Starobinsky-Type Inflation (from α'-Corrections)

1411.6010 & 1509.00024

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VSI Lunch Seminar

with D. Ciupke, F. Pedro, and A. Westphal

I will talk about . . .

Inflation, the CMB and Power-loss

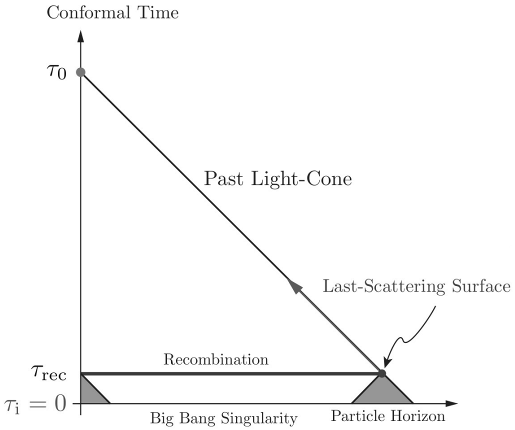
String Inflation from α'-corrections

$f(R)$ - Theory

$f(R)$ **beyond** R^2

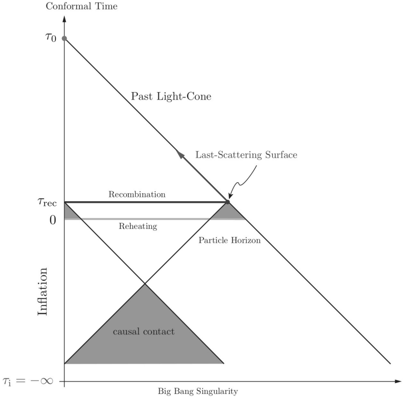
Part I

Why Inflation? I



$$d\tau = \frac{1}{a(t)} dt \,, \quad \chi(\tau)_{ds^2=0} = \int_{t_i}^{t_f} \frac{1}{a(t)} dt$$

Why Inflation? II



Idea:

$$\ddot{a} > 0\,, a \propto e^{Ht} \quad \Rightarrow \quad \int_{-\infty}^{t_f} \frac{1}{a(t)} dt \to \infty$$

Also: Flattnes, LSS ...

De-Sitter & Inflation I

Consider

$$\mathcal{L} = \frac{1}{2}\left(R - 2\Lambda\right)$$

This has

$$P_\Lambda = -\rho_\Lambda$$

Recall that for scalar field

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi), \quad \rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

Thus seek

$$\dot{\phi}^2 \ll V(\phi)$$

To realise above, need

$$\epsilon_V = \frac{1}{2}\left(\frac{V'}{V}\right)^2 \ll 1$$

De-Sitter & Inflation II

To have $\epsilon_V \ll 1$ for sufficiently long time, need

$$\eta_V = \frac{V''}{V} \ll 1$$

To cut long story short...

