmospheric angle, as well as lower bounds for the effective mass parameter $|m_{ee}|$ characterising $0\nu\beta\beta$ decay for both neutrino mass orderings.

2. The model

Here we consider a model with the multiplet content in Table 1 where the SM electroweak gauge symmetry is crossed with a global flavour symmetry group T_7. The down-type quarks and leptons (left- and right-handed) transform as triplets, RH up-type quarks transform as singlets while the SM Higgs is blind, as shown in Table 1. Then the Yukawa Lagrangian for the charged sector is given by,

$$\mathcal{L} = \frac{Y^\ell}{\Lambda}\bar{L}\ell_R H_d + \frac{Y^d}{\Lambda}\overline{Q}d_R H_d + \frac{Y^u}{\Lambda}\overline{Q}u_R H_u + h.c. \tag{2}$$

Here for simplicity we have omitted the flavour indices, and have defined $H_d \equiv H\varphi_d$, $H_u \equiv \tilde{H}\varphi_u$ and $\tilde{H} \equiv i\sigma_2 H^*$, where φ_a are T_7 flavon triplets and Λ is the scale at which these fields get their vacuum expectation values (vevs), $\langle\varphi_a\rangle$.

On the other hand, let us assume the existence of four RH-neutrinos accommodated as $\mathbf{3}\oplus\mathbf{1}_0$ under T_7 so that the Lagrangian for the neutrino sector becomes,

$$\mathcal{L}_\nu = \frac{Y_1^\nu}{\Lambda}\bar{L}N_R\tilde{H}_d + \frac{Y_2^\nu}{\Lambda}\bar{L}\nu_R H_u + \kappa_1 N_R N_R\varphi_\nu + \kappa_2\nu_R\nu_R\xi_\nu \tag{3}$$

where, $\tilde{H}_d \equiv \tilde{H}\overline{\varphi}_d$. Notice that the additional Abelian symmetry $\mathbb{Z}_7$ couples each T_7 flavon triplet with only one fermion sector (down-type, up-type or neutrino sector), so that, flavons transform non-trivially under the discrete Abelian group and their charges are unrelated to each other by conjugation. Therefore, in some sense, the order of the $\mathbb{Z}_n$ symmetry is fixed by the Yukawa sector.

In what follows we will study the flavon potential for three distinct triplets under T_7. The second column of Table 2 shows the vacuum expectation value alignments allowed in T_7 [24,31], with small deviations from those alignments shown in the third column.

2.1. Flavon potential

The Higgs scalar potential for a single T_7 flavon triplet, i.e. $\varphi \simeq \mathbf{3}$, is given by [24,31]

$$V_s = -\mu_s^2\sum_{i=1}^{3}\varphi_i^\dagger\varphi_i + \lambda_s\left(\sum_{i=1}^{3}\varphi_i^\dagger\varphi_i\right)^2 + \kappa_s\sum_{i=1}^{3}\varphi_i^\dagger\varphi_i\varphi_i^\dagger\varphi_i, \tag{4}$$

where the possible vacuum expectation value alignments are, see Appendix A,

$$\langle\varphi\rangle \sim \frac{1}{\sqrt{3}}(1,1,1) \quad \text{for } \kappa_s > 0 \quad \text{and} \quad \langle\varphi\rangle \sim (1,0,0), (0,1,0), (0,0,1) \quad \text{for } \kappa_s < 0. \tag{5}$$

In our case, ignoring the singlet ξ_ν, there are three triplets, φ_u, φ_d and φ_ν, with an additional $\mathbb{Z}_7$ charge so that the flavon potential is given as

$$V' = V_\nu + V_d + V_u + V_{\text{mix}}, \tag{6}$$

[1] In an early paper [22] Wilczek and Zee found, by using an $SU(2)_H$ symmetry, an extended mass relation $\frac{m_b}{\sqrt{m_d m_s}} = \frac{m_\tau}{\sqrt{m_e m_\mu}} = \frac{m_t}{\sqrt{m_u m_c}}$ which is now evidently ruled out.

