

Starobinsky-Type Inflation (from α' -Corrections)

1411.6010 & 1509.00024

Benedict J. Broy

Deutsches Elektronen-Synchrotron DESY, Theory Group,
Hamburg, Germany

VSI Lunch Seminar

with D. Ciupke, F. Pedro, and A. Westphal



I will talk about ...

Inflation, the CMB and Power-loss



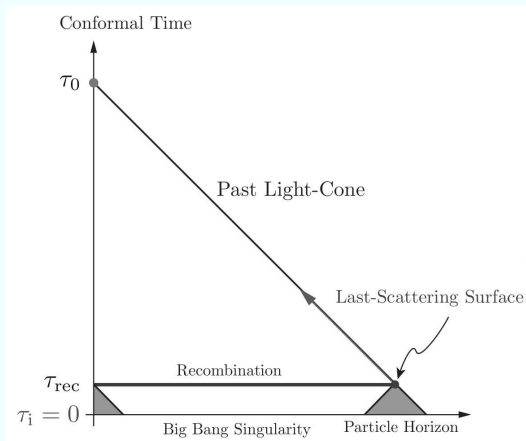
String Inflation from α' -corrections

$f(R)$ - Theory

$f(R)$ **beyond** R^2

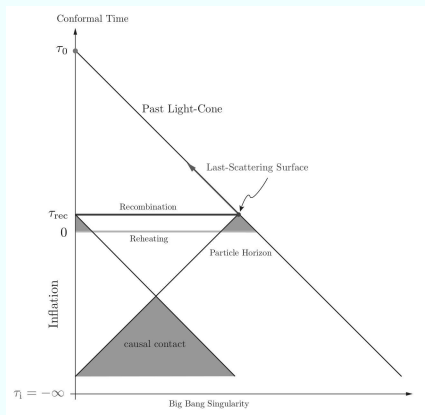
Part I

Why Inflation? I



$$d\tau = \frac{1}{a(t)} dt, \quad \chi(\tau)_{ds^2=0} = \int_{t_i}^{t_f} \frac{1}{a(t)} dt$$

Why Inflation? II



Idea:

$$\ddot{a} > 0, a \propto e^{Ht} \Rightarrow \int_{-\infty}^{t_f} \frac{1}{a(t)} dt \rightarrow \infty$$

Also: Flattnes, LSS ...

De-Sitter & Inflation I

Consider

$$\mathcal{L} = \frac{1}{2} (R - 2\Lambda)$$

This has

$$P_\Lambda = -\rho_\Lambda$$

Recall that for scalar field

$$P_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi), \quad \rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi)$$

Thus seek

$$\dot{\phi}^2 \ll V(\phi)$$

To realise above, need

$$\epsilon_V = \frac{1}{2} \left(\frac{V'}{V} \right)^2 \ll 1$$

De-Sitter & Inflation II

To have $\epsilon_V \ll 1$ for sufficiently long time, need

$$\eta_V = \frac{V''}{V} \ll 1$$

To cut long story short...