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 \sim \frac{V(\phi)}{3}$$

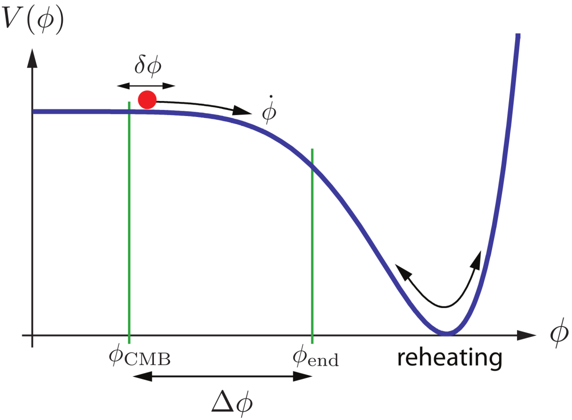
and hence

$$a(t) \propto e^{Ht}$$

Thus for inflation, need

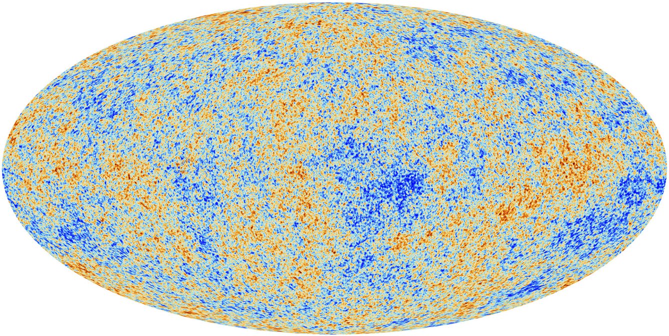
$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}\left(\partial\phi\right)^2 - V(\phi)\,, \quad \epsilon_V\,, \eta_V \ll 1$$

De-Sitter & Inflation III



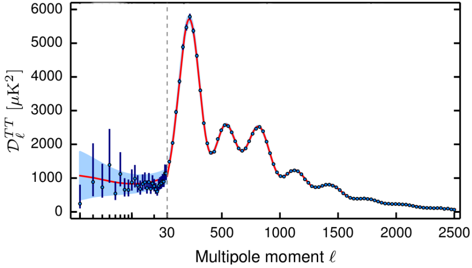
Inflation & The CMB

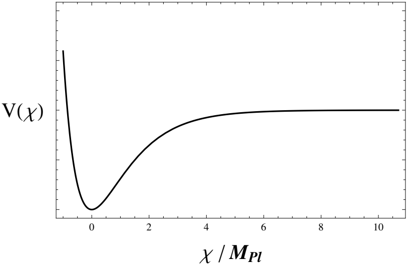
Quantum fluctuations $\delta\phi(t,\vec{x})$ of the inflaton $\Rightarrow$ Inflation ends at slightly different times $\Rightarrow$ spacetime is distorted $\Rightarrow$ gravitational potentials $\Rightarrow$ over and underdense regions/fluctuations in the primordial plasma $\Rightarrow$ universe cools & expands $\Rightarrow$ free streaming photons carry imprint of plasma's fluctuations $\Delta T/T \sim 10^{-5}\ldots$



$$\langle 0|\delta\hat{\phi}^\dagger_k(t,\vec{x})\delta\hat{\phi}_{k'}(t,\vec{x})|0\rangle \sim H^2$$

The CMB



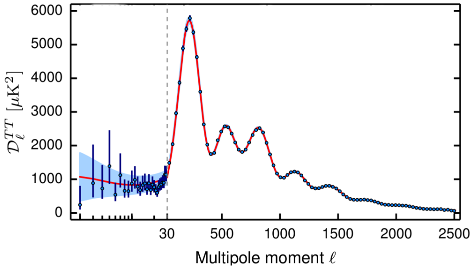


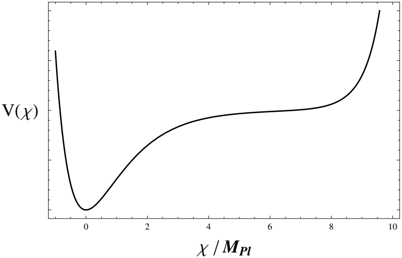
$$V_{inf} = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\right)^2 \quad \Longleftrightarrow \quad f(R) = R + \alpha R^2, \quad \alpha = \frac{1}{8V_0}$$

[1502.02114]

Hints for power suppression at low ℓ

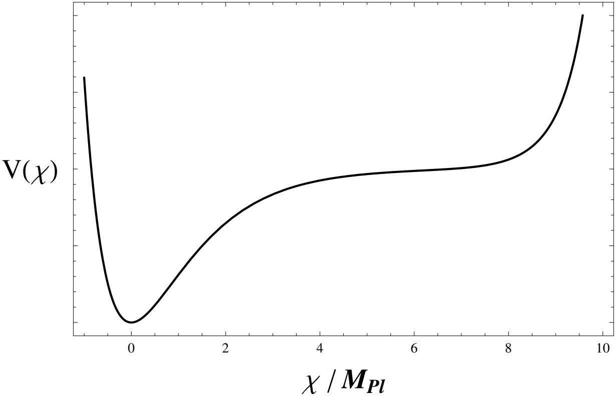
$$\Delta_s^2(k) \sim (k/k^*)^{n_s - 1}$$





$$V_{inf} = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\chi}\right)^2 + \varepsilon\, e^{\sqrt{\frac{2}{3}}\chi} \quad \Longleftrightarrow \quad f(R) = R + \alpha R^2 + \ldots ?$$