where V_α (with $\alpha = \nu, d, u$) are given by Eq. (4). Then, in components, V_α contain the triplet elements φ_{α_i} and the parameters μ_α^2, λ_α and κ_α. The mixing part of the potential is the following

$$V_{\text{mix}} = \kappa_{12}\left|\sum_{i=1}^{3}\varphi_{\nu_i}^\dagger \varphi_{u_i}\right|^2 + \kappa_{13}\left|\sum_{i=1}^{3}\varphi_{\nu_i}^\dagger \varphi_{d_i}\right|^2 + \kappa_{23}\left|\sum_{i=1}^{3}\varphi_{d_i}^\dagger \varphi_{u_i}\right|^2 + \kappa_{123}(\varphi_\nu\varphi_u\varphi_d + \text{h.c.}). \tag{7}$$

The vev configuration written down in the second column of Table 2 is a minimum of the potential Eq. (6) when $\kappa_\nu > 0$, $\kappa_u < 0$ and $\kappa_d < 0$ and the terms κ_{13} and κ_{123}, are suppressed.[2] Notice that some vevs are orthogonal (namely, $\langle\varphi_\nu\rangle \perp \langle\varphi_u\rangle$ and $\langle\varphi_u\rangle \perp \langle\varphi_d\rangle$). This property of the vevs has been described in [31,32]. In order to ensure a realistic model we assume small deviations from the vev canonical alignments in the middle column in Table 2. Such deviations can be induced by adding soft breaking terms in the flavon potential, Eq. (6).

2.2. Mass relation in down-type sector

As usual, one obtains the fermion mass matrices after electroweak symmetry breaking from the Lagrangian in Eq. (2). Given the T_7 multiplication rules (see Appendix B), one finds that the down-type quarks and the charged lepton mass matrices turn out to have the form

$$M_f = \begin{pmatrix} 0 & e^{i\theta_f} y_1^f v_3 & y_2^f v_2 \\ y_2^f v_3 & 0 & e^{i\theta_f} y_1^f v_1 \\ e^{i\theta_f} y_1^f v_2 & y_2^f v_1 & 0 \end{pmatrix}, \tag{8}$$

where $f = \ell, d$ and θ_f are unremovable phases contributing to CP-violation in the lepton and quark sector. In addition, we have used the following parameterisation,

$$\frac{\langle\varphi_d\rangle\langle H\rangle}{\Lambda} \approx (v_1, v_2, v_3). \tag{9}$$

It should be noticed that the matrices M_f in Eq. (8) have the same structure as those obtained with A_4 as flavour symmetry [18–20,33]. It is useful to rewrite Eq. (8) in the following way,

$$M_f = \begin{pmatrix} 0 & e^{i\theta_f} a^f \alpha^f & b^f \\ b^f \alpha^f & 0 & e^{i\theta_f} a^f r^f \\ e^{i\theta_f} a^f & b^f r^f & 0 \end{pmatrix}, \tag{10}$$

where

$$a^f = y_1^f v_2, \qquad b^f = y_2^f v_2, \qquad \alpha^f = v_3/v_2 \quad \text{and} \quad r^f = v_1/v_2. \tag{11}$$

Following the reasoning in [18–20] we consider the invariants of $M_f M_f^\dagger$ and obtain, at leading order in the limit $r^f \gg \alpha^f, 1$ and $r^f \gg b^f/a^f$,

$$\left(b^f r^f\right)^2 \approx m_3^2, \tag{12}$$

$$b^{f\,6} r^{f\,2} \alpha^{f\,2} \approx m_1^2 m_2^2 m_3^2, \tag{13}$$

$$a^{f\,2} b^{f\,2} r^{f\,4} \approx m_2^2 m_3^2. \tag{14}$$

Then, solving the last system of equations, Eqs. (12)–(14), one gets

$$a^f \approx \frac{m_2}{m_3}\sqrt{\frac{m_1 m_2}{\alpha^f}}, \qquad b^f \approx \sqrt{\frac{m_1 m_2}{\alpha^f}}, \quad \text{and} \quad r^f \approx m_3\sqrt{\frac{\alpha^f}{m_1 m_2}}. \tag{15}$$

From Eq. (15) and the fact that the same flavon is coupled to the down-type quarks and charged leptons we are led to the mass relation in Eq. (1),

$$\frac{m_b}{\sqrt{m_d m_s}} \approx \frac{m_\tau}{\sqrt{m_e m_\mu}}.$$

It is worth mentioning that even when the phases θ_f appear in the invariant $\det|M_f M_f^\dagger|$ with $f = \ell, d$, that is in Eq. (13), the mass relation is preserved.

