On the interpretation of dark matter self-interactions in Abell 3827

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ABSTRACT

Self-interactions of dark matter (DM) particles can potentially lead to an observable separation between the DM halo and the stars of a galaxy moving through a region of large DM density. Such a separation has recently been observed in a galaxy falling into the core of the galaxy cluster Abell 3827. We estimated the DM self-interaction cross-section needed to reproduce the observed effects and find that the sensitivity of Abell 3827 has been significantly overestimated in a previous study. Our corrected estimate is $\tilde{\sigma}/m_{\rm DM} \sim 3~{\rm cm^2~g^{-1}}$ when self-interactions result in an effective drag force and $\sigma/m_{\rm DM} \sim 1.5~{\rm cm^2~g^{-1}}$ for the case of contact interactions, in some tension with previous upper bounds.

Key words: astroparticle physics – galaxies: clusters: individual: Abell 3827 – galaxies: kinematics and dynamics – dark matter.

1 INTRODUCTION

Recently Massey et al. (2015) have studied elliptical galaxies falling into the core of the galaxy cluster Abell 3827, following previous such studies (Williams & Saha 2011; Mohammed et al. 2013). Using several strongly-lensed images of background objects, it is possible to reconstruct the positions of the dark matter (DM) subhaloes of each of the galaxies. One of these is observed to be significantly separated from the galaxy's stars by $\Delta=1.62^{+0.47}_{-0.49}$ kpc. While noting that an astrophysical origin of this separation cannot be excluded, the authors interpret this in terms of DM self-interactions and infer: $\sigma_{\rm DM}/m_{\rm DM} \sim (1.7 \pm 0.7) \times 10^{-4} \, {\rm cm^2 \, g^{-1}}$. If correct this would rule out many attractive DM candidates, in particular axions, neutrinos and 'weakly interacting massive particles' (WIMPs) such as supersymmetric neutralinos.

The cross-section estimated by Massey et al. (2015) is well below the weak upper bounds set by other astrophysical objects such as the 'Bullet Cluster' (1E 0657-56) which are typically: $\sigma_{\rm DM}/m_{\rm DM} \lesssim 1~{\rm cm^2~g^{-1}}$ (Markevitch et al. 2004; Randall et al. 2008; Peter et al. 2012; Rocha et al. 2013; Harvey et al. 2015). Massey et al. (2015) argue that A3827 is uniquely sensitive because the infall time for the galaxies is very long, hence the effects of DM self-interactions add up over a long period. They adopt a simple model for the effect of DM self-interactions following Williams & Saha (2011) which predicts that the separation between the stars and the DM subhalo should grow proportional to $t_{\rm infall}^2$ thus vastly amplifying the effect over time.

In this Letter, we argue that the model used to interpret the observations in terms of DM self-interactions is based on two questionable assumptions:

- (i) the stars and the associated DM subhalo are assumed to develop completely independently such that even a tiny difference in the acceleration they experience can lead to sizeable differences in their trajectories. This neglects the crucial fact that initially the stars are gravitationally bound to the DM subhalo and can only be separated from it if external forces are comparable to the gravitational attraction within the system. The effect of DM self-interactions must therefore be at least comparable to the relevant gravitational effects in order to lead to an observable separation.
- (ii) The effective drag force on the DM subhalo is assumed to be *constant* throughout the evolution of the system. This is in contradiction with the fact that the rate of DM self-interactions typically depends both on the velocity of the subhalo relative to the cluster and the DM density of the cluster (at the position of the subhalo), both of which will vary along the trajectory of the subhalo. In particular, the rate of DM self-interactions will be negligibly small as long as the subhalo is far away from the core of the cluster.

These issues have previously been discussed by Kahlhoefer et al. (2014, hereafter Ka14) in the context of merging clusters such as Abell 520, the 'Bullet Cluster' or the 'Musket Ball Cluster', but they apply equally in the present context. We apply the arguments from Ka14 to provide a corrected estimate of the DM self-interaction cross-section necessary to explain the observations of A3827. We find values that are intriguingly close to existing bounds from other systems, implying that such systems can potentially be used to confirm or rule out this interpretation.