Take home message - Part I [1509.00024]



Possible to obtain above potential from recently computed higher derivative $(\alpha')^3$-corrections in combination with string loop effects.
Caveat: Terms and Conditions may apply (ie. tuning)

Large Volume Scenario in a nutshell...

Cicoli, Conlon, Burgess, Quevedo

IIB Flux compactifications with K3-fibred $\mathcal{V}(\tau_1, \tau_2, \tau_3)$, where $\tau_1, \tau_2 \gg \tau_3$

$$K = -2\log\left(\mathcal{V} + \frac{\hat{\xi}}{2}\right) \qquad \text{and} \qquad W = W_0 + Ae^{-a\tau_3},$$

F-term scalar potential is generated for the Kähler moduli:

$$V^{LVS}(\mathcal{V}, \tau_3) = g_s\left[\frac{8a_3^2A_3^2}{3\alpha\gamma}\frac{\sqrt{\tau_3}}{\mathcal{V}}\mathrm{e}^{-2a_3\tau_3} - 4W_0 a_3 A_3\frac{\tau_3}{\mathcal{V}^2}\mathrm{e}^{-a_3\tau_3} + \frac{3\hat{\xi}W_0^2}{4\mathcal{V}^3}\right]$$

Potential does not depend on τ_1 and τ_2. V^{LVS} has minima to stabilise

$$\langle\tau_3\rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma}\right)^{2/3} \quad , \quad \langle\mathcal{V}\rangle = \frac{3\alpha\gamma}{4a_3A_3}W_0\sqrt{\langle\tau_3\rangle}\mathrm{e}^{a_3\langle\tau_3\rangle}$$

Four-Superspace-Derivative Correction [1505.03092]

Ciupke, Louis, Westphal

$$\boldsymbol{V_{(1)} = -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \Pi_i t^i} \quad \Rightarrow \quad V_{eff} = V^{LVS} + V_{(1)}$$

Recall

$$\mathcal{V} \sim k_{ijk} t^i t^j t^k \,, \quad \tau_i = \frac{\partial \mathcal{V}}{\partial t^i}$$

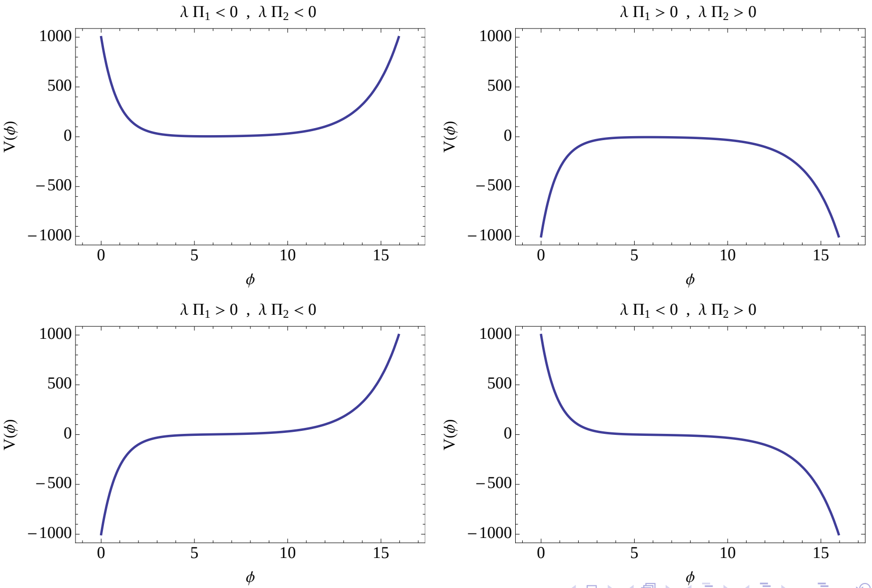
Non-trivial to find t^i as function of τ_i. Choose a geometry