[2] The term proportional to κ_{13} in the potential could be suppressed by adding a term like $-\mu_{13}^2(\varphi_\nu^\dagger\varphi_d + \text{h.c.})$ which softly breaks Z_7. The trilinear term can be forbidden by invoking an additional parity transformation over the fields.

Table 3
Parameters characterising the quark sector.

10 free parameters	a^d	b^d	r^d	y_1^u	y_2^u	y_3^u	α^d	α_1	α_2	θ_d

2.3. Quark mixing

From the Yukawa Lagrangian in Eq. (2) we have that after electroweak symmetry breaking the mass matrices for up- and down-type quarks are, respectively,

$$M_u = \begin{pmatrix} y_1^u u_1 & y_2^u u_1 & y_3^u u_1 \\ y_1^u u_2 & \omega y_2^u u_2 & \omega^2 y_3^u u_2 \\ y_1^u u_3 & \omega^2 y_2^u u_3 & \omega y_3^u u_3 \end{pmatrix} \quad \text{and} \quad M_d = \begin{pmatrix} 0 & e^{i\theta_d} a^d \alpha^d & b^d \\ b^f \alpha^d & 0 & e^{i\theta_d} a^d r^d \\ e^{i\theta_d} a^d & b^d r^d & 0 \end{pmatrix}, \tag{16}$$

where the parameters a^d, b^d and r^d are given by Eq. (15), with $\omega^3 = 1$ and the vevs u_i $(i = 1, 2, 3)$ defined through the parameterisation

$$\frac{\langle \varphi_u \rangle \langle H \rangle}{\Lambda} \approx (u_1, u_2, u_3). \tag{17}$$

It is useful to rewrite the vevs as follows,

$$(u_1, u_2, u_3) = u_3 \left(\frac{u_1}{u_3}, \frac{u_2}{u_3}, 1 \right) = u_3(\alpha_1, \alpha_2, 1), \tag{18}$$

in that way there are 10 free parameters in the quark sector, listed in Table 3. These parameters determine the six quark masses, the three CKM mixing angles and the quark CP-violating phase.

In Ref. [20] an A_4 flavour symmetry model was built leading to our mass formula in Eq. (1). The mass and CKM mixing parameters describing the quark sector, very similar to those in Eq. (16), were successfully reproduced, as seen in Table II in [20], assuming trivial phases, namely $\theta_d = 0, \pi$ in Eq. (16). However, even in this trivial case there is CP-violation due to the complex phase ω. Here for simplicity we just take advantage of the results given in [20] for the quark sector of our current T_7 model. Therefore we use the following values, given in the aforementioned A_4 model,

$$\begin{aligned} r^d &= 263.44 \text{ MeV}, \qquad y_1^u u_3 = -297393 \text{ MeV}, \\ a^d &= 0.21 \text{ MeV}, \qquad y_2^u u_3 = -15563 \text{ MeV} \\ b^d &= 10.73 \text{ MeV}, \qquad y_3^u u_3 = 277 \text{ MeV} \\ \alpha^d &= \frac{v_3}{v_2} = 1.58, \qquad \alpha_1 = \frac{u_1}{u_3} = 2.14\lambda^4, \\ \theta_d &= \pi, \quad \text{and} \quad \alpha_2 = \frac{u_2}{u_3} = 1.03\lambda^2, \end{aligned} \tag{19}$$

and where $\lambda = 0.2$ the Cabibbo angle. The parameters r^d, a^d and b^d can be computed by carrying out a substitution of (m_1, m_2, m_3) with the actual values of the down-type quark masses (m_d, m_s, m_b) in Eq. (15). One can verify with ease that the predictions for the CKM mixing matrix, quark masses and CP-violation are in agreement with the experimental data [34]. Now we proceed to study the lepton sector, for which some of the parameters will be fixed by the fit in the quark sector, namely the parameters α^d and r^d.