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¹ The assumption of a constant drag force would be justified if the subhalo were on a circular orbit around the centre of A3827. However, the observed separation between stars and DM points approximately in the radial direction, so this is disfavoured.

First we provide a simple estimate of the magnitude of the drag force and the relevant gravitational force to extract the relevant self-interaction cross-section. We refine this estimate by considering a realistic trajectory for the subhalo and evaluating its velocity and the background DM density along this trajectory. We then run numerical simulations of a subhalo falling towards the core of A3827, accounting for the motion of a large number of DM test particles undergoing self-interactions in a time-dependent gravitational potential. Finally, we discuss alternatives to a simple drag force that are motivated from a particle physics perspective.

2 ESTIMATES OF DM SELF-INTERACTIONS

Let us assume that DM particles moving with velocity v through an ambient DM density ρ experience a drag force of the form

$$\frac{F_{\text{drag}}}{m_{\text{DM}}} = \frac{1}{4} \frac{\tilde{\sigma}}{m_{\text{DM}}} v^2 \rho , \qquad (1)$$

where $\tilde{\sigma}$ is the effective DM self-interaction cross-section, $m_{\rm DM}$ is the DM mass and we have chosen the normalization of $\tilde{\sigma}$ in such a way that the momentum transfer cross-section is given by $\sigma_{\rm T} = \tilde{\sigma}/2$ in analogy to the case of isotropic scattering (see Ka14). Such a drag force can be obtained by averaging over a large number of DM scattering processes with small scattering angle, assuming that the differential cross-section has no velocity dependence but a strong angular dependence such that the probability of scattering is peaked in the forward direction.² The presence of such a drag force will slow down any DM subhalo falling into a larger DM halo compared to objects experiencing only gravitational forces, such as the stars bound in the subhalo.

The stars inside the subhalo, however, do not just experience the gravitational attraction of the cluster but also the attraction of the subhalo itself. The latter contribution aims to reduce any separation between the subhalo and the stars and will therefore oppose the effect of the drag force. At some point, the gravitational attraction between DM subhalo and stars will become sufficiently large to balance the drag force on the DM subhalo. Denoting the (average) separation between the subhalo and the stars by Δ , the restoring force will be given by

$$\frac{F_{\rm sh}}{m_{\rm star}} = \frac{G_{\rm N} \, M_{\rm sh}(\Delta)}{\Delta^2} \tag{2}$$

with $G_{\rm N}$ Newton's constant and $M_{\rm sh}(\Delta)$ the subhalo mass within radius Δ . As a rough estimate, we assume a constant density core with radius $a_{\rm sh}=2.7$ kpc and mass $M_{\rm sh}=7\times10^{10}\,{\rm M}_{\odot}$ (consistent with the mass estimate by Massey et al. 2015), so that $M_{\rm sh}(\Delta)=M_{\rm sh}\Delta^3/a_{\rm sh}^3$. In order for the drag force to result in a sizeable separation, we must require $F_{\rm sh}/m_{\rm star}< F_{\rm drag}/m_{\rm DM}$. This inequality leads to

$$\frac{\tilde{\sigma}}{m_{\rm DM}} > \frac{4}{v^2 \rho} \frac{G_{\rm N} M_{\rm sh} \Delta}{a_{\rm sh}^3} \ . \tag{3}$$

Based on our mass model for A3827 (Appendix A), we estimate $\rho \sim 4\, {\rm GeV\,cm^{-3}}$ and $v \sim 1500\, {\rm km\,s^{-1}}$ at $r=15\, {\rm kpc}$. Substituting these values and requiring $\Delta=1.6\, {\rm kpc}$ yields

$$\frac{\ddot{\sigma}}{m_{\rm DM}} \gtrsim 2 \, {\rm cm}^2 \, {\rm g}^{-1} \; , \tag{4}$$

which is in some tension with the upper bound from the Bullet Cluster: $\tilde{\sigma}/m_{\rm DM} \lesssim 1.2 \, {\rm cm}^2 \, {\rm g}^{-1}$ (Ka14).

If the self-interaction cross-section is much smaller than the estimate obtained above, the stars will remain closely bound to the subhalo due to the overwhelming gravitational restoring force. In particular, it should be clear that *no visible separation* can result from a cross-section as small as $\tilde{\sigma}/m_{\rm DM}\sim 10^{-4}\,{\rm cm}^2\,{\rm g}^{-1}$.