$$\mathcal{V} = \alpha \left(\sqrt{\tau_1} \tau_2 - \gamma \tau_3^{3/2} \right)$$

$$V_{(1)} \simeq -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \left(\Pi_1 \frac{\mathcal{V}}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right)$$

$$V_{eff} = V^{LVS} - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\langle \mathcal{V} \rangle^4} \left(\Pi_1 \langle \mathcal{V} \rangle e^{-2/\sqrt{3}\varphi} + \Pi_2 \lambda_1^{-1/2} e^{\varphi/\sqrt{3}} \right)$$

Possible Inflationary Potentials



String Loop Corrections

$$V_{eff} + \delta V_{(g_s)} \simeq V^{LVS} + V_{(1)} + \frac{g_s |W_0|^2}{\mathcal{V}^2} \left(g_s^2 \frac{(C_1^{KK})^2}{\tau_1^2} + 2 g_s^2 (\alpha C_2^{KK})^2 \frac{\tau_1}{\mathcal{V}^2} \right)$$

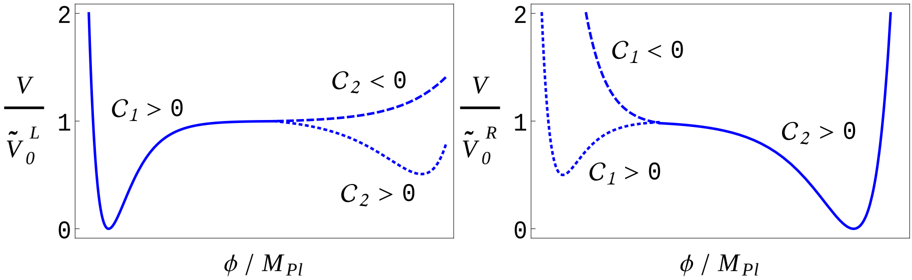
$$V = V_{\delta_{up}}^{LVS} + V_0 \left(-\mathcal{C}_1 e^{-2/\sqrt{3}\varphi} - \mathcal{C}_2 e^{\varphi/\sqrt{3}} + \mathcal{C}_1^{loop} e^{-4/\sqrt{3}\varphi} + \mathcal{C}_2^{loop} e^{2\sqrt{3}\varphi} \right)$$

where we have defined

$$V_0 = g_s^2 \frac{|W_0|^4}{\mathcal{V}^4}\,, \quad \mathcal{C}_1 = \hat{\lambda}\,\Pi_1 \mathcal{V}\,, \quad \mathcal{C}_2 = \hat{\lambda}\,\Pi_2 \lambda_1^{-1/2}\,,$$

$$\mathcal{C}_1^{loop} = \frac{\mathcal{V}^2}{|W_0|^2} g_s (C_1^{KK})^2 > 0\,, \quad \mathcal{C}_2^{loop} = \frac{2 g_s}{|W_0|^2} (\alpha C_2^{KK})^2 > 0$$

Viable Inflationary Potentials



$$V_{inf}^L \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} + \frac{\mathcal{C}_1^{loop}}{\tau_1^2} \right) \qquad V_{inf}^R \sim V_0 \left(-\mathcal{C}_2 \sqrt{\tau_1} + \mathcal{C}_2^{loop} \tau_1 \right)$$

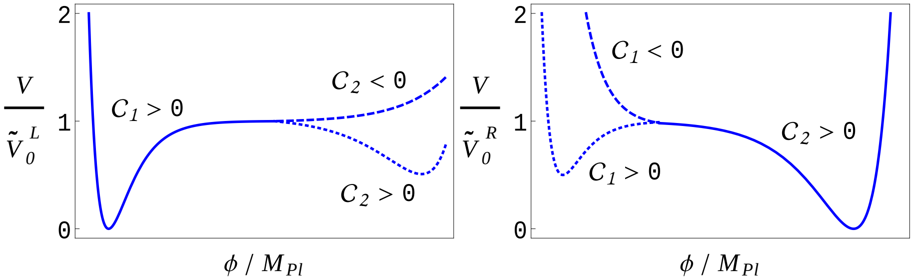
$$V_{inf}^L = \tilde{V}_0^L \left(1 - e^{-\kappa\phi} \right)^2 \qquad V_{inf}^R = \tilde{V}_0^R \left(1 - e^{\frac{\kappa}{2}\phi} \right)^2$$

