

Three-Point Functions of Short Operators

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Introduction

- Due to AdS/CFT, there has been a huge development in the study of strongly coupled conformal field theories. The best understood example is $\mathcal{N} = 4$ SYM, which is dual to type IIB string theory on $\text{AdS}_5 \times S^5$.
- Using the TBA, Y-system, or the more recent Quantum Spectral Curve, **dimensions** of operators can in principle be computed at any value of the coupling, at least numerically.
- The obvious next step is to investigate the **structure constants** of the theory, for which there is some limited knowledge at both weak and strong coupling.
- In this work, we focus on the strong coupling limit of three-point functions of operators dual to **short strings** (small compared to the curvature scale).

The Operators

- We consider operators with dimensions $\sqrt{\lambda} \gg \Delta \gg 1$, such that the dual string states can be treated as semiclassical **point particles**. (String length $\sim \Delta\alpha' = \Delta/\sqrt{\lambda}$ by AdS/CFT.)
 - We specialize to scalar operators dual to **massless** and **first massive level** string states. In the conformal field theory, these correspond to:
 - Chiral primaries with no anomalous dimension and R -charge J which equals their bare dimension:

$$\mathcal{O}_{\text{CP}} = C_{I_1 \dots I_J} \text{Tr}[\Phi^{I_1} \dots \Phi^{I_J}].$$
 - Operators with R -charge J and dimension $\Delta \sim \lambda^{1/4}$ at strong coupling, e.g. $\mathcal{O}_J = \text{Tr}[\Phi^J Z^J] + \dots$
- $\mathcal{O}_0$ is the **Konishi operator**.

Semiclassical Limit

- Given the boundary to bulk propagator

$$K = \frac{\Gamma(\Delta)}{\pi^2 \Gamma(\Delta - 2)} \left(\frac{z_0}{z_0^2 + (\vec{z} - \vec{x})^2} \right)^\Delta,$$
 the three-point function of scalar operators has been computed, up to the supergravity **couplings**.
- To obtain these couplings, we consider the path integral for the propagation in AdS of particles dual to scalars with dimensions Δ_i .
- Taking a semiclassical limit (saddle point) we notice two important facts:
 1. All three propagators as well as the intersection point are completely localized, and the propagator's **canonical momenta are conserved** at the intersection point.
 2. The size of the overlaps of trajectories in AdS is much smaller than the radius of AdS for operators with dimensions $\sqrt{\lambda} \gg \Delta_i \gg 1$.