2.1 One-dimensional simulations

For the estimate above, we have assumed both the velocity of the subhalo v and the background density of the cluster ρ to be constant in time. In this case, the separation between the DM subhalo and the stars is expected to remain constant in time, with the equilibrium value determined by the condition $F_{\rm drag}/m_{\rm DM}=F_{\rm sh}/m_{\rm star}$. However, as long as the subhalo is far away from the central region of the cluster, both v and ρ are expected to be small, so that no significant separation will occur. As the subhalo accelerates towards the central region, the separation will grow, provided that the drag force is always sufficiently larger than the restoring gravitational force. To be more realistic, we should therefore calculate the position and velocity of the subcluster as a function of time. To this end, we need to also include the gravitational force of the cluster, acting on both the subhalo and the stars:

$$\frac{F_{\text{cluster}}}{m} = \frac{G_{\text{N}} M_{\text{cluster}}(r)}{r^2} \,, \tag{5}$$

where $M_{\rm cluster}(r)$ is the cluster mass within radius r. Assuming a radial orbit, we can write

$$\ddot{r}_{\rm sh} = -\frac{F_{\rm cluster}}{m_{\rm DM}} + \frac{F_{\rm drag}}{m_{\rm DM}}
= -\frac{G_{\rm N} \, M_{\rm cluster}(r_{\rm sh})}{r_{\rm sh}^2} + \frac{1}{4} \frac{\tilde{\sigma}}{m_{\rm DM}} \, \dot{r}_{\rm sh}^2 \, \rho(r_{\rm sh}), \tag{6}$$

$$\ddot{r}_{\text{star}} = -\frac{F_{\text{cluster}}}{m_{\text{star}}} + \frac{F_{\text{sh}}}{m_{\text{star}}}$$

$$= -\frac{G_{\text{N}} M_{\text{cluster}}(r_{\text{star}})}{r_{\text{star}}^2} + \frac{G_{\text{N}} M_{\text{sh}}(r_{\text{sh}} - r_{\text{star}})}{a_{\text{sh}}^3}, \qquad (7)$$

where we assume the gravitational pull of the stars on the subhalo to be negligible. As before, we will assume a constant density core for the subhalo, such that $F_{\rm sh}$ is proportional to $r_{\rm sh}-r_{\rm star}$. For the cluster, however, we will use a more refined mass model as discussed in Appendix A and calculate the cluster mass within radius r according to $M_{\rm cluster}(r) = 4\pi \int_0^r r'^2 \rho(r') \, {\rm d} r'$.

The set of differential equations introduced above is readily solved for given initial conditions. We assume that the subhalo starts falling towards the cluster from rest at an initial distance of $100 \, \rm kpc$ and that its trajectory lies in the plane of the sky. It will then take approximately $t_{\rm infall} \sim 10^8 \, \rm yr$ to reach the central region of the cluster. In the absence of a drag force, the subhalo will reach a velocity of around $1900 \, \rm km \, s^{-1}$ at a distance of $15 \, \rm kpc$, which is similar to the velocity assumed above.

Our results for $\Delta = r_{\rm sh} - r_{\rm star}$ are shown in Fig. 1 as a function of $r_{\rm sh}$ taking $\tilde{\sigma}/m_{\rm DM} = 2$ and $3~{\rm cm^2g^{-1}}$. The observed separation at $r_{\rm sh} = 15~{\rm kpc}$ is found to be $\Delta = 0.8$ and 1.2 kpc, respectively, showing that our simple estimate above was quite accurate. It is worth emphasizing that the separation only becomes large once the subhalo comes close to the central region of the cluster.

2.2 Three-dimensional simulations

So far we have been using a simple model of the infalling subhalo with a constant density core which leads to a comparably shallow

² We note that such a cross section is very difficult to motivate from the particle physics picture. This issue will be discussed in more detail below.

