Internal Report

DESY M-80/14

August 1980

CHROMATICALLY CORRECTED MINI BETA INTERACTION REGION WITH
LONGITUDINAL ELECTRON SPIN

K. Steffen

CHROMATICALLY CORRECTED MINI BETA INTERACTION REGION WITH LONGITUDINAL ELECTRON SPIN

K. Steffen

Deutsches Elektronen-Synchrotron DESY, Hamburg, W.Germany.

ABSTRACT

In colliding beam facilities, the small beta functions required for high luminosity are often limited by the chromaticity they induce in the adjacent quadrupoles. A scheme that locally corrects this chromaticity and thus potentially permits to further decrease the cross section of the colliding beams was first designed by K. Brown and D. Ritson. This note comprises suggestions for simplifying and extending the original scheme and illustrates them by linear design examples. In the electron design example, the bending magnets required for chromaticity compensation are simultaneously used for generating a longitudinal electron spin direction at the interaction point. In the second example, suitable for protons, the bending magnets are in the collision region and may be used also as particle analysers.

INTRODUCTION

In an electron storage ring with current i and the beam emittance adjusted to give a beam-beam tune shift ΔQ , the luminosity scales as¹⁾

$$L = \text{const.} \cdot \gamma \Delta Q \frac{i \ell}{\epsilon}$$

where ℓ is the length of the interaction region, γ the Lorentz factor and ϵ the chromaticity of the insertion quadrupoles. This chromaticity may possibly be carried to higher values when carefully corrected in situ, resulting in a higher luminosity.

In a single pass collider, the luminosity also increases with chromaticity up to the point where the chromatic broadening of the beam size at the interaction point becomes equal to the monoenergetic beam size²⁾. For higher luminosities, a local chromaticity correction is again required.

PRINCIPLES

K. Brown and D. Ritson³⁾ have proposed a special corrective insertion that provides a local compensation of the chromaticity caused by the high beta quadrupoles. Their original proposal for an achromatic low beta interaction geometry contains two of these corrective insertions, one before and one after the interaction region; they each essentially consist of two quadrupole doublets which, in a symmetric optical arrangement, produce a beam waist between them. At this waist, a bending magnet produces a dispersion that is also symmetric with respect to this point. Chromaticity compensation, then, is provided by two symmetric pairs of sextupoles that have one of their members placed near each of the quadrupole doublets. Since the betatron phase advance between the sextupoles of each pair is thus very close to 180° , the geometric aberrations caused by one member

of the pair are compensated by the other member.

In order to somewhat simplify this scheme and reduce the large straight section length required, the author suggested the modification

- (1) to incorporate each of the interaction region quadrupole doublets into one of the two corrective insertions and use them as part of their symmetric focusing arrangement.

This feature was adopted in the SPLC proposal²⁾ which, however, has 4 additional "field lenses" in each corrective insertion to permit an exact phase advance of 180° between pairs of sextupoles. This point will be discussed in section 4; it is ignored in the design examples of section 3 which intend to illustrate principles rather than give solutions.

For proton colliders, in the absence of synchrotron radiation, bending magnets in the immediate neighbourhood of the interaction point may be tolerable or even desirable as particle analysers. This possibility further simplifies the correction scheme:

- (2) With bending magnets in the collision region, only one of the above corrective insertions is needed. It is symmetric about the interaction point and consists of the adjacent bending magnets which produce the symmetric dispersion and of the quadrupole doublets (or multiplets) which produce the low beta functions.

In electron colliders, vertical bending may be employed for rotating the vertical electron spin into the forward direction. This suggests the following feature:

- (3) The vertical bending magnet which, as part of the corrective insertion, produces the symmetric dispersion is used, at the same time, to rotate the electron spin into the longitudinal direction.

The features (1), (2), (3) may help to design chromatically corrected interaction regions that are relatively short and simple. (1) and (2) will be illustrated in the first linear design example, and (3) in the second.

LINEAR DESIGN EXAMPLES

The interaction scheme for electrons, with longitudinal spin at the I.P., is given in Fig. 1, showing lattice, beam envelopes and vertical dispersion for one half of the system. The bending radius is ≥ 450 m and the beam has a vertical slope of 23 mrad at the I.P. which corresponds to 90° spin rotation at 30 GeV. With beta functions of $\beta_x^* = 15$ cm and $\beta_z^* = 1$ cm at the I.P., the maximum beta functions are about 5000 m in both directions. The dispersion is matched to vanish outside the system; its maximum value is 0.6 m. The sextupole pairs are indicated as S_x , S_z . With a beam emittance of $\epsilon = 1 \times 10^{-9}$ m and a relative beam energy spread of 0.5 %, the dispersed beam size is about equal to the geometric beam size at the sextupole position, which is a requirement for optimum correction^{2,3)}.

Fig. 1: Chromatically corrected interaction scheme for electrons

The interaction scheme for protons, with bending magnets around the I.P., is shown in Fig. 2. The bending radius of 25 m corresponds to a magnetic field of 4 T at 30 GeV. Since proton beams tend to be "round", the scheme employs quadrupole quadruplets instead of the usual doublets for focusing. Then, a beta ratio near to one can be chosen without obtaining an excessive horizontal maximum. With beta functions of $\beta_x^* = 10\text{cm}$ and $\beta_z^* = 5\text{cm}$, the maximum values are about 5000 m again. The beam deflections in the magnets give a matched vertical beam bump, and the dispersion, reaching a maximum value of 6.3 m, is matched to vanish outside the system.

Fig. 2: Chromatically corrected interaction scheme for protons

The lattice parameters for both examples are given in Tab. 1.