First Order Observables

$$V_{inf} = V_0 \left(1 - e^{\pm\nu\phi}\right)^2 \quad \Rightarrow \quad n_s = 1 - \frac{2}{N}\,, \quad r \sim \nu^{-2}\frac{8}{N^2}$$

	$n_s(50)$	$n_s(60)$	$r(50)$	$r(60)$
right	0.960	0.967	0.0077	0.0055
left	0.960	0.967	0.0043	0.0016

Second Order Observables I



Second Order Observables II

$$V_{inf}^{L} \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} + \frac{\mathcal{C}_1^{loop}}{\tau_1^2} + \mathcal{C}_2 \sqrt{\tau_1} \right) \quad \Rightarrow \quad \tilde{V}_0^L \left(1 - 2e^{-\kappa\phi} + \varepsilon^2 e^{\frac{\kappa}{2}\phi} \right)$$

$$n_s = 1 - \frac{2}{N} - \frac{3\sqrt{2}\,\varepsilon^2\kappa}{\sqrt{N}} + \frac{\varepsilon^2\kappa^3}{\sqrt{2}}\sqrt{N} - \frac{3}{2}\varepsilon^4\kappa^4 N + \dots$$

Considering the 2-σ bounds by PLANCK, $\delta n_s \lesssim 0.008$ at $N = 55$, obtain

$$\varepsilon^2 \sim \lambda^{-3/2}\,\mathcal{V}^{-1}\left(g_s^{5/2}\mathcal{V}\right)^{3/2}\,\Pi_2\,\Pi_1^{-5/2}(C_1^{KK})^{3/2} \lesssim 10^{-3}$$

Second Order Observables III

Inflation to the Right

$$V_{inf}^R \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} - \mathcal{C}_2\sqrt{\tau_1} + \mathcal{C}_2^{loop}\tau_1 \right) \quad \Rightarrow \quad V_0^R \left(1 - 2e^{\frac{\kappa}{2}\phi} + \varepsilon^2 e^{-\kappa\phi} \right)$$

$$n_s = 1 - \frac{2}{N} - 3\,\varepsilon^2\kappa^4 N + \frac{\varepsilon^2\kappa^6}{2}N^2 + \dots$$

$$\lambda^{-3}\,\mathcal{V}\,g_s^{15/2}\,\Pi_1\,\Pi_2^{-4}(C_2^{KK})^6 \lesssim 2.4 \times 10^{-6}$$

Power-loss: $n_s = d\ln P/d\ln k = P^{-1}dP/dN$ and hence

$$\left.\frac{\Delta P(\delta n_s)}{P}\right|_{N+\Delta N}^{N} = \int_{N+\Delta N}^{N} \delta n_s \sim \delta n_s\,\Delta N \quad \rightarrow \quad \mathbf{4\%}$$

What you have to ensure...

To the Left	*resulting bound*
minimum at $\tau_1 \gtrsim 1$	$g_s^{5/2}\,\mathcal{V}(C_1^{KK})^2 \gtrsim 1$
$\tau_1^{min} < \tau_1^c$	$\mathcal{C}_1^{loop} < \frac{1}{2}\left(\frac{2}{\mathcal{C}_2}\right)^{2/3}\mathcal{C}_1^{5/3}$
PLANCK	$\lambda^2\,\lvert W_0\rvert^6\,\mathcal{V}^{-4} g_s^{-2}(C_1^{KK})^{-2} \sim 10^{-9}$