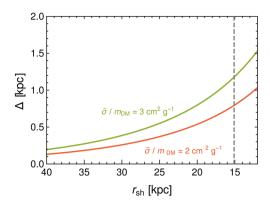


Figure 1. The separation between the DM subhalo and the stars as a function of the subhalo distance from the centre based on a simple one-dimensional simulation. The vertical line indicates the currently observed position of the subhalo.

potential and a relatively small binding energy. For a more realistic halo profile, stars in the central region will be much more tightly bound and will therefore always remain very close to the peak of the DM mass distribution. Indeed, the drag force on the DM subhalo can be interpreted as a tilt of the potential, so that loosely bound stars are typically more affected and will preferentially travel ahead of the DM subhalo (or even escape from the potential) and also the minimum of the potential will be slightly shifted. Both effects lead to an apparent separation between the distribution of stars and the DM subhalo.

Clearly, the motion of the stars in the combined potential of the subhalo and the cluster cannot be modelled in a one-dimensional simulation. In Ka14, a three-dimensional simulation was developed to address this problem in the context of major mergers like the Bullet Cluster. Our approach was to treat the gravitational potential of the cluster as time-independent, while for the subhalo the central density and scale radius are allowed to vary with time and determined self-consistently from the simulation. Assuming an initial density profile, the simulation chooses a representative set of DM particles and stars bound to the subhalo and then calculates the motion of all these particles in the combined gravitational potential of the cluster and the subhalo. It is then straightforward to add in an additional drag force affecting only the DM particles, based on the velocity of these particles and the background DM density.

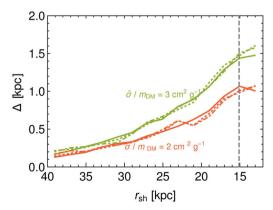
We use a Hernquist (1990) profile to model both the cluster and the subhalo (see Appendix A), the advantage being that it has a finite central potential and the velocity-distribution function can be described analytically. Since our results are quite independent of the DM density at large radii, we expect very similar results for the alternative Navarro–Frenk–White profile. We use the same initial conditions for the position and velocity of the subhalo as in the one-dimensional simulation.

A significant complication of the three-dimensional simulation is the need to determine the centroid of the DM subhalo and the distribution of stars. As discussed in Ka14, it is inconsistent to just calculate these centroids including all particles which were initially bound to the subhalo, because doing so would include particles that have escaped from the DM subhalo and are now far away from the peak of the distribution. A realistic estimate of the separation can be obtained by including only particles within the projected isodensity contour containing 20 per cent of the total mass of the original DM subhalo and ignoring regions where the surface density of DM particles originating from the subhalo is smaller than the background surface density of the cluster. This procedure corresponds roughly to including particles within 4 kpc of the peak of the distribution. The same procedure is applied to determine the centroid of the distribution of stars.

To illustrate how strongly our results depend on the definition of the centroid, we also show the predicted separation for different centroid definitions. In particular, we consider the case where only the innermost 5 per cent of the total mass are included in the calculation of the centroid (such that the tails of the distribution are mostly ignored and the position of the centroid is determined largely by the position of the peak). While this definition may be realistic for the case of the stars, where the peak of the distribution can be accurately determined, it may not be appropriate for the determination of the centroid of the DM subhalo. We therefore also consider an alternative definition, where the inner 20 per cent of the DM subhalo but only the inner 5 per cent of the stars are included in the calculation of the centroids.

Including the tails of the DM distribution has a further important advantage: we will see below that these tails can contain relevant information on the nature of DM self-interactions and should therefore not be neglected.

Our results for the separation Δ are shown in Fig. 2 as a function of $r_{\rm sh}$ for $\tilde{\sigma}/m_{\rm DM}=2$ and $3\,{\rm cm^2\,g^{-1}}$. The observed separation at



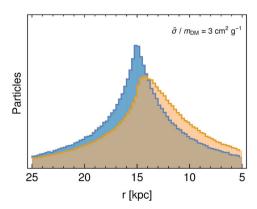
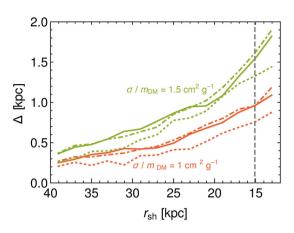


Figure 2. Left: predicted separation between the centroid of the DM subhalo and the centroid of the stars resulting from an effective drag force on the DM subhalo as a function of the subhalo position $r_{\rm sh}$. The solid, dashed and dot-dashed lines correspond to centroid calculations using the inner (20 per cent, 20 per cent), (5 per cent, 5 per cent) and (20 per cent, 5 per cent) of the DM subhalo and the distribution of stars, respectively (see text for details). Right: histogram of the distribution of DM particles (blue) and stars (orange) as a function of radial distance r from the centre of the cluster at the moment in time where the radial position of the peak of the subhalo is $r_{\rm sh} = 15$ kpc (indicated by the vertical dashed line in the left-hand panel).