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 \sim \frac{V(\phi)}{3}$$

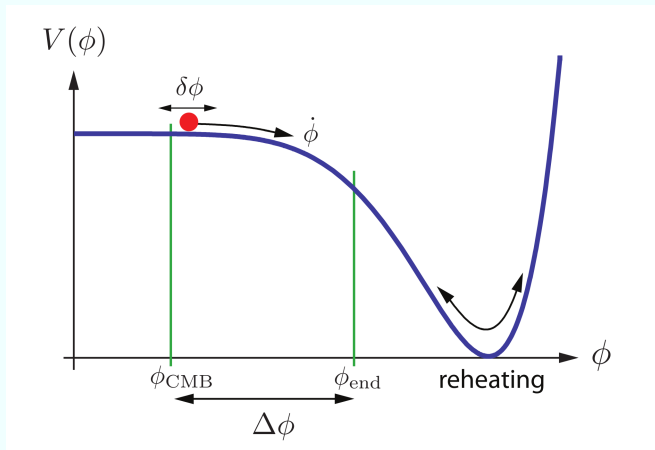
and hence

$$a(t) \propto e^{Ht}$$

Thus for inflation, need

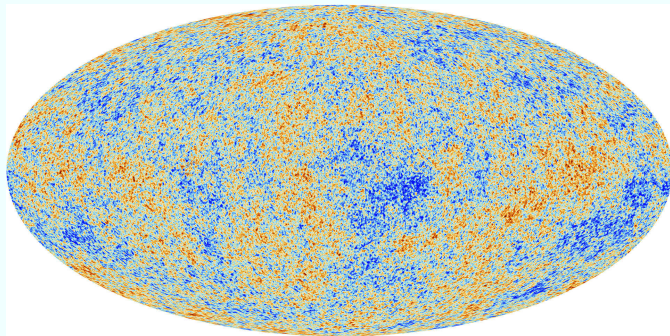
$$\mathcal{L} = \frac{R}{2} - \frac{1}{2}(\partial\phi)^2 - V(\phi), \quad \epsilon_V, \eta_V \ll 1$$

De-Sitter & Inflation III



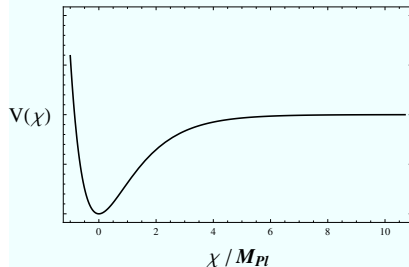
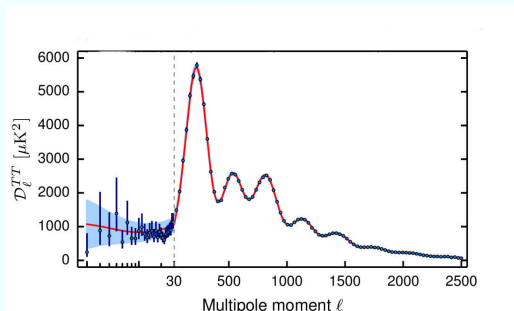
Inflation & The CMB

Quantum fluctuations $\delta\phi(t, \vec{x})$ of the inflaton $\Rightarrow$ Inflation ends at slightly different times $\Rightarrow$ spacetime is distorted $\Rightarrow$ gravitational potentials $\Rightarrow$ over and underdense regions/fluctuations in the primordial plasma $\Rightarrow$ universe cools & expands $\Rightarrow$ free streaming photons carry imprint of plasma's fluctuations $\Delta T/T \sim 10^{-5} \dots$



$$\langle 0 | \delta \hat{\phi}_k^\dagger(t, \vec{x}) \delta \hat{\phi}_{k'}(t, \vec{x}) | 0 \rangle \sim H^2$$

The CMB

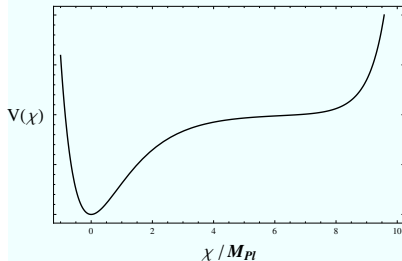
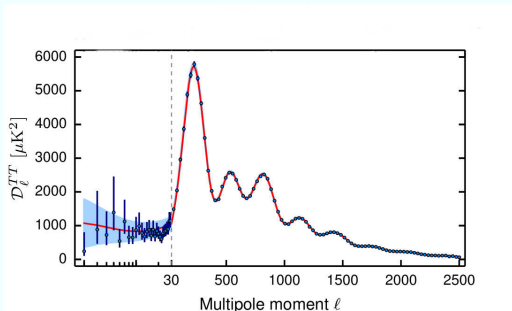


$$V_{inf} = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\chi} \right)^2 \iff f(R) = R + \alpha R^2, \quad \alpha = \frac{1}{8V_0}$$

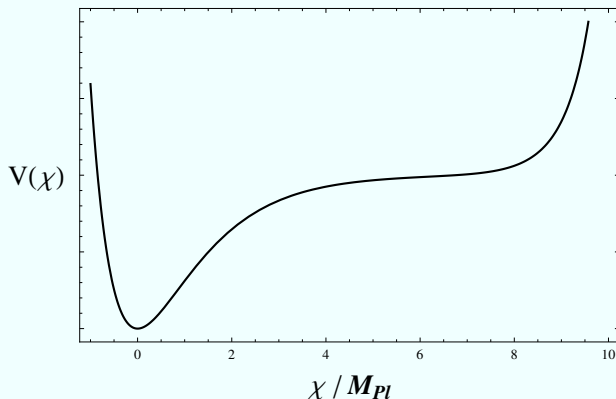
[1502.02114]