To the Right	*resulting bound*
plateau at $\tau_1 \gtrsim 1$	$2\,\frac{g_s^{5/2}(C_2^{KK})^2}{\lambda\,\lvert W_0\rvert^2\Pi_2} \ll 1$
$\tau_1^{min} > \tau_1^c$	$\mathcal{C}_2^{loop} < \left(\frac{\mathcal{C}_2^4}{\lvert\mathcal{C}_1\rvert}\right)^{1/3}$
PLANCK	$\lambda^2\,\lvert W_0\rvert^6\,\mathcal{V}^{-4} g_s^{-2}(C_2^{KK})^{-2} \sim 5\times 10^{-9}$

It's not a free lunch :(... What about masses?

Mass Hierarchy

Remaining scalars must be heavier than Hubble scale H during inflation

$$m_{cs}^2\,,\, m_S^2\,,\, m_{\tau_3}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^2}$$

Overall volume $\mathcal{V}$

$$m_{\mathcal{V}}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^3}$$

One must therefore make sure that

$$m_{\mathcal{V}}^2 \gg H^2 \sim V$$

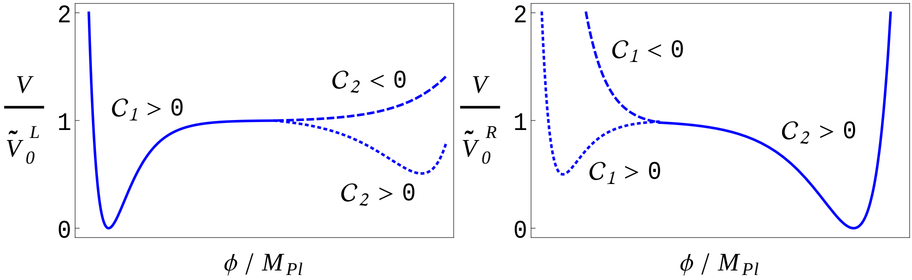
such that no other field than the fibre modulus τ_1 plays a role in inflation.

Numerical Examples

	W_0	g_s	$\mathcal{V}$	τ_1^{min}	Π_1	Π_2	C_1^{KK}	C_2^{KK}	n_s
$\mathcal{R}_1$	5	0.2	625.5	3000	0	100	0.00242	0.799	0.968
$\mathcal{R}_2$	25	0.3	1886.2	3500	0	10	0.000859	0.732	0.967
$\mathcal{L}_1$	2	0.3	460	3	100	1	0.163	0.0288	0.966
$\mathcal{L}_2$	5	0.4	1031.6	6	50	0	0.189	0.0266	0.969

Table: Examples of compactifications parameters and inflationary observables for inflation to the left ($\mathcal{L}_1$ and $\mathcal{L}_2$) and to the right ($\mathcal{R}_1$ and $\mathcal{R}_2$).

Conclusions I



Possible to obtain above potentials from recently computed higher derivative $(\alpha')^3$-corrections in combination with string loop effects.

Part II

$f(R)$ in a Nutshell

Consider

$$R \to f(R)$$

Weyl-transform via

$$\tilde{g}_{\mu\nu} = \frac{\partial f}{\partial R}\, g_{\mu\nu}$$

and obtain

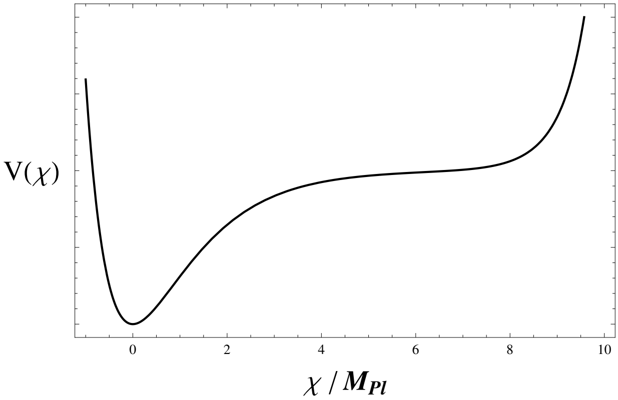
$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{\tilde{R}}{2} - \frac{1}{2}\,(\partial\chi)^2 - V(\chi),$$

for $\chi \equiv \sqrt{3/2}\,\ln f'$ with potential

$$V(\chi) = \frac{f'(R)\,R - f(R)}{2f'(R)^2}$$

Take home message - Part II

[1411.6010]
JCAP03 (2015) 029



Corresponding $f(R)$ - dual is to leading order R^n with $1 < n < 2$.