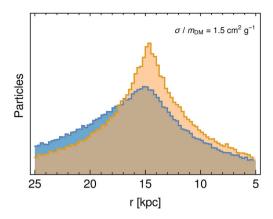


Figure 3. Same as Fig. 2 but for the case of rare self-interactions. In this case, the DM subhalo develops a tail in the backward direction, while the distribution of stars remains largely unchanged. The peaks of the two distributions remain largely coincident and therefore the separation depends sensitively on whether the tails of the distributions are included in the centroid calculation.

 $r_{\rm sh}=15$ kpc is found to be $\Delta\sim1.0$ and ~1.5 kpc, in good agreement with our previous estimate. We also note that the predicted separation varies by only about 5 per cent as we consider different definitions of the centroids. This indicates that not only the tails but also the position of the peaks differ for the two distributions.

To confirm this expectation, we show in the right-hand panel of Fig. 2 the distribution of the stars and the DM particles along the radial direction. Indeed, one can see that the peaks of the two distributions are slightly shifted. Furthermore the tail of the distribution of stars is enhanced in the forward direction due to stars that have escaped from the gravitational potential of the subhalo. Most of the remaining stars, however, remain bound to the subhalo and would return to their equilibrium position if the subhalo survived the passage through the central region of the cluster.

2.3 Alternative interpretations

We briefly discuss alternative particle physics models for DM self-interactions and their interpretation in the present context. The two most natural forms of DM self-interactions are long-range interactions, which can arise for example from a massless mediator (Feng et al. 2009), and contact interactions. Long-range interactions typically lead to a drag force of the form

$$\frac{F_{\text{drag}}}{m_{\text{DM}}} = \frac{1}{4} \frac{\tilde{\sigma}}{m_{\text{DM}}} \frac{c^4}{v^2} \rho \tag{8}$$

with $\tilde{\sigma}/m_{\rm DM} < 10^{-11}\,{\rm cm^2\,g^{-1}}$ from the observation that Milky Way satellites have survived up to the present day (Gnedin & Ostriker 2001). Using an estimate similar to the one discussed above, one can immediately see that for the DM densities and the velocities under consideration such a drag force cannot give an observable effect. Moreover, the inverse velocity dependence implies that DM self-interactions would actually be suppressed as the subhalo approaches the central region of the cluster, in contrast to what is required in order to obtain large effects from DM self-interactions. However a massless mediator can induce collisionless shocks, which Heikinheimo et al. (2015) argue can potentially explain the features observed in A3827.

For contact interactions, an effective description in terms of a drag force is not possible, because each individual DM particle will only experience a small number of collisions (if any). The reason is that in each collision the momentum transfer is so large that these collisions must be rare to avoid observational bounds from

the survival of subhaloes and halo ellipticity (Peter et al. 2012; Rocha et al. 2013). Nevertheless, a separation between the DM subhalo and stars can in principle also occur in the case of contact interactions. In this case, the separation is not due to stars leaving the DM subhalo in the forward direction, but due to DM particles receiving large momentum transfer and leaving the DM subhalo in the *backward direction*. In other words, rare DM self-interactions can affect a tail of DM particles, which shifts the centroid of the DM distribution relative to the distribution of stars, while the peak of the DM distribution, which is dominated by DM particles that have not experienced any self-interactions, remains coincident with the peak of the distribution of stars.