Table 1

Lattice parameters of chromatically corrected interaction schemes

Electron system:

drift length [m]	5.8	2.5	1.1	0.3	0.3	1.6	4.5	0.5	7.5	7.1
magnet type	Q1	Q2	Q1	Q2	$\frac{1}{2}Q2$	Q3	Q4	Q5	Q6	Q3
magnet length [m]	3	3	5.2	5.2	2.6	1.5	5	3	1.5	5.2
strength (m^{-1}) or (m^{-2})	$-.02560$	$-.07166$	$-.25 \times 10^{-5}$	2.05×10^{-5}	2.05×10^{-5}	$-.02687$	$-.02835$	$-.02451$	$-.00993$	$-.21 \times 10^{-5}$

Proton system:

drift length [m]	1.3	0.3	1.1	0.3	1.1	0.3	4	6.5	8.3
magnet type	B	B	Q1	Q2	-B	Q3	Q4	Q5	-B
magnet length [m]	5.2	5.2	3	3	5.2	3	3	3	5.2
strength (m^{-1}) or (m^{-2})	.04	.04	$-.06055$	$-.06625$	$-.04$	$-.04519$	$-.03192$	$-.07015$	$-.04$

DESIGN ASPECTS FOR CORRECTIVE INSERTION

In order to assess some properties of the corrective insertion, consider the symmetric thin lens example shown in Fig. 3.

Fig. 3: Symmetric thin lens example of corrective insertion

The trajectory transformation from point B to point 0 is given by the matrices

$$M_x = \begin{pmatrix} a & Lb \\ -\frac{1}{L}c & d \end{pmatrix} ; \quad M_z = \begin{pmatrix} a & Lc \\ -\frac{1}{L}b & d \end{pmatrix}$$

$$\left. \begin{array}{l} \text{with } a = 0 ; \quad b = 1 + 2y + 2\sqrt{y(1+y)} \\ c = 1 + 2y - 2\sqrt{y(1+y)} = \frac{1}{b} ; \quad d = 1 + 2\sqrt{y(1+y)} \end{array} \right\} \quad y = \frac{v}{L}$$

The lens strength is

$$\frac{1}{f} = \frac{1}{v} \sqrt{y(1+y)} = \frac{d-1}{2v}$$

Assuming the same maximum amplitude function B in x and z , we have at the interaction point 0

$$B_x^* = \frac{b}{c} \frac{L^2}{v} ; \quad B_z^* = \frac{L^2}{v} ; \quad \sqrt{B_x^* B_z^*} = \frac{bL^2}{v}$$

In a single pass collider, the chromatic increase in spot size for a relative

momentum deviation δ is

$$2\delta\sqrt{\epsilon\beta} \quad \text{in } x \quad \text{and} \quad 2\delta c\sqrt{\epsilon\beta} \quad \text{in } z.$$

For given δ , the sum of geometrical spot size $\sqrt{\epsilon\beta}$ and chromatic increase assumes a minimum when both contributions are equal, i.e. for

$$\beta = \frac{bL}{2\delta}. \quad \text{Setting } \delta_0 = \frac{bL}{2\beta},$$

no chromatic correction is needed for $\delta < \delta_0$.

For chromatic correction, the sextupoles require an integrated strength s of approximately

$$s = \frac{1}{Df}$$

where D is the dispersion at the sextupole. Due to the finite beam size $\sqrt{\epsilon\beta}$, the particles at the edge of the beam obtain by the sextupole field a faulty kick

$$\alpha = \frac{s}{2} \epsilon\beta = \frac{\epsilon\beta}{2Df}$$

If this kick is smaller than the beam divergence, i.e. for

$$\alpha < \epsilon' = \sqrt{\frac{\epsilon}{\beta}} \quad \text{or} \quad \epsilon < \epsilon_0 = 4 \frac{D^2 f^2}{\beta^3},$$

no correction of geometric aberration is needed.

If the corrective insertion were ideal, with a linear transformation of exactly 180° between sextupoles, the trajectory would reach the second sextupole with exactly the same, but negative amplitude independent of α , and its geometric aberration would be exactly compensated. Our example is not ideal since the transformation in x between points B' and B is

$$\begin{pmatrix} -1 & 2L \\ 0 & -1 \end{pmatrix}$$

and in z between points A' and A

$$\begin{pmatrix} d & Lb \\ -\frac{1}{L}c & a \end{pmatrix} \begin{pmatrix} a & Lb \\ -\frac{1}{L}c & d \end{pmatrix} = \begin{pmatrix} -1 & 2Lbd \\ 0 & -1 \end{pmatrix}$$

The off-axis matrix element in z can be made to vanish by inserting, at the symmetry point O' , a "field lens" of strength $1/f_0 = 2d/Lb$ which, at the same time, increases the off-axis element in x to

$$2L \left(1 + \frac{d}{b}\right)$$

With this field lens, then, the trajectory arrives at B with a relative offset

$$\frac{\Delta x}{x} = \frac{\alpha \cdot 2L \left(1 + \frac{d}{b}\right)}{\sqrt{\epsilon\beta}} = \sqrt{\epsilon\beta} \frac{L}{Df} \left(1 + \frac{d}{b}\right)$$

In cases where the product of beam emittance and maximum amplitude function is small enough to keep $\Delta x/x$ in the range of a few percent, one should expect that the compensation of geometric aberration will be sufficient with a single field lens only.

ACKNOWLEDEMENT

I wish to thank D. Ritson for explaining to me at the Les Diablerets workshop the principles of the achromatic interaction scheme that he and K. Brown were designing for the single pass collider.

* * *

REFERENCES

- 1) K. Steffen, Cornell Internal Report CBN 80-10, May 1980
- 2) Single Pass Collider Conceptual Design Report, SLAC, June 1980
- 3) K. Brown and D. Ritson, priv. comm., Oct. 1979