2.4. Lepton mixing

As we saw above, the spontaneous breaking of the electroweak symmetry yields the following form for the charged lepton mass matrix,

$$M_\ell = \begin{pmatrix} 0 & e^{i\theta_\ell} a^\ell \alpha^\ell & b^\ell \\ b^\ell \alpha^\ell & 0 & e^{i\theta_\ell} a^\ell r^\ell \\ e^{i\theta_\ell} a^\ell & b^\ell r^\ell & 0 \end{pmatrix}, \tag{20}$$

where, from the T_7 multiplication rules in Appendices A and B one finds,

$$a^\ell = y_1^\ell v_2, \qquad b^\ell = y_2^\ell v_2, \qquad \alpha^\ell = v_3/v_2 \quad \text{and} \quad r^\ell = v_1/v_2. \tag{21}$$

On the other hand, as mentioned in the introduction, here we adopt a Type-I seesaw approach for generating the neutrino masses. This is in contrast to previous models leading to the mass formula in Eq. (1) from the A_4 group. In those schemes an effective dimension-five operator approach was employed. In the present case the neutrino mass matrix is given by,

$$M_\nu = -M_D M_{RR}^{-1} M_D^T \tag{22}$$

where,

$$M_D = \begin{pmatrix} Y_1^\nu v_2 & 0 & 0 & e^{i\theta_1} Y_2^\nu u_1 \\ 0 & Y_1^\nu v_3 & 0 & e^{i\theta_1} Y_2^\nu u_2 \\ 0 & 0 & Y_1^\nu v_1 & e^{i\theta_1} Y_2^\nu u_3 \end{pmatrix} \quad \text{and} \quad M_{RR} = \begin{pmatrix} 0 & M_3 & M_2 & 0 \\ M_3 & 0 & M_1 & 0 \\ M_2 & M_1 & 0 & 0 \\ 0 & 0 & 0 & e^{i\theta_2} M_4 \end{pmatrix}, \tag{23}$$

Table 4
Parameters in the lepton sector.

Parameters in the lepton sector	a^ℓ	b^ℓ	r^d	α^d	α_1	α_2	ϵ_1	ϵ_2	ϵ_3	θ_ℓ	θ_ν
Fixed			✓	✓	✓	✓					
Free	✓	✓					✓	✓	✓	✓	✓

where $M_i = \kappa_1 \langle \varphi_\nu \rangle_i$ (for $i = 1, 2, 3$) and $M_4 = \kappa_2 \langle \xi_\nu \rangle$. The real matrix elements M_i satisfy $M_1 \sim M_2 \gg M_3$, Table 2. Notice that for complex Yukawas the mass matrices M_D and M_{RR} in Eq. (23) only depend on one unremovable phase.

In order to implement the vev alignments in Table 2 we assume that the vevs u_i and v_i in Eq. (23) satisfy $u_3 \gg u_{1,2}$ and $v_1 \gg v_{2,3}$. The former vev hierarchy has to do with the fit in the quark sector and the latter comes from the mass relation $r^d \gg \alpha^d, 1$. Then, the vev alignments can be rewritten as follows,

$$u_3\left(\frac{u_1}{u_3}, \frac{u_2}{u_3}, 1\right) = u(\alpha_1, \alpha_2, 1) \propto (\delta_{u_1}, \delta_{u_2}, 1),$$
$$v_2\left(\frac{v_1}{v_2}, 1, \frac{v_3}{v_2}\right) = v\left(r^d, 1, \alpha^d\right) \propto (1, \delta_{d_1}, \delta_{d_2}),$$
$$M_3\left(\frac{M_1}{M_3}, \frac{M_2}{M_3}, 1\right) = M(\epsilon_1 R, R, 1) \propto (1 + \delta_{\nu_1}, 1, \delta_{\nu_2}) \tag{24}$$

where $\alpha_1 = 2.14\lambda^4$, $\alpha_2 = 1.03\lambda^2$, $\lambda = 0.2$ and we have defined $u_3 = u$, $v_2 = v$ and $M_3 = M$.