This scenario can also be investigated using the simulations discussed above (for details, see Ka14). Our results are shown in Fig. 3 for $\sigma/m_{\rm DM}=1.0$ and $1.5\,{\rm cm^2\,g^{-1}}$. For larger self-interaction cross-sections, the evaporation rate of the DM subhalo becomes so large that it is unlikely to have survived up to its present position. Including the inner 20 per cent of the DM subhalo and the stars in the centroid calculation, the observed separations at $r_{\rm sh}=15\,{\rm kpc}$ are found to be $\Delta=1.0$ and $1.6\,{\rm kpc}$, respectively. As in the case of a drag force, there is tension between the observation of a separation in A3827 and other constraints on the DM self-interaction cross-section, which are typically $\sigma_{\rm DM}/m_{\rm DM}\lesssim 1\,{\rm cm^2\,g^{-1}}$ (Markevitch et al. 2004; Randall et al. 2008; Peter et al. 2012; Rocha et al. 2013; Harvey et al. 2015).

In contrast to the case of an effective drag force we find that the predicted separation depends sensitively on the definition of the centroids. Including only the innermost 5 per cent of the subhalo and the stars (in order to determine the approximate position of the respective peaks) yields a somewhat smaller predicted separation of 0.8 and 1.3 kpc. This observation indicates that the separation is mainly due to differences in the shapes of the two respective distributions, while the peaks of the distributions remain coincident. This expectation is confirmed by the histograms shown in the right-hand panel of Fig. 3.

An important conclusion is that the case of contact interactions can potentially be distinguished from the case of an effective drag force by studying in detail the shape of the DM subhalo and the relative position of the peaks of the two distributions. In the case of contact interactions, the DM subhalo is expected to be deformed due to the scattered DM particles leaving the subhalo in the backward direction, such that the position of the centroid depends sensitively on which particles are included in the calculation. For an effective

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drag force, on the other hand, we expect the DM subhalo to retain its shape, while the distribution of stars will be both shifted and deformed.

3 DISCUSSION

In this Letter we have discussed a possible interpretation in terms of DM self-interactions of an observed separation of about 1.6 kpc between the DM halo of a galaxy and its stars. Using several increasingly refined methods we estimate that the self-interaction cross-sections necessary to explain this effect are of the order of $\tilde{\sigma}/m_{\rm DM}\sim 3\,{\rm cm^2~g^{-1}}$ for the case of an effective drag force (proportional to the square of the velocity) or $\sigma/m_{\rm DM}\sim 1.5\,{\rm cm^2~g^{-1}}$ for the case of contact interactions. Both of these values are in tension with the upper bounds on DM self-interactions from other astrophysical observations.

We have modelled the system under consideration rather simply, but our derived estimates are conservative since a more refined analysis (e.g. considering other possible trajectories for the subhalo) would predict a *higher* self-interaction cross-section for the same observed separation. It should therefore be clear that A3827 is no more sensitive to DM self-interactions than other systems considered in this context and can certainly not be used to probe cross-sections as small as $\sigma_{\rm DM}/m_{\rm DM}\sim 10^{-4}~{\rm cm^2~g^{-1}}$. Further studies of such systems are imperative to establish if the indication from A3827 for a non-zero self-interaction cross-section (of the order of 1 cm² g⁻¹) is indeed correct.

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APPENDIX A: A CRUDE MODEL OF A3827

For simplicity, we model both the central region of cluster A3827 and the galactic subhalo called N1 by Williams & Saha (2011) and Massey et al. (2015) using a Hernquist (1990) profile:

$$\rho(r) = \frac{M}{2\pi} \frac{a}{r(a+r)^3} \ . \tag{A1}$$

For the cluster, we determine $M_{\rm cluster}$ and $a_{\rm cluster}$ by fitting the mass distribution to the one given in fig. B2 of Massey et al. (2015). This procedure leads to $M_{\rm cluster} = 7 \times 10^{13}\,{\rm M}_{\odot}$ and $a_{\rm cluster} = 60\,{\rm kpc}$. As suggested in Massey et al. (2015) we assume a constant-density core with radius 8 kpc. We have checked that this model reproduces the observed projected mass in the central region and the observed velocity dispersion with reasonable accuracy.

For the subhalo, we find that taking $M_{\rm sh} = 5 \times 10^{11} \, \rm M_{\odot}$ and $a_{\rm sh} = 7 \, \rm kpc$ enables us to reproduce satisfactorily the observed velocity dispersion and the projected mass in the central region of the subhalo.

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