The Potential at Large Fields

If potential at large field values is

$$V(\chi) \sim V_0 e^{n\,\kappa\,\chi},$$

with $n \geq 1$, have to solve differential equation

$$V_0 f'^{\,n} = \frac{f'R - f}{2f'^{\,2}}.$$

Obtain asymptotic solution

$$\boxed{f(R) \sim R^{(n+2)/(n+1)} + \dots}$$

An exact $f(R)$ - Toy Model

Consider

$$V(\chi) = V_0 \left[\left(1 - e^{-\sqrt{2/3}\chi} \right)^2 + \varepsilon \, e^{\sqrt{2/3}\chi} \right] - \varepsilon \, V_0$$

to obtain exact

$$f(R) = \frac{\varepsilon - 1}{3\varepsilon} R + 4\varepsilon V_0 \left[\frac{(1-\varepsilon)^2}{9\varepsilon^2} + \frac{2}{3\varepsilon} + \frac{R}{6\varepsilon V_0} \right]^{3/2} + K$$

Taylor expanding for $\varepsilon \to 0$ recovers Starobinsky coefficients, e.g.

$$\lim_{\varepsilon \to 0} c_2 = \frac{1}{8\, V_0}$$

Need $\varepsilon \lesssim \mathcal{O}\left(10^{-4}\right)$ for $n_s \sim 0.97$

A non-zero Λ for free?

It is easy to show that when $\boldsymbol{V(0) = 0}$

$$f(R|_{\boldsymbol{\chi=0}}) \;=\; R|_{\boldsymbol{\chi=0}} \;=\; 2\,\varepsilon\,V_0$$

Checklist for $f(R)$

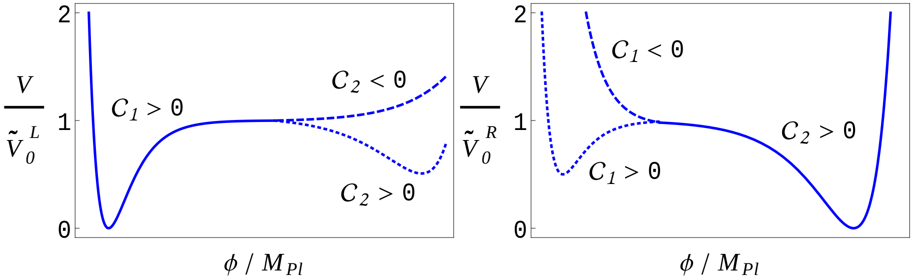
$$f(0) = 0\,, \quad f(R)\,,\, f'(R) > 0 \quad \forall\, R > 0$$

At first attempt, **not possible** to have both

$$f(R|_{\boldsymbol{\chi=0}})\,,\, f(0) = 0$$

at the same time (e. g. shifting $\chi \to \chi + \chi_0$ or adjusting K)

Recap: Conclusions I



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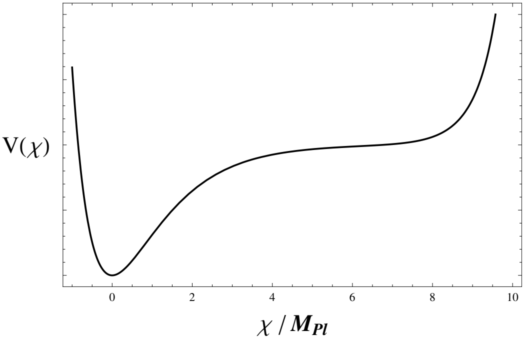
Conclusions II



Corresponding $f(R)$ - dual is to leading order R^n with $1 < n < 2$.

Thank you very much for your attention!