Therefore, using Eqs. (23)–(24), the light neutrino mass matrix after the seesaw mechanism turns out to be

$$M_\nu = \kappa \begin{pmatrix} \epsilon_1 - 2e^{-i\theta_\nu}\alpha_1^2\epsilon_2 & -\alpha^d - 2e^{-i\theta_\nu}\alpha_1\alpha_2\epsilon_2 & -\epsilon_3 - 2e^{-i\theta_\nu}\alpha_1\epsilon_2 \\ \cdot & \frac{\alpha^{d\,2}}{\epsilon_1} - 2e^{-i\theta_\nu}\alpha_2^2\epsilon_2 & -\frac{\alpha^d\epsilon_3}{\epsilon_1} - 2e^{-i\theta_\nu}\alpha_2\epsilon_2 \\ \cdot & \cdot & -2e^{-i\theta_\nu}\epsilon_2 + \frac{\epsilon_3^2}{\epsilon_1} \end{pmatrix}, \tag{25}$$

which is symmetric and $\alpha_1 = 2.14\lambda^4$, $\alpha_2 = 1.03\lambda^2$, $\lambda = 0.2$ and we have defined,

$$\kappa \equiv \frac{(Y^\nu v)^2}{M}, \qquad \epsilon_2 \equiv \frac{M(Y_2^\nu u)^2}{M_4(Y_1^\nu v)^2}, \qquad \epsilon_3 \equiv \frac{r^d}{R} \quad \text{and} \quad \theta_\nu \equiv -2\theta_1 + \theta_2. \tag{26}$$

It is important to note that some parameters in the neutrino mass matrix are fixed by the fit in the quark sector. In Table 4 we list the parameters in the lepton sector denoting as “fixed” those determined by the fit in the quark sector. Bear in mind that down-type quarks and charged leptons couple to the same flavon φ_d and hence, $\alpha^d = \alpha^\ell$ and $r^d = r^\ell$. This is the origin of the mass relation in Eq. (1).

Gathering all we have in the lepton sector we can compute the lepton mixing matrix,

$$U = U_\ell^\dagger U_\nu \tag{27}$$

where U_ℓ and U_ν are the matrices that diagonalise the charged and neutral mass matrices, $M_\ell^2 \equiv M_\ell M_\ell^\dagger$ and $M_\nu^2 \equiv M_\nu M_\nu^\dagger$, respectively. Remind that M_ℓ is the matrix in Eq. (20) with one unremovable phase θ_ℓ.

3. Results

In our analysis, we have varied for instance ϵ_i in the range $[0, 5]$ and the phases $\theta_{\ell,\nu}$ in the range $[0, 2\pi]$. We make use of the neutrino mass matrix invariants $\mathrm{tr}M_\nu^2$, $\det M_\nu^2$ and $(\mathrm{tr}M_\nu^2)^2 - \mathrm{tr}(M_\nu^4)$ and choose to rewrite the three neutrino masses in terms of the square mass differences $\Delta m^2_{\mathrm{atm}}$ and $\Delta m^2_{\mathrm{sol}}$ and the lightest neutrino mass, m_1 for the case of normal hierarchy and m_3 for inverted hierarchy. We now sum up all our results.

The panel on the left in Fig. 1 shows the correlation between the atmospheric angle for normal hierarchy (NH, i.e. $|m_3| > |m_2| > |m_1|$) and the sum of neutrino masses (defined as $\Sigma \equiv |m_1| + |m_2| + |m_3|$). We find that there is a lower bound for the lightest neutrino mass and that the first octant is favoured by lighter neutrino masses. For reference we also display the constraint coming from the combination of cosmological CMB data from Planck and WMAP, including baryon acoustic oscillations (BAO) data from [35]. If taken at face value such stringent cosmological bound would disfavour not only heavy neutrinos but also the best fit value for the atmospheric angle lying in second octant [5].