Hints for power suppression at low ℓ

$$\Delta_s^2(k) \sim (k/k^*)^{n_s-1}$$



$$V_{inf} = V_0 \left(1 - e^{-\sqrt{\frac{2}{3}}\chi} \right)^2 + \varepsilon e^{\sqrt{\frac{2}{3}}\chi} \iff f(R) = R + \alpha R^2 + \dots?$$



Possible to obtain above potential from recently computed higher derivative $(\alpha')^3$ -corrections in combination with string loop effects.
Caveat: Terms and Conditions may apply (ie. tuning)

Large Volume Scenario in a nutshell. . .

Cicoli, Conlon, Burgess, Quevedo

IIB Flux compactifications with K3-fibred $\mathcal{V}(\tau_1, \tau_2, \tau_3)$, where $\tau_1, \tau_2 \gg \tau_3$

$$K = -2 \log \left(\mathcal{V} + \frac{\hat{\xi}}{2} \right) \quad \text{and} \quad W = W_0 + A e^{-a\tau_3},$$

F-term scalar potential is generated for the Kähler moduli:

$$V^{LVS}(\mathcal{V}, \tau_3) = g_s \left[\frac{8a_3^2 A_3^2}{3\alpha\gamma} \frac{\sqrt{\tau_3}}{\mathcal{V}} e^{-2a_3\tau_3} - 4W_0 a_3 A_3 \frac{\tau_3}{\mathcal{V}^2} e^{-a_3\tau_3} + \frac{3\hat{\xi} W_0^2}{4\mathcal{V}^3} \right]$$

Potential does **not** depend on τ_1 and τ_2 . V^{LVS} has minima to stabilise

$$\langle \tau_3 \rangle = \left(\frac{\hat{\xi}}{2\alpha\gamma} \right)^{2/3}, \quad \langle \mathcal{V} \rangle = \frac{3\alpha\gamma}{4a_3 A_3} W_0 \sqrt{\langle \tau_3 \rangle} e^{a_3 \langle \tau_3 \rangle}$$

$$V_{(1)} = -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \Pi_i t^i \quad \Rightarrow \quad V_{eff} = V^{LVS} + V_{(1)}$$

Recall

$$\mathcal{V} \sim k_{ijk} t^i t^j t^k, \quad \tau_i = \frac{\partial \mathcal{V}}{\partial t^i}$$

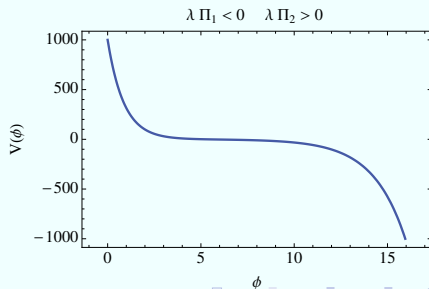
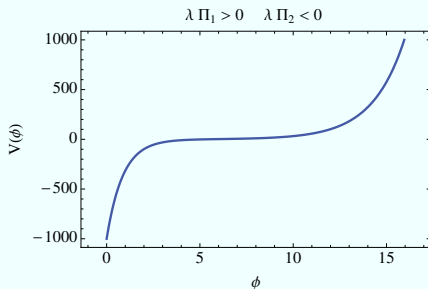
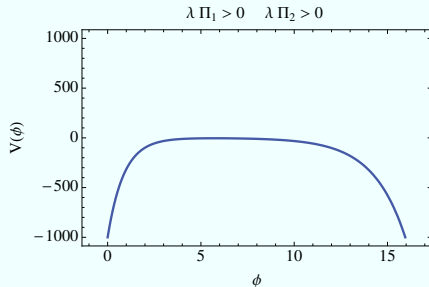
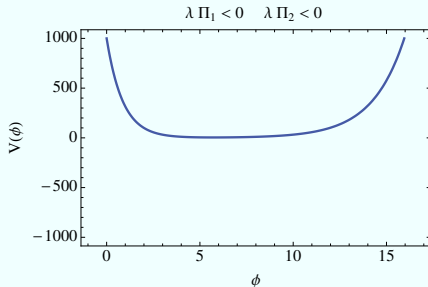
Non-trivial to find t^i as function of τ_i . Choose a geometry

