Integrable Deformations for $\mathcal{N} = 4$ SYM and ABJM Amplitudes

Till Bargheer
IAS Princeton

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Based on 1407.4449 with
Yu-tin Huang, Florian Loebbert, Masahito Yamazaki

Graßmannian Geometry of Scattering Amplitudes
Walter Burke Institute Workshop, California Institute of Technology

Motivation

On-shell integrand for planar $\mathcal{N} = 4$ and ABJM well understood

How to integrate?

Regularization breaks conformal symmetry

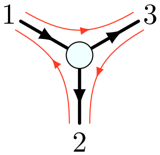
But even for finite ratio function: No practical way to integrate

$\Rightarrow$ Try to deform integrand, preserving as much symmetry as possible

Deformed On-Shell Diagrams in $\mathcal{N} = 4$ SYM (I)

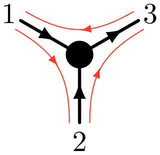
Deformed three-point amplitudes:

[Ferro, Łukowski, Meneghelli Plefka, Staudacher, 2012]



$$= \int \frac{d\alpha_2}{\alpha_2^{1+a_2}} \frac{d\alpha_3}{\alpha_3^{1+a_3}} \, \delta^{4|4}(C_\circ \cdot \mathcal{W}) \simeq \frac{\delta^4(P)\,\delta^4(\tilde{Q})}{[12]^{1+a_3}[23]^{1-a_1}[31]^{1+a_2}}$$

$$c_1 = a_1 \equiv a_2 + a_3, \quad c_2 = -a_2, \quad c_3 = -a_3$$



$$= \int \frac{d\alpha_1}{\alpha_1^{1+a_1}} \frac{d\alpha_2}{\alpha_2^{1+a_2}} \, \delta^{8|8}(C_\bullet \cdot \mathcal{W}) \simeq \frac{\delta^4(P)\,\delta^8(Q)}{\langle 12\rangle^{1-a_3}\langle 23\rangle^{1+a_1}\langle 31\rangle^{1+a_2}}$$

$$c_1 = a_1, \quad c_2 = a_2, \quad c_3 = -a_3 \equiv -a_1 - a_2$$

Yangian: $\mathfrak{J}^a \in \mathfrak{psu}(2,2|4)\,, \quad \widehat{\mathfrak{J}}^a = f^a{}_{bc} \sum_{1\le i<j\le n} \mathfrak{J}_i^b \, \mathfrak{J}_j^c + \sum_{i=1}^{n} u_i \, \mathfrak{J}_i^a$

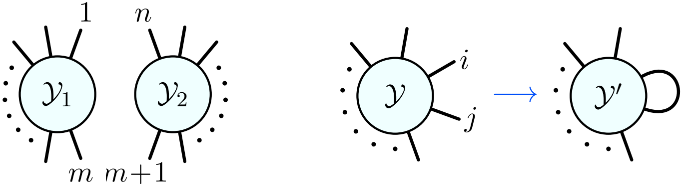
Invariance conditions: $u_j^+ = u_{\sigma(j)}^-, \quad u_j^\pm = u_j \pm c_j$

[Beisert, Broedel Rosso, 2014]

Deformed helicities: $\mathfrak{h}_j = 1 - \mathfrak{C}_j, \quad h_j = 1 - c_j$

Deformed On-Shell Diagrams in $\mathcal{N} = 4$ SYM (II)

Bigger on-shell diagrams can be obained by gluing: Products & Fusion



Reduced diagram $\simeq$ permutation σ
Invariance conditions from gluing: $u_j^+ = u_{\sigma(j)}^-$ $\quad u_j^\pm = u_j \pm c_j$

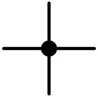
General deformed on-shell diagram: [Beisert, Broedel Rosso, 2014]

$$\mathcal{Y}(\mathcal{W}_i, a_i) = \int \prod_{j=1}^{n_{\mathrm{F}}-1} \frac{\mathrm{d}\alpha_j}{\alpha_j^{1+a_j}} \, \delta^{4k|4k}(C \cdot \mathcal{W}) \, ,$$

Edge deformation parameters α_i equal central charges on internal lines, fully determined by external u_i, c_i via left-right paths

Deformations in ABJM: Four-Vertex

The Basic diagram in ABJM is the four-vertex:

$\mathcal{A}_4 =$  $= \dfrac{\delta^3(P)\,\delta^6(Q)}{\langle 12\rangle\,\langle 23\rangle}$

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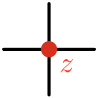
$$\widehat{\mathcal{A}}_4(z) =$$ 

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[TB, Huang, Loebbert Yamazaki, 2014]

Admits a deformation, parameter z

Can show invariance under level-zero $\mathfrak{osp}(6|4)$

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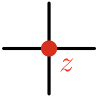

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Level-one generators: $$\widehat{\mathfrak{J}}^a = f^a{}_{bc} \sum_{1\le i<j\le n} \mathfrak{J}_i^b\,\mathfrak{J}_j^c + \sum_{i=1}^{n} u_i\,\mathfrak{J}_i^a$$

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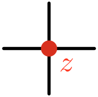
$$\widehat{\mathcal{A}}_4(z) = $$ 

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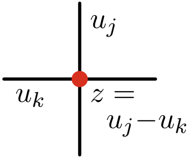
$$\widehat{\mathfrak{P}}^{\alpha\beta} = \sum_{1\leq j<k\leq n} \tfrac{1}{2}\Big[\big(\mathfrak{L}_{j\gamma}^{(\alpha} + \delta_\gamma^{(\alpha}\mathfrak{D}_j\big)\mathfrak{P}_k^{\gamma\beta)} - \mathfrak{Q}_j^{(\alpha A}\mathfrak{Q}_k^{\beta)}{}_A - (j\leftrightarrow k)\Big] + \sum_k u_k \mathfrak{P}_k^{\alpha\beta}$$

Invariance under $\widehat{\mathfrak{P}}$ implies full $\mathrm{Y}(\mathfrak{osp}(6|4))$ invariance.

Constraints:

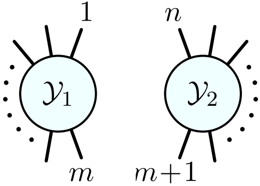
$$u_1 = u_3\,, \qquad u_2 = u_4\,, \qquad z = u_1 - u_2\,.$$



Deformations in ABJM: Gluing

Construct bigger deformed diagrams from four-vertex

Products: Invariance trivial, no constraints



$$\mathcal{Y}(1\dots n) = \mathcal{Y}_1(1\dots m)\,\mathcal{Y}_2(m+1\dots n)$$

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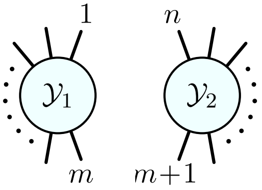

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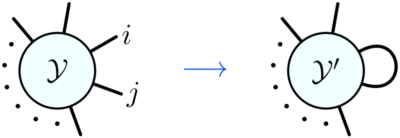
Fusing: $\mathcal{Y}'(\ldots) = \int \mathrm{d}^{2|3}\Lambda_i\,\mathrm{d}^{2|3}\Lambda_j\,\delta^{2|3}(\Lambda_i - i\Lambda_j)\,\mathcal{Y}(\ldots,i,j,\ldots)$



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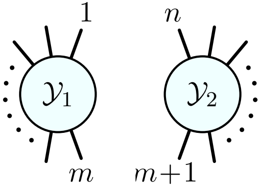

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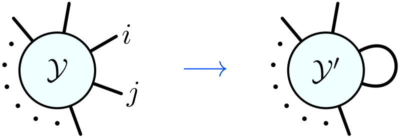


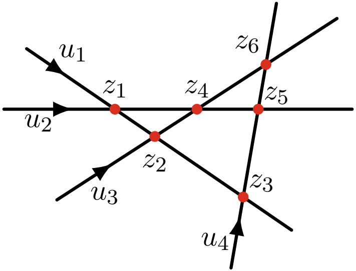
Constraint: $u_i = u_j$

Can construct all deformed diagrams by iterated product and fusion

Deformations in ABJM: General Diagrams

Fundamental vertex is four-valent

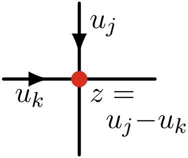
$\Rightarrow$ General $2k$-point diagram can be drawn with k straight lines