On the other hand, a similar correlation between the atmospheric angle and the sum of neutrino masses, Σ, is also found for the inverted hierarchy case (IH, i.e. $|m_2| > |m_1| > |m_3|$). This is shown on the right panel of Fig. 1 where the dot-dashed vertical line is the constraint coming from the same combination of cosmological data [35]. Taking the most stringent cosmological (BAO) bound into account as well as the oscillation results one sees that, at 1σ, this case would be disfavoured. Indeed, if this cosmological bound is taken at face value, the second octant would be excluded for inverse hierarchy. However, as seen in Fig. 2, at 3σ the second octant is certainly allowed for inverted hierarchy. The resulting lower bound for the lightest neutrino mass is much tighter than the one that holds for normal hierarchy. For comparison we also display the future sensitivity of the KATRIN experiment on tritium beta decay, $\Sigma \simeq 0.6$ eV, [36].

In summary, one sees that for both hierarchies our model implies a correlation between the atmospheric angle and the lightest neutrino mass. The current neutrino oscillation experiments lead to a lower bound for m_1.

Such a lower bounds have implications for the effective mass parameter $|m_{ee}|$ specifying the neutrinoless double beta – $0\nu\beta\beta$ – decay amplitude.

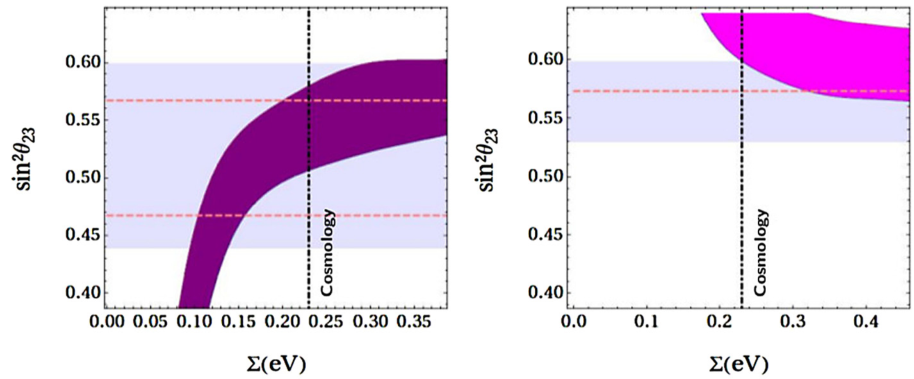

Fig. 1. Left panel: correlation between the atmospheric angle and the sum of neutrino masses Σ for the normal hierarchy case. Right panel: correlation between the atmospheric angle and Σ when assuming inverted hierarchy. The horizontal dotted lines denote the best fit values for the atmospheric angle [5] while the horizontal bands are allowed at 1σ. The vertical dot-dashed line is the cosmological bound from the combination of CMB and BAO data [35].

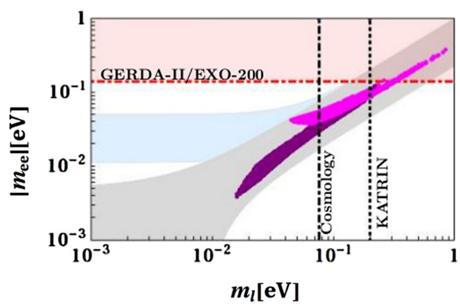

Fig. 2. Effective neutrino mass parameter $|m_{ee}|$ versus the lightest neutrino mass for normal (purple/dark region) and inverted (magenta/light region) hierarchies. The vertical dot-dashed line and labelled as "Cosmology" denotes the bound from the combination of CMB and BAO data [35]. The vertical dotted line is the future sensitivity of KATRIN, [36]. Here the oscillation constraints are taken at 3σ [5]. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Let us now turn to the implications for $0\nu\beta\beta$. In Fig. 2 we plot the effective parameter $|m_{ee}|$ as function of the lightest neutrino mass. The NH case corresponds to the purple/dark region, while the IH case is denoted by the magenta/light region, respectively. The vertical dot-dashed line and labelled as "Cosmology" represents the constraint coming from the combination of CMB data [35], as well as the future sensitivity of KATRIN [36] indicated by the vertical dotted line.

4. Conclusions

In this paper we have suggested a model based on the flavour symmetry group T_7 leading to a very successful canonical mass relation between charged leptons and down-type quarks proposed in [18–20]. Previous papers predicting this mass relation have adopted the A_4 flavour symmetry and assumed that neutrino masses were generated through higher order operators. In our T_7 model the neutrino masses are generated through the conventional Type-I seesaw mechanism.