$$\mathcal{V} = \alpha \left(\sqrt{\tau_1} \tau_2 - \gamma \tau_3^{3/2} \right)$$

$$V_{(1)} \simeq -g_s^2 \hat{\lambda} \frac{|W_0|^4}{\mathcal{V}^4} \left(\Pi_1 \frac{\mathcal{V}}{\tau_1} + \Pi_2 \lambda_1^{-1/2} \sqrt{\tau_1} \right)$$

$$V_{eff} = V^{LVS} - g_s^2 \hat{\lambda} \frac{|W_0|^4}{\langle \mathcal{V} \rangle^4} \left(\Pi_1 \langle \mathcal{V} \rangle e^{-2/\sqrt{3} \varphi} + \Pi_2 \lambda_1^{-1/2} e^{\varphi/\sqrt{3}} \right)$$

Possible Inflationary Potentials



String Loop Corrections

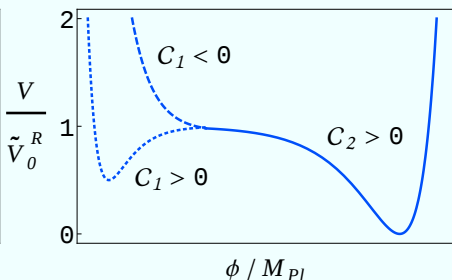
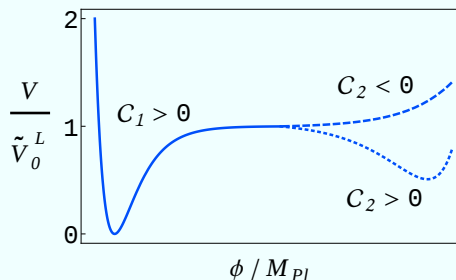
$$V_{eff} + \delta V_{(g_s)} \simeq V^{LVS} + V_{(1)} + \frac{g_s |W_0|^2}{\mathcal{V}^2} \left(g_s^2 \frac{(C_1^{KK})^2}{\tau_1^2} + 2g_s^2 (\alpha C_2^{KK})^2 \frac{\tau_1}{\mathcal{V}^2} \right)$$

$$V = V_{\delta_{up}}^{LVS} + V_0 \left(-\mathcal{C}_1 e^{-2/\sqrt{3}\varphi} - \mathcal{C}_2 e^{\varphi/\sqrt{3}} + \mathcal{C}_1^{loop} e^{-4/\sqrt{3}\varphi} + \mathcal{C}_2^{loop} e^{2\sqrt{3}\varphi} \right)$$

where we have defined

$$V_0 = g_s^2 \frac{|W_0|^4}{\mathcal{V}^4}, \quad \mathcal{C}_1 = \hat{\lambda} \Pi_1 \mathcal{V}, \quad \mathcal{C}_2 = \hat{\lambda} \Pi_2 \lambda_1^{-1/2},$$
$$\mathcal{C}_1^{loop} = \frac{\mathcal{V}^2}{|W_0|^2} g_s (C_1^{KK})^2 > 0, \quad \mathcal{C}_2^{loop} = \frac{2g_s}{|W_0|^2} (\alpha C_2^{KK})^2 > 0$$