Characterized by order-two permutation σ, $\sigma^2 = 1$

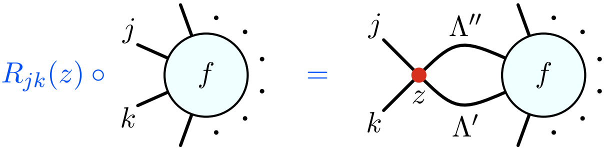
One evaluation parameter on each line, $u_j = u_{\sigma(j)}$

Vertex parameters z_i completely fixed by constraint



Deformations in ABJM: R-Matrix Formalism (I)

Reformulate invariants in terms of R-matrix construction.



$$(R_{jk}(z) \circ f)(\Lambda_j, \Lambda_k) \equiv \int \mathrm{d}\Lambda' \, \mathrm{d}\Lambda'' \, \mathcal{A}_4(z)(\Lambda_j, \Lambda_k, i\Lambda', i\Lambda'') \, f(\Lambda'', \Lambda')$$

R-matrix kernel is four-point amplitude

Just a reformulation of gluing $\Rightarrow R_{jk}(z)$ trivially preserves invariance

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R-operator intertwines two single-particle representations

Permutes evaluation parameters:

$$\widehat{\mathfrak{J}}^a(\ldots, u_j, u_k, \ldots)\, R_{jk}(z) = R_{jk}(z)\, \widehat{\mathfrak{J}}^a(\ldots, u_k, u_j, \ldots)$$

(on space of invariants)

Deformations in ABJM: R-Matrix Formalism (II)

R-operator permutes evaluation parameters:

$$\widehat{\mathfrak{J}}^a(\ldots, u_j, u_k, \ldots)\, R_{jk}(z) = R_{jk}(z)\, \widehat{\mathfrak{J}}^a(\ldots, u_k, u_j, \ldots)$$

Invariants are chains of R-matrices acting on vacuum Ω_{2k}:

$$\mathcal{Y}_{2k} = R_{i_\ell, j_\ell}(z_\ell) \ldots R_{i_1, j_1}(z_1)\, \Omega_{2k}\,, \qquad \Omega_{2k} = \prod_{j=1}^{k} \delta^{2|3}(\Lambda_{2j-1} + i\Lambda_{2j})$$

Vacuum permutation: $\sigma = [1,2][3,4]\ldots[2k-1,2k]$

Action of R-matrices conjugate the permutation:

$$\mathcal{Y}_{2k} \to R_{ij}\mathcal{Y}_{2k} \quad \Longrightarrow \quad \sigma \to [i,j]\cdot\sigma\cdot[i,j]$$

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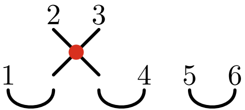

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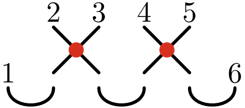

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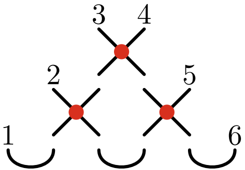

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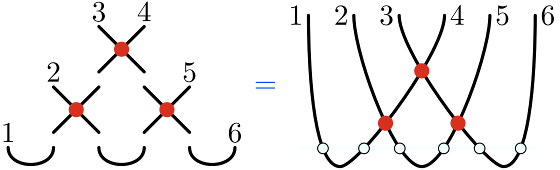

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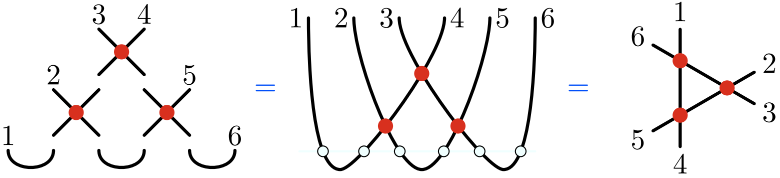

Yang–Baxter Equation

Invariants are chains of R-matrices acting on vacuum $\mathit{\Omega}_{2k}$:

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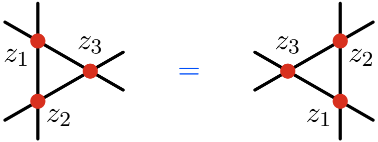
Invariance requires that parameters z_i satisfy constraints.

R-operator satisfies Yang–Baxter equation (triangle equality):



$$R_{ij}(w-v)R_{j\ell}(w-u)R_{ij}(v-u) = R_{j\ell}(v-u)R_{ij}(w-u)R_{j\ell}(w-v)$$

Deformed Amplitudes?