The model leads to a correlation between the lightest neutrino mass and the atmospheric angle. This correlation implies lower bounds for the lightest neutrino mass which come from applying the neutrino oscillation constraints. These bounds on the lightest neutrino mass also translate to lower bounds on the effective amplitude parameter $|m_{ee}|$ characterising $0\nu\beta\beta$ decay for both neutrino mass hierarchies.

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Appendix A. Vacuum alignments

Let us assume that the vev of the T_7 flavon triplet is real and that the field is shifted as,

$$\varphi_i = u_i + \phi_i. \tag{A.1}$$

The flavon potential is given by [24],

$$V_s = -\mu_s^2 \sum_{i=1}^{3} \varphi_i^\dagger \varphi_i + \lambda_s \left(\sum_{i=1}^{3} \varphi_i^\dagger \varphi_i \right)^2 + \kappa_s \sum_{i=1}^{3} \varphi_i^\dagger \varphi_i \varphi_i^\dagger \varphi_i, \tag{A.2}$$

where $\lambda_s > 0$. The minimisation conditions are obtained by taking,

$$\left.\frac{\partial V_s}{\partial \varphi_i}\right|_{\varphi_i \to 0} = 0, \tag{A.3}$$

which leads to the following system of equations,

$$\begin{aligned} &-\mu^2 + 2(\kappa_s + \lambda_s)u_1^2 + 2\lambda_s\left(u_2^2 + u_3^2\right) = 0 \\ &-\mu^2 + 2(\kappa_s + \lambda_s)u_2^2 + 2\lambda_s\left(u_1^2 + u_3^2\right) = 0 \\ &-\mu^2 + 2(\kappa_s + \lambda_s)u_3^2 + 2\lambda_s\left(u_1^2 + u_2^2\right) = 0. \end{aligned} \tag{A.4}$$

One set of minimisation conditions is obtained by solving (A.4) for instance for μ^2, u_2 and u_3,

$$\begin{aligned} &\text{a)}\quad \mu^2 = 2(\kappa_s + 3\lambda_s)u_1^2, \quad u_2 = u_3 = u_1; \\ &\text{b)}\quad \mu^2 = 2(\kappa_s + \lambda_s)u_1^2, \quad u_2 = u_3 = 0; \\ &\text{c)}\quad \mu^2 = 2(\kappa_s + 2\lambda_s)u_1^2, \quad u_2 = u_1 \text{ and } u_3 = 0, \end{aligned} \tag{A.5}$$

which can be translated in the following alignments, $\langle\varphi\rangle \equiv (u_1, u_2, u_3) \sim (1, 1, 1)$, $\langle\varphi\rangle \sim (1, 0, 0)$ and $\langle\varphi\rangle \sim (1, 1, 0)$, respectively. In order to characterise each case in (A.5) as a local minimum we compute the Hessian matrix,

$$\mathcal{H} = \left.\frac{\partial^2 V_s}{\partial \varphi_i \partial \varphi_j}\right|_{\varphi_i \to 0}, \tag{A.6}$$

and verify its positivity, that is all its eigenvalues are positive. For case a) the Hessian matrix turns out to be,

$$\mathcal{H}_a = 8u_1^2 \begin{pmatrix} (\kappa_s + \lambda_s) & \lambda_s & \lambda_s \\ \lambda_s & (\kappa_s + \lambda_s) & \lambda_s \\ \lambda_s & \lambda_s & (\kappa_s + \lambda_s) \end{pmatrix}. \tag{A.7}$$

The eigenvalues of $\mathcal{H}_a$ are, $8u_1^2(\kappa_s, \kappa_s, \kappa_s + 3\lambda_s)$ which are positive iff $\kappa_s > 0$. For b) we have,

$$\mathcal{H}_b = 4u_1^2 \begin{pmatrix} 2(\kappa_s + \lambda_s) & 0 & 0 \\ 0 & -\kappa_s & 0 \\ 0 & 0 & -\kappa_s \end{pmatrix}, \tag{A.8}$$

which is positive definite if $-\lambda_s < \kappa_s < 0$. Finally, in the last case we have,

$$\mathcal{H}_c = 4u_1^2 \begin{pmatrix} 2(\kappa_s + \lambda_s) & 2\lambda_s & 0 \\ 2\lambda_s & 2(\kappa_s + \lambda_s) & 0 \\ 0 & 0 & -\kappa_s \end{pmatrix}. \tag{A.9}$$