Viable Inflationary Potentials



$$V_{inf}^L \sim V_0 \left(-\frac{C_1}{\tau_1} + \frac{C_1^{loop}}{\tau_1^2} \right)$$

$$V_{inf}^L = \tilde{V}_0^L \left(1 - e^{-\kappa\phi} \right)^2$$

$$V_{inf}^R \sim V_0 \left(-C_2\sqrt{\tau_1} + C_2^{loop}\tau_1 \right)$$

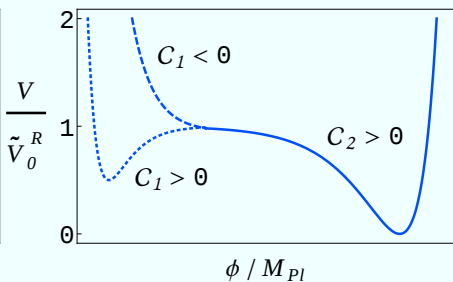
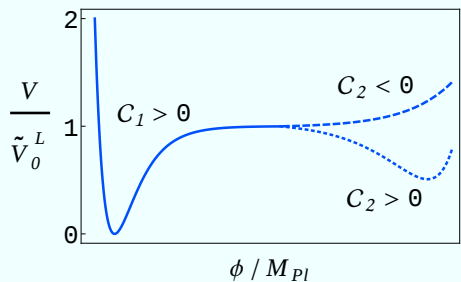
$$V_{inf}^R = \tilde{V}_0^R \left(1 - e^{\frac{\kappa}{2}\phi} \right)^2$$

First Order Observables

$$V_{inf} = V_0 \left(1 - e^{\pm \nu \phi}\right)^2 \quad \Rightarrow \quad n_s = 1 - \frac{2}{N}, \quad r \sim \nu^{-2} \frac{8}{N^2}$$

	$n_s(50)$	$n_s(60)$	$r(50)$	$r(60)$
<i>right</i>	0.960	0.967	0.0077	0.0055
<i>left</i>	0.960	0.967	0.0043	0.0016

Second Order Observables I



Second Order Observables II

$$V_{inf}^L \sim V_0 \left(-\frac{C_1}{\tau_1} + \frac{C_1^{loop}}{\tau_1^2} + \mathcal{C}_2 \sqrt{\tau_1} \right) \Rightarrow \tilde{V}_0^L \left(1 - 2e^{-\kappa\phi} + \varepsilon^2 e^{\frac{\kappa}{2}\phi} \right)$$

$$n_s = 1 - \frac{2}{N} - \frac{3\sqrt{2}\varepsilon^2\kappa}{\sqrt{N}} + \frac{\varepsilon^2\kappa^3}{\sqrt{2}} \sqrt{N} - \frac{3}{2}\varepsilon^4\kappa^4 N + \dots$$

Considering the 2- σ bounds by PLANCK, $\delta n_s \lesssim 0.008$ at $N = 55$, obtain

$$\varepsilon^2 \sim \lambda^{-3/2} \mathcal{V}^{-1} \left(g_s^{5/2} \mathcal{V} \right)^{3/2} \Pi_2 \Pi_1^{-5/2} (C_1^{KK})^{3/2} \lesssim 10^{-3}$$

Second Order Observables III

Inflation to the Right

$$V_{inf}^R \sim V_0 \left(-\frac{\mathcal{C}_1}{\tau_1} - \mathcal{C}_2 \sqrt{\tau_1} + \mathcal{C}_2^{loop} \tau_1 \right) \Rightarrow V_0^R \left(1 - 2e^{\frac{\kappa}{2}\phi} + \varepsilon^2 e^{-\kappa\phi} \right)$$

$$n_s = 1 - \frac{2}{N} - 3\varepsilon^2 \kappa^4 N + \frac{\varepsilon^2 \kappa^6}{2} N^2 + \dots$$

$$\lambda^{-3} \mathcal{V} g_s^{15/2} \Pi_1 \Pi_2^{-4} (C_2^{KK})^6 \lesssim 2.4 \times 10^{-6}$$

Power-loss: $n_s = d \ln P / d \ln k = P^{-1} dP / dN$ and hence

$$\left. \frac{\Delta P(\delta n_s)}{P} \right|_{N+\Delta N}^N = \int_{N+\Delta N}^N \delta n_s \sim \delta n_s \Delta N \rightarrow 4\%$$

What you have to ensure...