Tree amplitudes are linear combinations of on-shell diagrams (BCFW)

$\mathcal{N} = 4$ SYM

Each n-point diagram:

n central charges, n evaluation parameters, n constraints

Sensible Yangian representation:

Requires same parameters on every diagram

Number of BCFW terms grows factorially

$\Rightarrow$ Constraints almost always outnumber parameters [Beisert, Broedel Rosso, 2014]

Exception: n-point MHV and 6-point NMHV

Loops: Four-point integrand admits deformation $\rightarrow$ [Johannes' talk]

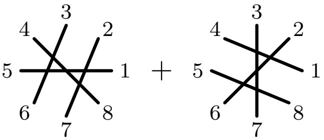
Integration non-trivial; result not easy to interpret

No admissible deformations for higher-point BCFW integrands.

Deformed Amplitudes?

ABJM

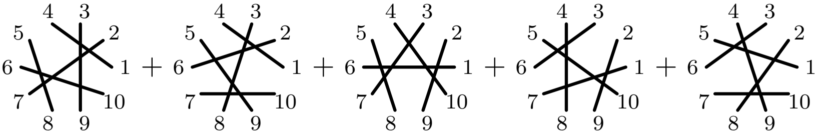
Four-point and six-point amplitudes are single diagrams. ✓

Eight points: 

[Huang, Wen]

⇒ Allows for a one-parameter deformation: $u_1 - u_2$. ✓

Ten points:



⇒ No non-trivial deformation

[TB, Huang, Loebbert, Yamazaki, 2014]

General $(2p+4)$-point amplitude: $(2p)!/(p!(p+1)!)$ diagrams. [Huang, Wen]

⇒ No non-trivial deformation beyond eight points

Deformed Graßmannian Integral

Directly deforming BCFW decomposition fails in general

$\Rightarrow$ Need deformed "parent" object

Natural parent object: Graßmannian integral

Parametrized by top cell diagram $\Rightarrow$ deformation straightforward

$\mathcal{N} = 4$ SYM: [TB, Huang, Loebbert Yamazaki, 2014] [Ferro, Łukowski Staudacher, 2014]

$$\mathcal{G}_{n,k}(\mathcal{W}_i, b_i) = \int \frac{\mathrm{d}^{k\cdot n}C}{|\mathrm{GL}(k)|} \frac{1}{M_1 \dots M_n} \delta^{4k|4k}(C \cdot \mathcal{W})$$

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Parameters $u_j^\pm = u_j \pm c_j$ constrained by top-cell permutation:

$$u_j^+ = u_{\sigma(j)}^- , \qquad \sigma(j) = j + k$$

Exponents b_j related to central charges:

$$\mathfrak{C}_j = -\mathcal{W}_j^\mathcal{C} \frac{\partial}{\partial \mathcal{W}_j^\mathcal{C}} \qquad \Longrightarrow \qquad b_i = \tfrac{1}{2}(u_i^- - u_{i-1}^-) = \tfrac{1}{2}(u_{i-k}^+ - u_{i-k-1}^+)$$

Deformed integral enjoys the full Yangian symmetry

Deformed Graßmannian Integral

ABJM:

$$\mathcal{G}_{2k}(\Lambda_i, b_i) = \int \frac{d^{k\times 2k}C}{|\mathrm{GL}(k)|} \frac{\delta^{k(k+1)/2}(C \cdot C^{\mathrm{T}})\, \delta^{2k|3k}(C \cdot \Lambda)}{M_1 \ldots M_k}$$

Evaluation parameters fixed by permutation of top cell:

$$u_j = u_{j+k}$$

No central charges to fix $b_i \Rightarrow$ Act directly with generator:

$$\widehat{\mathfrak{J}}(u_i)\, \mathcal{G}_{2k}(b_i) = 0 \qquad \Rightarrow \qquad b_j = u_j - u_{j-1} \quad (1 \leq j \leq k)$$

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What to Do With It?

Deformed Graßmannian integral

$$\mathcal{G}_{n,k}(\mathcal{W}_i, b_i) = \int \frac{\mathrm{d}^{k\cdot n}C}{|\mathrm{GL}(k)|} \frac{1}{M_1^{1+b_1}\dots M_n^{1+b_n}} \delta^{4k|4k}(C\cdot\mathcal{W})$$

Interpret this directly as the deformed tree amplitude?