Flat Space Approximation

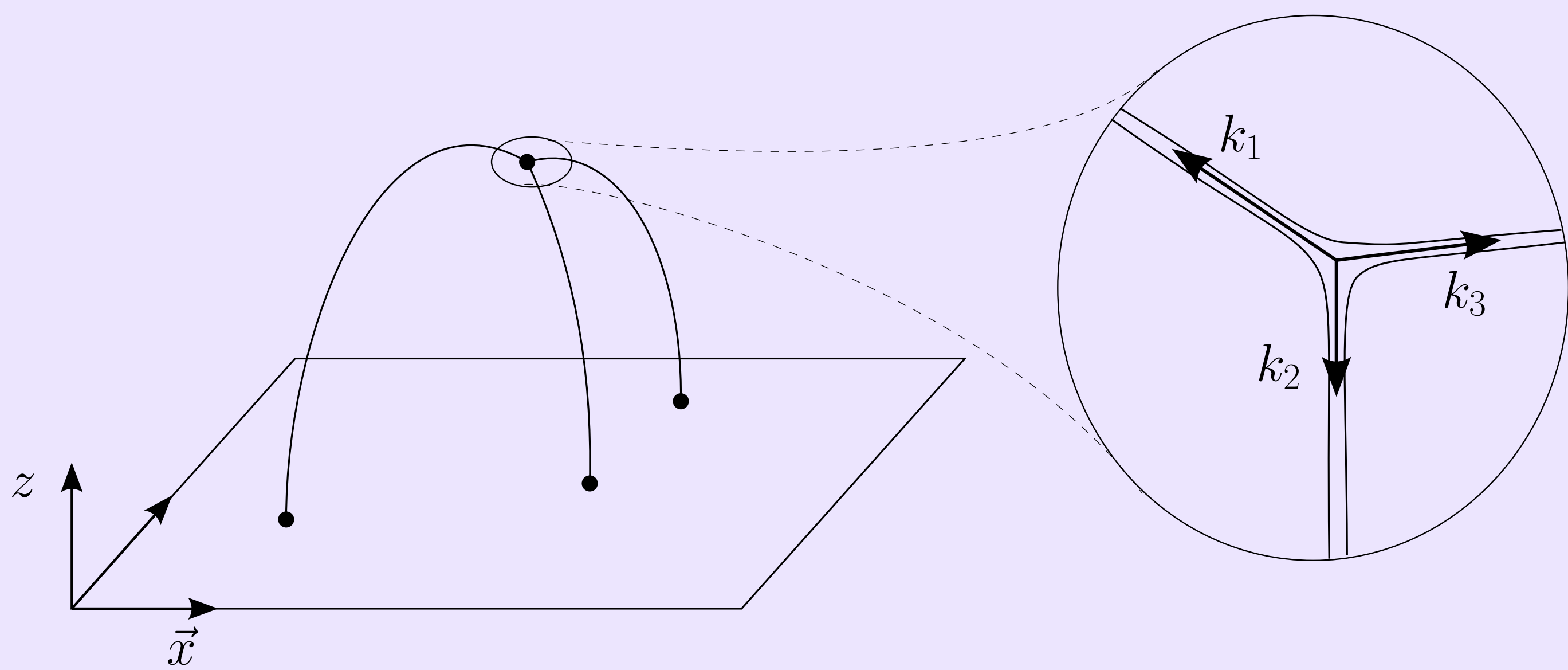


Figure 1: Conservation of momenta at the intersection point.

- Due to the **small overlap** for operators with dimensions $\sqrt{\lambda} \gg \Delta_i \gg 1$, one can ignore curvature and RR-flux effects at the intersection point.
- It is thus possible to use standard **flat-space techniques**, so that obtaining the coupling reduces to the simpler problem of computing string amplitudes in flat space using RNS vertex operators.

Vertex Operators

- One constructs a physical vertex operator by imposing $[Q_{\text{BRST}}, V_{\text{phys}}] = 0$. The momentum of the vertex operator is given by the canonical momentum at the intersection point. For the n 'th mass level, it obeys

$$k^2 \approx -\Delta^2 + J^2 = \frac{4}{\alpha'} n.$$

- At the massless level the NS and R sector left movers are:

$$g_c \varepsilon_M \psi^M e^{-\phi} e^{ik \cdot X}, \quad g_c t_A \Theta^A e^{-\frac{1}{2}\phi} e^{ik \cdot X}.$$

- For the first massive level there are two NS vertices

$$g_c \alpha_{MNP} \psi^M \psi^N \psi^P e^{-\phi} e^{ik \cdot X}, \quad g_c \varepsilon_{MN} \psi^M \partial X^N e^{-\phi} e^{ik \cdot X},$$

and one in the R sector

$$g_c t_{A,M} (i \partial X^M \Theta - \frac{1}{8} \psi^M (\not{k} \psi \Theta)) e^{-\frac{1}{2}\phi} e^{ik \cdot X}.$$

CFT Operators in the String Formalism

- Taking the flat-space limit, the $PSU(2, 2|4)$ algebra reduces to a ten dimensional super-Poincaré. The condition for primary operator $S_\alpha^a \mathcal{O}(0) = 0$ then becomes an algebraic condition on the vertex operators:

$$Q_{\alpha\bar{a}}^L V = i Q_{\alpha\bar{a}}^R V, \quad Q_{\dot{\alpha}}^{L\bar{a}} V = -i Q_{\dot{\alpha}}^{R\bar{a}} V,$$
 where $(\alpha, \dot{\alpha})$ defines the space orthogonal to the momentum in AdS.
- The equation implies that primary operators are linear combinations of NS-NS and R-R vertices.
- For the massless and first massive level, the solution is unique.
- The dilaton is given by the massless NS-NS scalar vertex operator.

Results

- We test our method by computing the three-point function for three chiral primaries, obtaining total agreement with previous supergravity calculation.
- For two chiral primaries with R -charge J and a Konishi-like operator we get

$$\mathcal{C}_{\text{CKK}} \approx \frac{\pi^{3/2}}{2\sqrt{2} N \Gamma(J) \Gamma(J-1)} \lambda^{J/2-1/4} 2^{-2\lambda^{1/4}}.$$

- We can also compute the structure constant of one chiral primary and two Konishi-like operators with R -charges $J_i \ll \lambda^{1/4}$, thus obtaining

$$\mathcal{C}_{\text{CKK}} \approx \frac{\pi^{1/2}}{N} \lambda^{1/4} 2^{-J}.$$

- For three Konishi-like operators: Horrific combinatorics, but **huge simplification**:

$$\langle VVV \rangle = g_c^3 \frac{3^8}{2^9}, \quad \mathcal{C}_{\text{KKK}} \approx \frac{64\pi^{1/2}}{N} \lambda^{1/4} \left(\frac{3}{4} \right)^{3\lambda^{1/4}+5/2}.$$

- In all cases, the structure constant is exponentially suppressed in λ (or J).

Conclusion

- We successfully computed structure constants for scalar operators dual to short string states. Our method works for massless states with R -charge $J \gg 1$, and for any operator at the first massive level.
- The procedure is in principle applicable to any CFT with a string dual, up to the construction of the right vertex operators.
- Big simplifications suggest that **integrability** plays a role.
- Method perhaps can help lead us to exact vertex operators in $\text{AdS}_5 \times S^5$.
- For future work it would be interesting to compute three-point functions of operators with spin, and to investigate four-point functions, as well as subleading corrections in the α' expansion.

References

- [1] T. Bargheer, J. A. Minahan and R. Pereira, "Computing Three-Point Functions for Short Operators", JHEP 1403, 096 (2014), arxiv:1311.7461.