<i>To the Left</i>	<i>resulting bound</i>
minimum at $\tau_1 \gtrsim 1$	$g_s^{5/2} \mathcal{V} (C_1^{KK})^2 \gtrsim 1$
$\tau_1^{min} < \tau_1^c$	$C_1^{loop} < \frac{1}{2} \left(\frac{2}{\bar{c}_2} \right)^{2/3} C_1^{5/3}$
PLANCK	$\lambda^2 W_0 ^6 \mathcal{V}^{-4} g_s^{-2} (C_1^{KK})^{-2} \sim 10^{-9}$
<i>To the Right</i>	<i>resulting bound</i>
plateau at $\tau_1 \gtrsim 1$	$2 \frac{g_s^{5/2} (C_2^{KK})^2}{\lambda W_0 ^2 \Pi_2} \ll 1$
$\tau_1^{min} > \tau_1^c$	$C_2^{loop} < \left(\frac{C_2^4}{ C_1 } \right)^{1/3}$
PLANCK	$\lambda^2 W_0 ^6 \mathcal{V}^{-4} g_s^{-2} (C_2^{KK})^{-2} \sim 5 \times 10^{-9}$

It's not a free lunch :(... What about masses?

Mass Hierarchy

Remaining scalars must be heavier than Hubble scale H during inflation

$$m_{cs}^2, m_S^2, m_{\tau_3}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^2}$$

Overall volume $\mathcal{V}$

$$m_{\mathcal{V}}^2 \sim g_s \frac{|W_0|^2}{\mathcal{V}^3}$$

One must therefore make sure that

$$m_{\mathcal{V}}^2 \gg H^2 \sim V$$

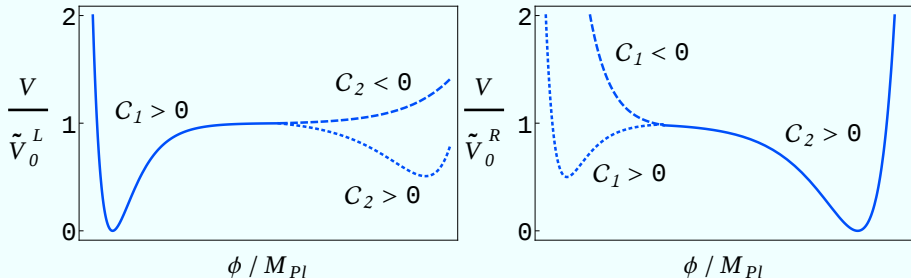
such that no other field than the fibre modulus τ_1 plays a role in inflation.

Numerical Examples

	W_0	g_s	$\mathcal{V}$	τ_1^{min}	Π_1	Π_2	C_1^{KK}	C_2^{KK}	n_s
$\mathcal{R}_1$	5	0.2	625.5	3000	0	100	0.00242	0.799	0.968
$\mathcal{R}_2$	25	0.3	1886.2	3500	0	10	0.000859	0.732	0.967
$\mathcal{L}_1$	2	0.3	460	3	100	1	0.163	0.0288	0.966
$\mathcal{L}_2$	5	0.4	1031.6	6	50	0	0.189	0.0266	0.969

Table: Examples of compactifications parameters and inflationary observables for inflation to the left ($\mathcal{L}_1$ and $\mathcal{L}_2$) and to the right ($\mathcal{R}_1$ and $\mathcal{R}_2$).

Conclusions I



Possible to obtain above potentials from recently computed higher derivative $(\alpha')^3$ -corrections in combination with string loop effects.

Part II

$f(R)$ in a Nutshell

Consider

$$R \rightarrow f(R)$$

Weyl-transform via

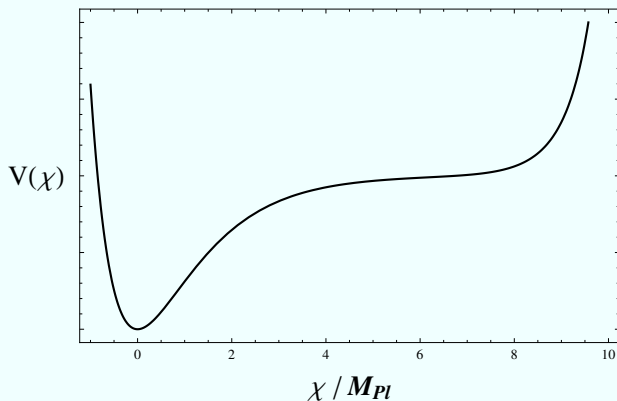
$$\tilde{g}_{\mu\nu} = \frac{\partial f}{\partial R} g_{\mu\nu}$$

and obtain

$$\frac{\mathcal{L}}{\sqrt{-\tilde{g}}} = \frac{\tilde{R}}{2} - \frac{1}{2} (\partial\chi)^2 - V(\chi),$$

for $\chi \equiv \sqrt{3/2} \ln f'$ with potential

$$V(\chi) = \frac{f'(R) R - f(R)}{2f'(R)^2}$$



Corresponding $f(R)$ - dual is to leading order R^n with $1 < n < 2$.