The eigenvalues of $\mathcal{H}_c$ are given by, $4u_1^2(2\kappa_s, 2(\kappa_s + 2\lambda_s), -\kappa_s)$. Therefore, we have that the only possible *global* minima are,

$$\begin{aligned} &\text{a)}\quad \langle\varphi\rangle \sim (\pm 1, \pm 1, \pm 1) \quad \text{for } \kappa_s > 0, \\ &\text{b)}\quad \langle\varphi\rangle \sim (\pm 1, 0, 0) \quad \text{for } -\lambda_s < \kappa_s < 0 \end{aligned}$$

up to sign permutations in the former and permutations of the non-zero value in the latter. These other possibilities lead to degenerate vacua. In the realistic case of our model there are other terms in the potential including T_7 symmetry breaking terms needed to generate δs in Table 2. In general these are expected to lift the degeneracies of the above minima.

Appendix B. T_7 group basics

The group T_7 is a subgroup of $SU(3)$ with 21 elements and isomorphic to $\mathbb{Z}_7 \rtimes \mathbb{Z}_3$. This group has five irreducible representations (i.e., $\mathbf{1}_0$, $\mathbf{1}_1$, $\mathbf{1}_2$, $\mathbf{3}$ and $\bar{\mathbf{3}}$) and is known as the smallest group containing a complex triplet. The multiplication rules in T_7 are the following,

$$\begin{aligned} &\mathbf{3} \otimes \mathbf{3} = \mathbf{3} \oplus \bar{\mathbf{3}} \oplus \bar{\mathbf{3}}, \qquad \mathbf{3} \otimes \mathbf{3} = \bar{\mathbf{3}} \oplus \mathbf{3} \oplus \mathbf{3}, \\ &\mathbf{3} \otimes \bar{\mathbf{3}} = \sum_{a=0}^{2} \mathbf{1}_a \oplus \mathbf{3} \oplus \bar{\mathbf{3}} \quad \text{and} \quad \mathbf{3} \otimes \mathbf{1} = \mathbf{3}. \end{aligned} \tag{B.1}$$

Let $\mathbf{X}^a = (x_1^a, x_2^a, x_3^a)^T$, $\bar{\mathbf{X}}^a = (\bar{x}_1^a, \bar{x}_2^a, \bar{x}_3^a)^T$, and $\mathbf{z}_i$ (with $i = 0, 1, 2$), be triplets, anti-triplets and singlets, respectively, under T_7 then these elements are multiplied as follows:

- $\mathbf{X} \times \mathbf{X}' = \mathbf{X}'' + \bar{\mathbf{X}} + \bar{\mathbf{X}}'$, where $\mathbf{X}'' = (x_3x'_3, x_1x'_1, x_2x'_2)$, $\bar{\mathbf{X}} = (x_2x'_3, x_3x'_1, x_1x'_2)$ and $\bar{\mathbf{X}}' = (x_3x'_2, x_1x'_3, x_2x'_1)$, (B.2)
- $\mathbf{X} \times \bar{\mathbf{X}} = \sum_{a=0}^{2} \mathbf{z}_a + \mathbf{X}' + \bar{\mathbf{X}}'$, where $z_a = x_1\bar{x}_1 + \omega^{2a}x_2\bar{x}_2 + \omega^a x_3\bar{x}_3$, $\mathbf{X}' = (x_2\bar{x}_1, x_3\bar{x}_2, x_1\bar{x}_3)$, and $\bar{\mathbf{X}}' = (x_1\bar{x}_2, x_2\bar{x}_3, x_3\bar{x}_1)$, (B.3)
- $\mathbf{z}_a \times \mathbf{X} = \mathbf{X}'$ where $\mathbf{X}' = (z_a x_1, \omega^a z_a x_2, \omega^{2a} z_a x_3)$. (B.4)

For more details about the group T_7 see for instance, Refs. [23–25].

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