Properties

Can no longer localize $n(k-2)-k^2+4$ integrations on residues

Can set some b_i to zero and still localize on tree contour

⇒ Known deformed BCFW sums for MHV and $n=6$ NMHV

⇒ In general requires all $b_i = 0$

⇒ No new deformations

Integrate this (not by residues) on some contour? → [Matthias' talk]

Deformed Momentum-Twistor Diagrams

For $\mathcal{N} = 4$ SYM:

Momentum-Twistor Diagrams $\rightarrow$ [Song's talk]

Mathematically the same as conventional diagrams, $\mathcal{W} \rightarrow \mathcal{Z}$

Can deform in exactly the same way

Yangian now based on dual superconformal symmetry, $\mathfrak{J}^{\mathcal{A}}{}_{\mathcal{B}} = \mathcal{Z}^{A} \frac{\partial}{\partial \mathcal{Z}^{B}}$

BCFW Decomposition

Amplitudes $\mathcal{A}_{n,k}/\mathcal{A}_{n,k}^{\mathrm{MHV,tree}}$: Sum of momentum-twistor diagrams

Deformation parameters v_i, c_i^{dual} again constrained by $v_j^{+} = v_{\sigma(j)}^{-}$

Simplest non-trivial example: Six-point NMHV. Three terms.
Admits two-parameter deformation.

Again no deformations admitted at higher points.

Also no deformations at loop level

Momentum-Twistor Graßmannian Integral

Momentum-twistor Graßmannian integral

$$\tilde{\mathcal{G}}_{n,k}(\mathcal{Z}_i, \tilde{b}_i) = \int \frac{\mathrm{d}^{k\cdot n}\tilde{C}}{|\mathrm{GL}(k)|} \frac{1}{\tilde{M}_1^{1+\tilde{b}_1} \ldots \tilde{M}_n^{1+\tilde{b}_n}} \delta^{4k|4k}(C \cdot \mathcal{Z})$$

Relation to conventional twistor integral

Reduce twistor to momentum-twistor Graßmannian: [Arkani-H., Bourjaily, Goncharov Cachazo, Postnikov, Trnka, 2012]

Reduction from C to $\tilde{C}$ reduces k to $k-2$. Relation between minors:

$$M_i = \langle i, i+1 \rangle \ldots \langle i+k-2, i+k-1 \rangle \, \tilde{M}_{i+1}$$

Induces relation between twistor and momentum-twistor invariants:

$$\mathcal{Y}(\mathcal{W}) \simeq \frac{\delta^4(P)\delta^8(Q)}{\langle 12 \rangle \langle 23 \rangle \ldots \langle n1 \rangle} \tilde{\mathcal{Y}}(\mathcal{Z}) \qquad \text{(undeformed)}$$

For deformed Graßmannian integrals:

$$\mathcal{G}_{n,k}(\mathcal{W}_i, b_i) \simeq \frac{\delta^4(P)\delta^8(Q)}{\langle 12 \rangle^{\gamma_1} \ldots \langle n1 \rangle^{\gamma_n}} \tilde{\mathcal{G}}_{n,k-2}(\mathcal{Z}_i, b_{i-1}), \quad \gamma_j = 1 + \frac{u_j^- - u_{j+1-k}^-}{2}$$

Summary

- Deformed on-shell graphs for ABJM
- R-matrix construction for $\mathfrak{osp}(6|4)$ Yangian invariants
- Deformed Graßmannian integral for $\mathcal{N} = 4$ SYM
- Deformed momentum-twistor Graßmannian integral
- Deformed Graßmannian integral for ABJM

Outlook/Questions

Can deformed BCFW expansion be rescued?
Different set of central charges for each diagram?

Integration contour for Graßmannian integral? → [Matthias' talk]

Lift to deformed amplituhedron?
Likely requires global coordinates → [Nima's talk]

Deformation at loop level?
Helpful for integrating the on-shell integrand?

Do the deformed amplitudes have any direct physical meaning?

What do the deformations mean for the geometric & differential structure of the Graßmannian?

Relation between Yangian & dual Yangian in deformed case?

Deformations compatible with exact symmetry?