The Potential at Large Fields

If potential at large field values is

$$V(\chi) \sim V_0 e^{n \kappa \chi},$$

with $n \geq 1$, have to solve differential equation

$$V_0 f'^n = \frac{f' R - f}{2 f'^2}.$$

Obtain asymptotic solution

$$f(R) \sim R^{(n+2)/(n+1)} + \dots$$

An exact $f(R)$ - Toy Model

Consider

$$V(\chi) = V_0 \left[\left(1 - e^{-\sqrt{2/3}\chi} \right)^2 + \varepsilon e^{\sqrt{2/3}\chi} \right] - \varepsilon V_0$$

to obtain exact

$$f(R) = \frac{\varepsilon - 1}{3\varepsilon} R + 4\varepsilon V_0 \left[\frac{(1 - \varepsilon)^2}{9\varepsilon^2} + \frac{2}{3\varepsilon} + \frac{R}{6\varepsilon V_0} \right]^{3/2} + K$$

Taylor expanding for $\varepsilon \rightarrow 0$ recovers Starobinsky coefficients, e.g.

$$\lim_{\varepsilon \rightarrow 0} c_2 = \frac{1}{8V_0}$$

Need $\varepsilon \lesssim \mathcal{O}(10^{-4})$ for $n_s \sim 0.97$

A non-zero Λ for free?

It is easy to show that when $V(0) = 0$

$$f(R|_{\chi=0}) = R|_{\chi=0} = 2\varepsilon V_0$$

Checklist for $f(R)$

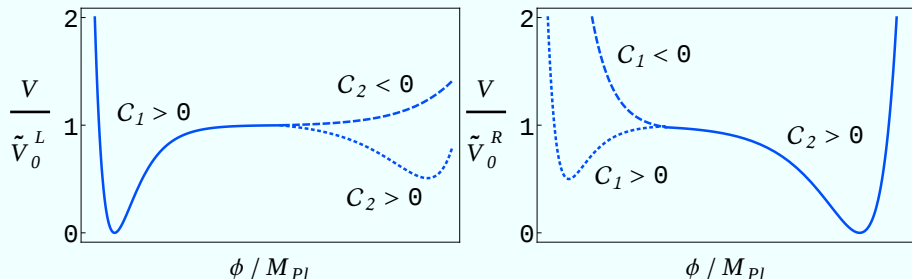
$$f(0) = 0, \quad f(R), f'(R) > 0 \quad \forall R > 0$$

At first attempt, **not possible** to have both

$$f(R|_{\chi=0}), f(0) = 0$$

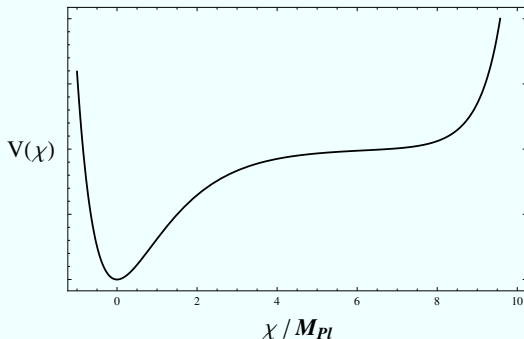
at the same time (e. g. shifting $\chi \rightarrow \chi + \chi_0$ or adjusting K)

Recap: Conclusions I



Possible to obtain above potentials from recently computed higher derivative $(\alpha')^3$ -corrections in combination with string loop effects.

Conclusions II



Corresponding $f(R)$ - dual is to leading order R^n with $1 < n < 2$.

Thank you very much for your attention!